Analytical design of the fractional order controller and robustness verification

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ABSTRACT

This paper proposes a fractional order controller (FOC) for the level control problem of the coupled tank system, using the desired time domain specifications. The coupled tank system is used in the chemical industries for the storage and mixing of liquids. The FOC is designed analytically using the direct synthesis method. In the direct synthesis method, the Bode's ideal loop transfer function is chosen as the desired transfer function. Bode's loop transfer function has the advantages like robustness to system gain variations, constant phase and very high gain margin. Performance of the proposed controller is compared with the state of the art literature. Simulation results showed that the proposed controller has the least peak overshoot. The robust performance of the proposed controller is also the best. Robust stability of the system with the proposed controller is verified, and the system is found to be robustly stable.

Keywords:
Bode's ideal transfer function
Coupled tank system
Direct synthesis
Fractional order controller
Robust stability

1. INTRODUCTION

The coupled tank system consists of two or more than two tanks interconnected. In this work, a cylindrical coupled tank system in the interacting mode is taken as the case study. Coupled tank system is used in the process control industry for storing the liquid and pumping the liquid to next stages of the batch process. The interacting mode comprises two cylindrical shaped tanks connected physically through a valve. The level control of the coupled tank system is a challenging problem in the control systems because of its nonlinearity and time delay. John et al., proposed a backstepping controller for the coupled tank system [1]. Sharma et al., discussed the modeling and identification for the coupled tank system and proposed a fuzzy logic controller for the level control problem [2]. Sathish Kumar et al., developed a nonlinear model of the three tank system and designed a linear quadratic controller [3]. Patel and Shah designed an integer order proportional integral differentiator (PID) and artificial intelligence based controllers for the level control of the coupled tank system [4]. Singla et al., proposed a modified PID controller for the level control of the coupled tank system [5]. The modified PID controller showed better performance than the conventional PID controller. Gouta et al., discussed the adaptive control of coupled tank system using an observer [6]. A multistage cost function is optimized to obtain the controller parameters. Mahapatro et al., designed a decentralized PI controller for the level control of a coupled tank system using a reference model [7]. A frequency domain based modeling of the system is also discussed. Nafea et al., discussed a neural network based level control of the coupled tank system [8]. They proposed a perturbed model of the system from the
Fractional order controllers (FOCs) are finding applications in all areas of control systems because of advantages like robustness to the system gain variations, wide stability margins, near infinite gain margin, and more tunable parameters. Podlubny proposed the fractional order proportional integral derivative (FOPID) controller [9]. After FOPID is developed, numerous fractional controllers are designed and implemented. Vinopraba et al., discussed two degrees of freedom (2-DOF) fractional order internal model controller (FOIMC) [10]. The parameters are tuned analytically using the desired closed loop bandwidth. Saxena and Hote discussed an internal model controller-proportional integral differentiator (IMC-PID) based fractional controller for the DC servomotor speed control [11]. The controller has less tunable parameters and the controller parameters are obtained using frequency domain specifications. Baruah et al., proposed a fractional order proportional integral (FOPI) controller for the coupled tank system. The controller parameters are tuned analytically using integral performance index and balanced state space methods [12]. Walid et al., developed a fractional order PID controller for the speed control of permanent magnet synchronous machine [13]. The controller is synthesized using the Bode's ideal open loop transfer function (OLTF). Lino et al., proposed a feed forward fractional controller for the speed and position control of DC motor [14]. Empirical formulae for the controller parameters are obtained using the frequency domain specifications. Zhenlog et al., devised a Smith predictor based fractional order proportional integral (PI) controller for the higher order system [15]. The controller gave good set point tracking and disturbance rejection. Vavilala and Vinopraba tuned the FOC parameters using particle swarm optimization (PSO) algorithm for a conical tank system level control [16]. A FO- [PI] controller is developed for the two input two output (TTTO) coupled tank system using the frequency domain specifications [17]. Fractional powers are approximated using Oustaloup recursive approximation (ORA). Tepljakov proposed a toolbox called fractional order modeling and control (FOMCON) in MATLAB for implementing fractional controllers and modeling the fractional order systems [18]. The FOMCON toolbox is used for simulations in this work.

