

## A Discrete Delay N-decomposition Approach for Delay-Dependent Stability of Generator Excitation Control System with Constant Communication Delays

S. Manikandan<sup>1</sup>, Priyanka Kokil<sup>2</sup>,

<sup>1</sup>Department of Electrical and Electronics Engineering, Pondicherry Engineering College, Puducherry, India.

<sup>2</sup>Department of Electronics and Communication Engineering, IIITDM, Kancheepuram, India.

---

### Article Info

#### Article history:

Received Juni 22, 2017

Revised Sep 5, 2017

Accepted Sep 19, 2017

#### Keyword:

Constant delay simulation  
Delay-dependent stability  
Discrete delay decomposition  
approach  
Generator excitation system

---

### ABSTRACT

This paper deals with problem of delay in stability analysis of network controlled generator excitation system. Delays exist in communication channel in network based control between system and controller. A discrete delay N-decomposition is used to compute delay margin for generator excitation system with constant delay which is easier when compared to analytical method. A Lyapunov krasovskii function is constructed for given time delay generator excitation system and linear matrix inequalities techniques are used. Generator excitation system is employed with proportional integral controller, delay margin calculated for various values of gain of proportional integral controller. Theoretically obtained results are verified using simulation studies.

Copyright © 2017 Institute of Advanced Engineering and Science.  
All rights reserved.

---

### Corresponding Author:

S. Manikandan,  
Department of Electrical and Electronics Engineering,  
Pondicherry Engineering College,  
Puducherry, India.  
Email: manicbse@gmail.com.

---

## 1. INTRODUCTION

In Power system control one of method is employed to limit the variation in magnitude of voltage and reactive power control is done using generator excitation method. Generator excitation system (GES) involves exciter supplying DC power to field of synchronous generator [1-2]. Regulator involves proportional integral (PI) controller and amplifier. PI controller provides dynamic performance and reduces steady state error [3]. Generator terminal voltage decreases due to excessive reactive power drawn by connecting heavy inductive loads. Under such a condition change in terminal voltage is sensed by potential transformer and phasor measurement unit (PMU). PMU deliver positive sequence of voltage to rectifier and filter. DC signal from rectifier is send through communication network to the controller [4].

Controller processes this information by comparing with reference. Then send back to the GES through same network. Stability of GES is maintained by proper tuning of PI controller [4-5]. In general control system PI controller is tuned with zero delay but network based control system involves tuning of PI controller with respect to delays. These paper deals with effect of delay in generator excitation control system. Communication delay happens when bit of data travelled from GES system to the controller and controller to GES system. Network based control of GES system involves delay in feedback path as well as forward path. These delay would degrade dynamic performance GES system. There are two approaches for ascertaining stability of the time delay system.

They are Delay-independent stability criteria and delay-dependent stability criteria. Delay-independent stability criteria does not take into account of delay. Since any practical system cannot be

stable for any magnitude of time-delay, this method is more conservative. Delay-dependent stability criteria gives upper bound of the time delay within which the delayed system remains stable. Hence, this method is less conservative. This criteria is derived based on Lyapunov Krasovski stability analysis [8]. Delay-dependent stability of GES is examined by Lyapunov-krasovskii functional (LKF) analysis which is less conservative. Delay margin is computed by solving chosen LKF for considered time delay system.

Delay margin is defined as maximum upper bound of delay within which closed loop system remains asymptotically stable. Delay margin is calculated for various values of  $K_p$  and  $K_i$  of PI controller [9-10]. This method can be used for tuning PI controller of GES. GES is modeled in state space equation with time delay. Many literatures discussed performance of GES with respect to delay using analytical method to determine delay margin of GES system [11-12]. These method provide accurate results but when system topology get altered it becomes afresh [4]. This motivated present research work. This problem can be eliminated by using discrete delay N-decomposition approach. It involves dividing discrete delay interval into several N-sub interval. When N increases, results becomes less conservative [14]. This paper discuss application of discrete delay N-decomposition approach for stability analysis of generator excitation system with constant delays.