The fractional order controller parameters can be tuned using analytical procedure, evolutionary algorithms and empirical formulae. In this work, controller structure is synthesized analytically using the direct synthesis procedure. The direct synthesis method is a conventional analytical method used for the controller design. A desired model of the system will be specified and using that model, controller will be synthesized. Vanavil et al., discussed the design of PID controller using the direct synthesis for the time delay systems [19]. The design used maximum sensitivity for obtaining the controller parameters. Ravishkore et al., used the direct synthesis method to derive the PID controller for the unstable first order and second order time delay systems [20]. Castillo-Garcia et al., proposed a FO- [PI] controller for the automation of the water canals [21]. Kumar and Anwar discussed a practical application using the direct synthesis [22]. The controller design is based on the maximum sensitivity. The design of the PID controller using the direct synthesis method is discussed in [23-25]. Anil and Padmasree used direct synthesis method to obtain the PID controller for different integrating systems with time delay [23]. Anwar and Pan utilized the direct synthesis method to get the PID controller parameters for the load frequency control (LFC) problem [24]. Kumar and Singh designed a PID controller for six different time delay systems using the direct synthesis method [25]. This work considers Bode's ideal OLTF as the desired reference model for the closed loop system. Bode [26] proposed the OLTF as a reference model, which gives robustness to system gain variations (iso-damping property). Iso-damping property means even though the system gain varies, the phase of the system remains constant and therefore the step response is same over a range of system gain values. Hence, if the model has some uncertainty or the system parameters (gain) change over time, it is taken care by the Bode's OLTF model.

Kashi and Rahali proposed a FOPI controller for the wind generation system using the Bode's ideal OLTF [27]. Ye and Bing discussed the design of the Smith predictor based 2-DOF IMC using the Bode's ideal OLTF method as a reference model [28]. The controller parameters are optimized using the integral absolute error. Yumuk et al., proposed a FOPID based on the Bode's ideal OLTF for the fractional order system [29]. Frequency domain specifications are used to tune the controller parameters. In a recent work, Keziz et al., discussed the design of FOC with Bode's ideal OLTF as a reference model and pole placement technique for controller tuning [30]. The controller is simple and robust. Safaei and Tavakoli proposed an analytical procedure for the FOC design using direct synthesis [31]. The controller parameters are tuned using the time domain specifications with empirical formulae. Arya and Chakrabarty designed a FOIMC for the nonminimum phase systems [32]. The controller parameters are optimized using the time domain specifications and integral performance indices. Palival et al., discussed the design of FOPID controller for the LFC problem [33]. PSO algorithm is used to get the controller parameters using different performance indices. A robust controller handles the uncertainty in the system parameters, rejects the disturbances, and performs well for changes in the reference input. The designed controller has to perform reasonably well for

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a change in the operating point. Robust performance and robust stability can be analyzed analytically using the $H_{\infty}$ frequency norm [34].

Purpose or motivation of the work: Most of the literature on the controller design, based on the Bode's ideal loop transfer function, tuned the controller parameters using the frequency domain specifications. But [31] proposed a Smith predictor based FOC using the time domain specifications. The empirical formulae are devised based on the Bode's ideal loop transfer function's step response. These formulae can be used for any linear system with a time delay. The controller is obtained analytically based on the direct synthesis approach. Motivated by the simple design procedure based on the time domain specifications, an attempt is made to design a FOC without the Smith predictor, for the first order plus delay time (FOPDT) model of the system. The proposed controller handled the time delay in the system without the Smith predictor. For the systems with large time delay and/or large time constants, tuning of the controller parameters using frequency domain specifications like gain margin, phase margin, will not give satisfactory results, hence time domain specifications based tuning is more apt.