## 2. SYSTEM DESCRIPTION AND PROBLEM FORMULATION

The block diagram of generator excitation system is shown in Figure 1. In this figure, delay exist between sensed signal from GES and controller is  $\tau_1$ . Delay exists between control signal from controller to GES is  $\tau_2$ . This two delay are combined as single delay  $\tau = \tau_1 + \tau_2$ . The mathematical model of GES is modeled in state space framework. The  $V_{ref}$  is set to zero. The state vector of GES is  $x(t) = [\Delta V_R(t) \ \Delta V_F(t) \ \Delta V_T(t) \ \Delta V_S(t) \ \int \Delta V_S(t)dt]^T$ . The state equation of delayed system is given by

$$\dot{x}(t) = Ax(t) + A_d x(t-h) \quad (1)$$

$$x(t) = \Phi(t), \ t \in [-\max h, 0] \quad (2)$$

where  $x(t) \in R^n$  is the state vector,  $B \in R^{n \times 1}$  is input matrix,  $K \in R^{1 \times n}$  is controller gain,  $A \in R^{n \times n}$  and  $A_d \in R^{n \times n}$  represent system matrix where,  $A_d = BK$ ,  $h$  is upper bound of constant delay.

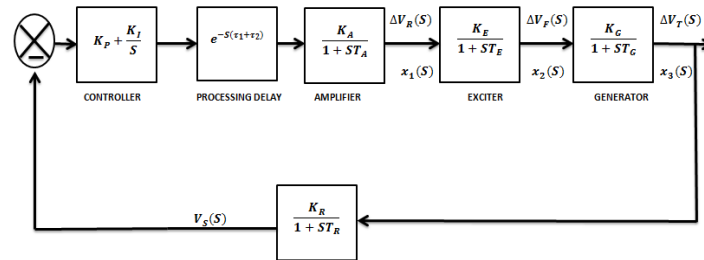


Figure 1. Mathematical model of Generator Excitation System

$$\dot{x}_1(t) = -\frac{1}{T_A}x_1(t) - \frac{K_p K_A}{T_A}x_4(t-h) - \frac{K_i K_A}{T_A}x_5(t-h) \quad (3)$$

$$\dot{x}_2(t) = \frac{K_E}{T_E}x_1(t) - \frac{1}{T_E}x_2(t) \quad (4)$$

$$\dot{x}_3(t) = \frac{K_G}{T_G}x_2(t) - \frac{1}{T_G}x_3(t) \quad (5)$$

$$\dot{x}_4(t) = \frac{K_R}{T_R}x_3(t) - \frac{1}{T_R}x_4(t) \quad (6)$$

$$\dot{x}_5(t) = x_4(t) \quad (7)$$

The above equations are represented in standard format (1) given below

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \\ \dot{x}_5(t) \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_A} & 0 & 0 & 0 & 0 \\ \frac{K_E}{T_E} & -\frac{1}{T_E} & 0 & 0 & 0 \\ 0 & \frac{K_G}{T_G} & -\frac{1}{T_G} & 0 & 0 \\ 0 & 0 & \frac{K_R}{T_R} & -\frac{1}{T_R} & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & -\frac{K_P K_A}{T_A} & -\frac{K_I K_A}{T_A} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t-h) \\ x_2(t-h) \\ x_3(t-h) \\ x_4(t-h) \\ x_5(t-h) \end{bmatrix} \quad (8)$$

The parameters are  $K_A = 5$ ,  $K_E = 1$ ,  $K_G = 1$ ,  $K_R = 1$ ,  $T_A = 0.1$ ,  $T_E = 0.4$ ,  $T_G = 1$ ,  $T_R = 0.05$  [11-12]. Delay margin is obtained by solving theorem 1. Condition for maximum delay is  $0 \leq \tau \leq h$ .

### 3. THEOREM

The delay-dependent stability of consider time delay GES is given (1) and (2) Theorem 1 [14]. For a given scalar  $h > 0$  and a positive integer  $N \geq 2$ , the system is asymptotically stable if there exist real  $n \times n$  matrices  $P > 0$ ,  $Q > 0$ ,  $R > 0$ ,  $W \geq 0$  and  $S_{ii} = S_{ii}^T (i = 1, 2, \dots, N)$ ,  $S_{ij} (i < j; i = 1, 2, \dots, N-1; j = 2, 3, \dots, N)$  such that

$$S = S^T = \begin{bmatrix} S_{11} & S_{12} & \cdot & \cdot & S_{1N} \\ * & S_{22} & \cdot & \cdot & S_{2N} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ * & * & \cdot & \cdot & S_{NN} \end{bmatrix} \geq 0 \quad (9)$$

$$\omega = \begin{bmatrix} \omega^{(1)} & \omega^{(2)} & \omega^{(3)} \\ * & -W & 0 \\ * & * & -R \end{bmatrix} < 0 \quad (10)$$

where

$$\omega^{(1)} = \begin{bmatrix} \omega_{11}^{(1)} & \omega_{12}^{(1)} & S_{13} & \cdot & \cdot & S_{1N} & PB + R \\ * & \omega_{22}^{(1)} & \omega_{23}^{(1)} & \cdot & \cdot & S_{2N} - S_{1N-1} & -S_{1N} \\ * & * & \omega_{33}^{(1)} & \cdot & \cdot & S_{3N} - S_{2N-1} & -S_{2N} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \omega_{NN}^{(1)} & -S_{N-1N} \\ * & * & * & \cdot & \cdot & * & \omega_{N+1N+1}^{(1)} \end{bmatrix}$$

$$\omega^{(2)} = \begin{bmatrix} \frac{h}{N} A^T W \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \frac{h}{N} B^T W \end{bmatrix}, \quad \omega^{(3)} = \begin{bmatrix} h A^T W \\ 0 \\ 0 \\ \cdot \\ \cdot \\ h B^T W \end{bmatrix}$$

Elements of  $\omega^{(1)}$

$$\omega_{11}^{(1)} = A^T P + PA + Q - W - R + S_{11}$$

$$\omega_{22}^{(1)} = S_{22} - S_{11} - W$$

$$\omega_{33}^{(1)} = S_{33} - S_{22}$$

$$\omega_{NN}^{(1)} = S_{NN} - S_{N-1N-1}$$

$$\omega_{N+1N+1}^{(1)} = -S_{NN} - Q - R$$

$$\omega_{12}^{(1)} = W + S_{12}$$

$$\omega_{23}^{(1)} = S_{23} - S_{12}$$

Theorem 1 is applied to GES to determine delay margin for constant delays. Delay margin is obtained for various values of  $K_p$  and  $K_I$  of PI controller.

### 3.1. Delay-dependent stability problem

Consider a LKF for given time delay system (1)

$$\begin{aligned} v(t, x_t) = & x^T(t)Px(t) + \int_{t-h}^t x^T(\xi)Qx(\xi)d\xi + \int_{t-h}^t (h-t-\xi)\dot{x}^T(\xi)(hR)\dot{x}(\xi)d\xi \\ & + \int_{t-\frac{h}{N}}^t Z^T(\xi)Sx(\xi)d\xi + \int_{t-\frac{h}{N}}^t \left(\frac{h}{N}-t+\xi\right)\dot{x}^T(\xi)\left(\frac{h}{N}W\right)\dot{x}(\xi)d\xi \end{aligned} \quad (11)$$

where  $\left[ x^T(t) \quad x^T(t-\frac{h}{N}) \quad \dots \quad x^T(t-\frac{(N-1)h}{N}) \right]$

## 4. RESULTS AND DISCUSSION

Delay margin ( $h$ ) is obtained from theorem 1 for GES for various values of gain PI controller with  $N$  (subinterval) = 3, 4, 5. Table 1 gives delay margin for  $K_p = 0.3$  to 0.9 and  $K_I = 0.1$  to 0.5 with  $N=3$ . Table 1 and figure 4 shows delay margin ( $h$ ) decreases for increase in  $K_I$  for fixed  $K_p$ . Similarly delay margin decreases for increase in  $K_p$  for fixed  $K_I$  values shown in figure 3. These inferences are used in studying PI controller in performance of network control. From table 1, 2, 3 and figure 2 shows when sub-interval  $N$  increases delay margin increases, result becomes less conservative for increase in  $N$  for considered network controlled GES.