Main contributions and novelty: A novel fractional order controller configuration based on the Bode's ideal OLTF is obtained using the direct synthesis method. Servo response, disturbance responses are compared with the state of the art. Robust performance and robust stability of the proposed FOC are analyzed using $H_{\infty}$ norm. The fragility indices of the controllers are compared to check the fragility of the controller to changes in the controller parameters. The proposed controller has the simple structure of an integer order PI controller cascaded with a fractional filter. The controller parameters are tuned using peak overshoot and settling time specifications for the desired step response. Novelty in the controller design is; FOC is analytically synthesized without the Smith predictor, for a FOPDT model of the system. Organization of the paper: After the Introduction, the second section describes the problem formulation and preliminaries of the fractional order control. The third section gives the analytical design of the proposed fractional order controller. Robust stability, robust performance and fragility analysis methods are discussed in the third section. The fourth section discusses the time domain and frequency domain simulation results. The fifth section gives the conclusion of the work.

2. PROBLEM FORMULATION AND PRELIMINARIES
2.1. Problem formulation
Statement of the problem and the proposed solution: This work is aimed at addressing the level control problem of the coupled tank system based on the time domain specifications like peak overshoot and settling time. The work provides the solution for this problem by proposing a design based on the empirical formulae involving the peak overshoot and settling time. Further, the design involves the reference closed loop transfer function as modified Bode's ideal loop transfer function and direct synthesis method. The work also verifies whether the performance of the proposed FOC is robust enough and robustly stable. The study also discusses the existing literature on the FOC design using time domain specifications. For the FOPDT model of the coupled tank system, a FOC is designed such that the following controller objectives are satisfied. Controller objectives:
- Achieve good transient and steady state step responses ($M_p \leq 10\%$ and $t_s \leq 150$ sec)
- Achieve a satisfactory servo tracking and disturbance rejection properties simultaneously
- Obtain a stable and robust controller

Figure 1 shows the block diagram of the feedback controller. In Figure 1, the system $G(s)$ has the FOPDT model given by (1), and the FOC $C(s)$ design is discussed in Section 3.

![Feedback controller](image)

Figure 1. Feedback controller

\[
G(s) = \frac{Ke^{-Ls}}{7s+1}
\]
2.2. Preliminaries of fractional calculus and fractional control

Fractional calculus is 300 years old, but its applications became popular in the past three decades only. There are three definitions available for the fractional differentiation, Reimann-Liouville, Caputo, and Grunwald-Letnikov. Among these definitions, Caputo definition is used normally in the control and engineering applications. G-L definition is more amenable to digital implementation. The Caputo definition of the fractional order derivative is given by (2).

\[
D^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau
\]  

(2)

In this definition, the function \( f(t) \) is first differentiated and then integrated. Fractional derivatives are nonlocal and are suitable to represent the systems involving distributed parameters. A general form of the fractional PID controller developed by I. Podlubny is represented as (3).

\[
C(s) = K_p + \frac{K_i}{s^\alpha} + K_d s^\mu
\]  

(3)

where \( K_p, K_i, \) and \( K_d \) are proportional, integral and derivative gains respectively. After FOPID, many other fractional order controller configurations are proposed and are widely used.

3. RESEARCH METHOD

3.1. Design of the fractional order controller

Bode’s ideal open loop transfer function is the reference for many design procedures in the fractional controllers. It has properties like robustness to system gain variations and low pass filter characteristic. (4) shows the Bode’s OLT denoted by \( L(s) \).

\[
L(s) = \frac{1}{\tau_c^a}
\]  

(4)

\[
\tau_c = \frac{1}{(\omega_{gc})^a}
\]  

(5)

where,

- \( \tau_c = \) Time constant
- \( \omega_{gc} = \) Gain crossover frequency
- \( \alpha = \) Fractional order \((1 \leq \alpha < 2)\)

(5) gives the relation between \( \tau_c \) and \( \omega_{gc} \). The closed loop transfer function \( T(s) \) with \( L(s) \) in forward path is

\[
T(s) = \frac{1}{1+\tau_c s^a}
\]  