Table 1. Delay margin of GES with  $N=3$

$K_I$	$K_p$		
	0.3	0.5	0.7
0.1	1.4021	0.6239	0.3640
0.2	0.9604	0.5446	0.3340
0.3	0.6755	0.4649	0.3029
0.4	0.4820	0.3902	0.2716
0.5	0.3453	0.3229	0.2407

Table 2. Delay margin of GES with  $N=4$

$K_I$	$K_p$		
	0.3	0.5	0.7
0.1	1.4083	0.6255	0.3645
0.2	0.9666	0.5457	0.3344
0.3	0.6766	0.4656	0.3032
0.4	0.4824	0.3906	0.2718
0.5	0.3455	0.3232	0.2409

Table 3. Delay margin of GES with  $N=5$

$K_I$	$K_p$		
	0.3	0.5	0.7
0.1	1.4112	0.6262	0.3647
0.2	0.9678	0.5461	0.3346
0.3	0.6771	0.4659	0.3034
0.4	0.4827	0.3908	0.2719
0.5	0.3456	0.3233	0.2410

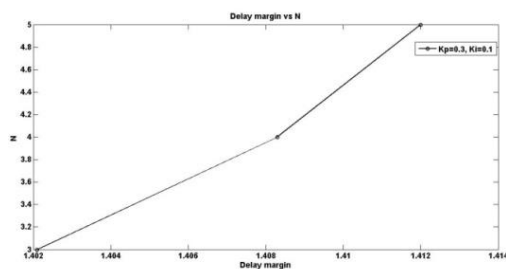


Figure 2. Delay margin vs.  $N$ (sub interval of delay) with  $K_p=0.3$  and  $K_I=0.1$

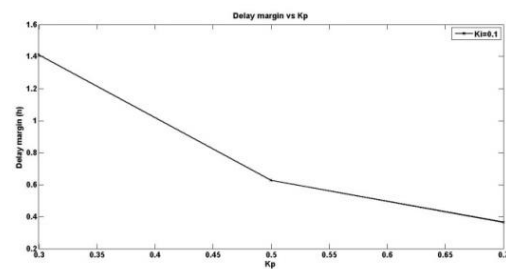
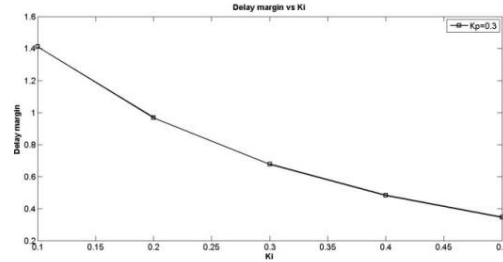


Figure 3.  $K_p$  vs. delay margin ( $h$ )

Figure 4.  $K_I$  vs. delay margin (h)

## 5. MATLAB SIMULATION FOR VERIFICATION OF THEORETICAL RESULTS

Simulation is done for generator excitation control system equipped with PI controller. Theoretically calculated delay margin values are used in simulation studies. For verification purpose  $K_p=0.3$  and  $K_I=0.1$  are chosen with  $N=5$  from table 3. From table 3 delay margin  $h$  is chosen such that  $h=1.4112$  s. Figure 5 shows that voltage response of GES without delay. System response reaches steady state (absolutely stable system). Figure 6 shows voltage response of GES with constant delay of 1 s throughout process. System reaches steady state with some oscillations (stable system). Figure 7 shows voltage response of GES with delay margin of  $h=1.4112$  s. Response of system with sustained oscillations (marginally stable system). Delay margin  $h=1.42$  and  $h=1.5$  chosen for same value of  $K_p$  and  $K_I$ . Figure 8 and 9 shows system becomes unstable.