(6)

Step responses are obtained using the (6) for different values of \( \alpha \) [31]. By analyzing the step responses the following relations are obtained among peak overshoot \((M_p)\), fractional order \((\alpha)\), settling time \((t_s)\) and GCF \((\omega_{gc})\):

\[
M_p = 73.9(\alpha^2 - 1.6739\alpha + 0.6756)
\]  

(7)

\[
t_s - 2\% \omega_{gc} = \frac{0.7085a^2 - 0.2693}{a - 0.8673}, \quad 1 < \alpha < 1.078
\]  

(8)

\[
t_s - 2\% \omega_{gc} = \frac{3.003a^2 - 2.981}{a^2 - 2.012a + 1.056}, \quad 1.078 < \alpha < 1.486
\]  

(9)

\[
t_s - 5\% \omega_{gc} = \frac{0.812a^2 - 0.2036}{a - 0.8007}, \quad 1 < \alpha < 1.15
\]  

(10)

\[
t_s - 5\% \omega_{gc} = \frac{7.156a^2 - 7.9}{a^2 - 1.033a + 0.2578}, \quad 1.15 < \alpha < 1.5
\]  

(11)

In (8-11) settling times with 2\% and 5\% tolerance band near steady state are shown. Among these four as shown in (10) is used because it gives peak overshoot less than 3\%. (6) is the modified with time delay incorporated and given by (12). Figure 2 shows the block diagram representation of the (12).
Figure 2. Desired closed loop reference transfer function

\[ T_{new}(s) = \frac{Y(s)}{R(s)}_{\text{desired}} = e^{-Ls} \frac{1 + \tau_c s^\alpha}{1 + \tau_c s^\alpha} \]  

(12)

Equations (13) and (14) are given below:

\[ \frac{Y(s)}{R(s)} = \frac{G(s)C(s)}{1 + G(s)C(s)} \]  

(13)

\[ \frac{Y(s)}{R(s)} = \frac{\frac{K e^{-Ls}}{\tau_c s^{-\alpha}}}{{1 + \frac{K e^{-Ls}}{\tau_c s^{-\alpha}}}} \]  

(14)

Equating the right hand sides of (13) and (14) and solving for \( C(s) \),

\[ C(s) = \left( \frac{T_s + 1}{K} \right) \left( \frac{1}{1 + \tau_c s^\alpha e^{-Ls}} \right) \]  

(15)

Expanding the delay term \( e^{-Ls} \) in the denominator, using the Taylor series of first order and simplifying further,

\[ C(s) = \left( \frac{T_s + 1}{K} \right) \left( \frac{1}{\tau_c s^\alpha + Ls} \right) \]  

(16)

Rearranging terms of (16),

\[ C(s) = K_c \left( 1 + \frac{1}{T_is} \right) \left( \frac{1}{1 + (\tau_c/L)s^{\alpha-1}} \right) \]  

(17)

The structure of \( C(s) \) is an integer order PI cascaded with a fractional filter of order \((\alpha-1)\) where,

- \( K_c = T/LK \) = Proportional gain constant
- \( T_i = T \) = Integral time constant
- \( T \) = Time constant

In the controller transfer function \( C(s) \) of (15) the unknown tunable parameters are \( \alpha, \tau_c \). From the desired time domain specifications of \( M_p, t_s \) and using (7) and (10) \( \alpha, \tau_c \) are obtained. The design procedure is explained in the following steps:

- Obtain FOPDT model of the system.
- Decide on the required time domain specifications, peak overshoot and settling time.
- Using the empirical formulae, get the controller parameters of FOC.
- Using the MATLAB/SIMULINK simulation, obtain the step response results of the closed loop system and verify if the time domain specifications are met?