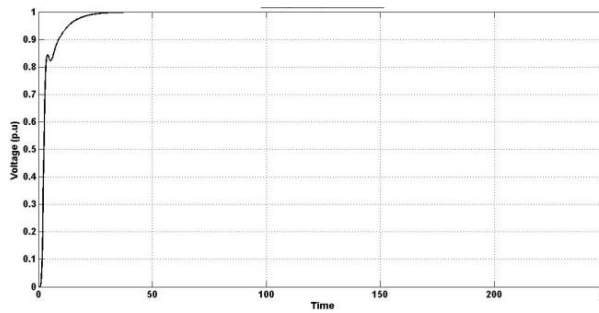
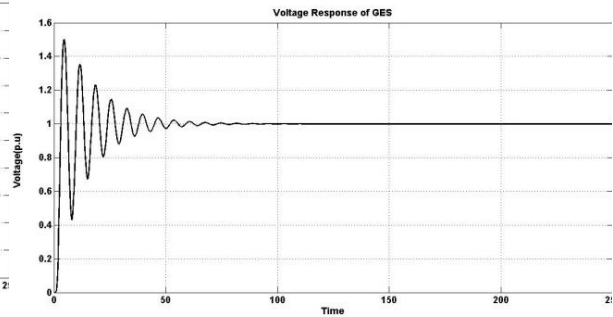
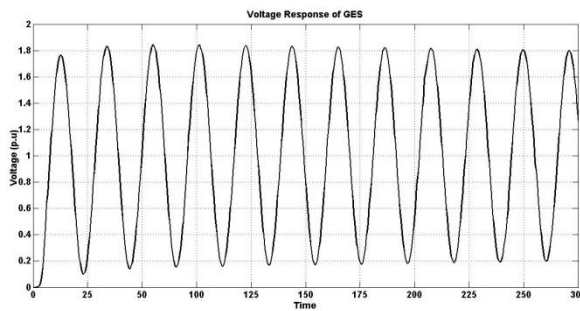
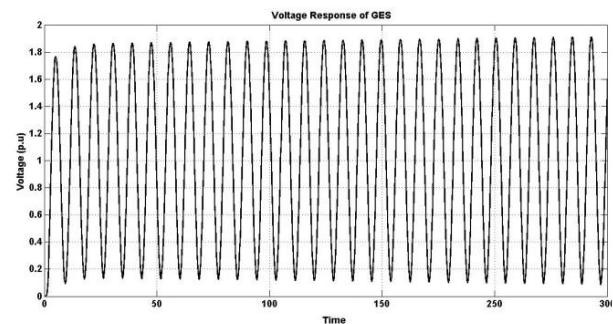


Figure 5. Voltage response of GES with zero delay

Figure 6. Voltage response of GES with delay margin  $h=1.0$  s with  $K_p=0.3$  and  $K_I=0.1$ Figure 7. Voltage response of GES with delay margin  $h=1.4112$  s with  $K_p=0.3$  and  $K_I=0.1$ Figure 8. Voltage response of GES with delay margin  $h=1.42$  s with  $K_p=0.3$  and  $K_I=0.1$

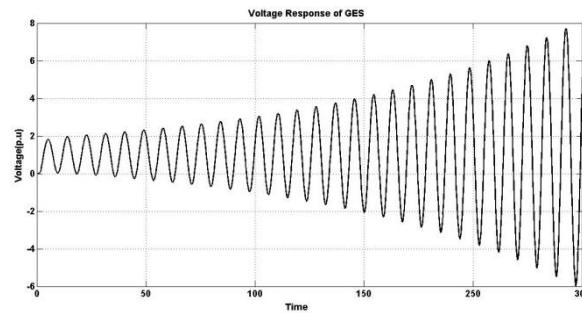


Figure 9. Voltage response of GES with delay margin  $h=1.5$  s with  $K_P=0.3$  and  $K_I=0.1$

## 6. CONCLUSION

A Discrete delay N-decomposition approach overcomes disadvantages of exact method provided in recent literatures. This method involves dividing delay interval into N sub-intervals. When N increases, chosen LKF becomes less conservative. Such that  $N=5$  provides sufficient conditions for system stability. Hence this method can be used to determine delay margin any systems with constant delays. This approach applied to GES for delay-dependent stability analysis for constant delays. Delay margin calculated for GES for various values of gain of PI controller. This method can be used to tune network controller to maintain system stability.