Advantages of the proposed FOC:

- Robustness to system gain variations or isodamping property.
- Infinite gain margin.
- Increased stability range.
3.2. Robust stability and robust performance analysis

The designed controller has to perform reasonably well for a change in the operating point. The closed loop system is robustly stable, if the following equation is satisfied [34].

$$20\log_{10}(|T_c(j\omega)||G_d(j\omega)|) < 0dB$$  \hspace{1cm} (18)

Where $T_c(s)$ is the complementary sensitivity transfer function of the closed loop system given by,

$$\tau_c(s) = \frac{G(s)C(s)}{1+G(s)C(s)}$$  \hspace{1cm} (19)

Expanding the system transfer function $G(s)$ using Taylor series of first order,

$$G(s) = \frac{-K_{L,s}+K}{T_s+1}$$  \hspace{1cm} (20)

Substituting (16) and (20) in (19), and simplifying,

$$\tau_c(s) = \frac{1-Ls}{1+Tc^a}$$  \hspace{1cm} (21)

Let the transfer function of the system at a different operating point be,

$$G_d(s) = \frac{K_1e^{-L_1s}}{T_{1s}+1} = \frac{-K_{L_1,s}+K_1}{T_{1s}+1}$$  \hspace{1cm} (22)

Uncertainty in the plant transfer function $G_d(s)$ is defined as,

$$G_d(s) = G(s)-\frac{G(s)}{G(s)}$$  \hspace{1cm} (23)

Substituting (20) and (22) in (23), $G_d(s)$ is obtained as shown in (24).

$$G_d(s) = \frac{(KL_1-K_{L_1}T_1)s^2+(K(L-\bar{T}_1)-K_{L}(L_1-\bar{T}_1)s+K_1}{KLs^2+(K_{L_1}-KL)s+K}$$  \hspace{1cm} (24)

The closed loop system meets the robust performance specification, if and only if the following equation is satisfied.

$$20\log_{10}(|T_c(j\omega)||G_d(j\omega)|+|S(j\omega)||W(j\omega)|) << 0dB$$  \hspace{1cm} (25)

The sensitivity transfer function $S(s)$ is given by,

$$S(s) = \frac{Y(s)}{D(s)} = \frac{1}{1+G(s)C(s)}$$  \hspace{1cm} (26)

Substituting (16) and (20) in (26),

$$S(s) = \frac{Ls+\tau_c^a}{1+\tau_c^a}$$  \hspace{1cm} (27)

$W(s)$ is a low frequency weighting function, since $S(s)$ needs to be minimized only in the low frequency disturbances. In the present work, the weight is selected as,

$$W(s) = \frac{0.01s+0.001}{s}$$  \hspace{1cm} (28)

The Bode plot of the weight is shown in Figure 3. It is selected to have a high magnitude in lower frequencies, and low magnitude in higher frequencies.
3.3. Fragility analysis

Apart from robustness to system parameter variation, robustness to controller parameter variation needs attention. While analog controller implementation suffers from physical parameter changes [32], digital controller implementation suffers from the inaccuracies in fixed word length and round offs in numerical calculation. The robustness of the controller parameter variation is analyzed using fragility index $RFI_{Δ20}$ [32] defined by (29).

$$RFI_{Δ20} = \frac{M_{Δ20}}{M_s} - 1$$

where $M_s$ is the nominal maximum sensitivity and $M_{Δ20}$ is the nominal maximum sensitivity obtained when all the controller parameters are varied by +20%. The controller is said to be resilient to controller parameter changes, for values of the $RFI_{Δ20} \leq 0.1$, is nonfragile for values of $0.1 < RFI_{Δ20} \leq 0.5$ and fragile for values of $RFI_{Δ20} > 0.5$.

4. RESULTS AND DISCUSSION

4.1. Case study: Coupled tank system

In this work coupled tank system case study is considered to verify the effectiveness of the proposed FOC. Figure 4 shows the block diagram of the coupled tank system and, Table 1 give the nominal parameters of the coupled tank system.