## 7. DECLARATION OF CONFLICTING INTERESTS

The authors declare that there is no conflict of interest

## 8. FUNDING

This research received no specific grant from any funding agency in the public, commercial or not-for-profit sectors.

## REFERENCES

1. Kunder P, "Power System Stability and Control," New Delhi: Third Edition. TATA McGraw-Hill Education Private Limited, 1994.
2. Kothari DP, Nagrath IJ, "Modern Power System Analysis," New Delhi: Third Edition. TATA McGraw-Hill Education Private Limited, 2003.
3. IEEE Standard 421.5, "IEEE recommended practice for excitation system models for power system stability studies," IEEE Power Engineering Society, 2005.
4. Ramakrishnan K, "Delay-dependent stability of networked generator-excitation control systems: An LMI based approach," *IFAC PapersOnline*, vol. 49(1), pp. 431-436, 2016.
5. Manikandan S, Ramakrishnan K, "Delay-dependent stability of network controlled DC position servo system," Anna University, *RTIC 2016*, pp. 48-54, 2016.
6. Gahinet P, Nemirovskii A, LaubAJ, Chilali M, "LMI control toolbox user guide," *Natick MA: Math works*, 2000.
7. Sonmez S, Ayasun S, Nwankpa C, "An exact method for computing delay margin for stability of load frequency control systems with constant communication delays," *IEEE Transactions on Power Systems*, vol. 31(2), pp. 370-377, 2016.
8. Liu Y and Li M, "An improved delay-dependent stability criterion of networked control systems," *Journal of the Franklin Institute*, vol. 351, pp.1540-1552, 2014.
9. Jiang L, Yao W, Wu QH, Wen JY, et al., "Delay-dependent stability for load frequency control with constant and time-varying delays," *IEEE Transactions on Power Systems*, vol. 27(2), pp. 932-941, 2012.
10. Ramakrishnan K, Ray G, "Stability criteria for nonlinearly perturbed load frequency systems with time-delay," *IEEE Journal on Emerging and Selected Topics in Circuits and Systems*, vol. 5(3), pp. 383-392, 2015.
11. Kokil P, Kar H, Kandanvli VKR, "Stability analysis of linear discrete-time systems with interval delay: A delay-partitioning approach," *ISRN Applied Mathematics*, 624127, pp. 1-10, 2011.
12. Ayasun S and Gelen A, "Stability analysis of a generator excitation control system with time delays," *Springer*, vol. 91 (6), pp. 347-355, 2009.
13. Ayasun S, Eminoglu U and Sonmez S, "Computation of stability delay margin of time-delayed generator excitation control system with a stabilizing transformer," *Mathematical Problems in Engineering*, 392535, pp.1-10, 2014.
14. Han QL, "A discrete delay decomposition to stability of linear retarded and neutral systems," *Automatica*, vol. 45, pp. 517-524, 2009.

15. Kokil P, Kar H, and Kandanvli VKR, "Delay-partitioning approach to stability of linear discrete-time systems with interval-like time-varying delay," *International Journal of Engineering Mathematics*, 291976), pp. 1-7, 2013.

## BIOGRAPHIES OF AUTHORS



**Subramanian Manikandan** did his bachelor's degree in Electrical and Electronics Engineering from University College of Engineering, Panruti, Tamilnadu, India in 2014. He was awarded University Gold Medal and University Rank in the year 2014 for academic excellence in Department of Electrical and Electronics Engineering by Anna University. He then completed the M.Tech degree in Electrical Drives and Control from Pondicherry Engineering College, Puducherry, India through TEQIP fellowship in 2016. His Area of interest is time-delay system, control system application to electrical drives and power system. E-mail: manicbse@gmail.com.



**Priyanka Kokil** did her bachelor's degree from Motilal Nehru National Institute of Technology, Allahabad, India and completed the M.Tech degree from IIT Delhi, India. She then completed the PhD from Motilal Nehru National Institute of Technology, Allahabad, India. She is working as assistant professor in IIITDM, Kancheepuram, Tamilnadu, India. Her area of interest is the nonlinear system, delayed system and multidimensional system. E-mail: pkokil1@gmail.com.