- $Q_1$ = Inlet flow rate
- $Q_{12}$ = Flow rate from tank one to tank two
- $Q_{d1}$ = Disturbance flow rate from tank one
- $Q_{d2}$ = Disturbance flow rate from tank two
- $h_1$ = Variable height of the tank one
- $h_2$ = Variable height of the tank two
- $D$ = Diameter of each tank
- $A$ = Area of each tank

The controller objective is to regulate the height of the tank 2, by changing the inlet flow rate to the tank 1. In the present work, the operating region of the level control is selected as a (5-15) cm. Figure 5 shows the open loop test results got from the two tank system by varying the inlet flow rate $Q_1$ from (50-225) LPH in steps of 25 LPH and the corresponding heights of the two tanks. The open loop input and output data shows that the coupled tank system level control in the operating height of (5-15) cm follows a first order FOPDT model (in the (5-15) cm range) dynamics. From the experimental open loop data, FOPDT model of the system in the operating height of (5-15) cm is obtained as,

$$G(s) = \frac{h_2(s)}{Q_1(s)} = \frac{0.268e^{-10s}}{975s+1}$$

(30)
Table 1. Parameters of the cylindrical tank system

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>100 cm</td>
</tr>
<tr>
<td>D</td>
<td>15 cm</td>
</tr>
<tr>
<td>A</td>
<td>176.7 cm²</td>
</tr>
<tr>
<td>Qₛ</td>
<td>175 LPH</td>
</tr>
</tbody>
</table>

4.2. Time domain results

As discussed in the Section 3.1 second step in the controller design is to select the desired time domain specifications of the closed loop response. The time domain specifications selected for the controller design are, \( M_p \leq 10 \% \) and \( t_s \leq 150 \text{ sec} \). In (7) by substituting \( M_p \) with 0.1 and solving for the \( \alpha \), fractional order is obtained 1.15. In (10), substituting \( t_s \) with 150 and \( \alpha \) with 1.15 and \( \omega_{gc} \) is obtained as 0.0208 rad/sec. Finally using (5) and substituting the values of \( \omega_{gc} \) and \( \alpha \), \( \tau_c \) is obtained as 85.56. The fractional controller derived for the coupled tank system is shown in (31).

\[
C_{FOC}(s) = \left(363.8 + \frac{0.373}{s}\right)\left(\frac{1}{1 + 8.55s^{0.15}}\right)
\]  

(31)

Using (21) and (27), \( T_C(s) \) and \( S(s) \) are obtained as,

\[
T_C(s) = \frac{1 - 10s}{1 + 85.65s^{1.15}}
\]  

(32)

\[
S(s) = \frac{10s + 293s^{1.15}}{1 + 293s^{1.15}}
\]  

(33)

Sometimes the controller has to accommodate the changes (be robust) in the plant parameters, changes in the setpoint, changes in the operating region. In this work, the controller’s robustness is verified by operating it in an adjacent operating region of (16-30) cm. The transfer function of the new region is (34).

\[
G_\Delta(s) = \frac{0.3e^{-20s}}{1 + 0.95s + 1}
\]  

(34)

Using (23) \( G_d(s) \) is obtained as,

\[
G_d(s) = \frac{-2915.4s^2 - 4.28s + 0.032}{-2934s^2 + 290.78s + 0.268}
\]  

(35)

Table gives the robust stability, robust performance norms and fragility indices of the controllers. Hence the closed loop system is robustly stable and satisfies robust performance criterion as well. The performance of the proposed controller is compared with two state of the art literature works. [32] shows a FOIMC designed using the frequency domain specifications and is denoted as Ref-1. The controller obtained for the coupled tank system is,

\[
e_{FOIMC-Ref-1}(s) = \frac{975s + 1}{3.85s^{1.23} + 2.68s}
\]  

(36)
Integer order internal model controller (IOIMC) for the coupled tank system is taken as Ref-2 and is obtained as,

\[ C_{\text{FOIMC-Ref-2}}(s) = 12.16 + \frac{0.012}{s} \]

To compare the performance of the FOMCON toolbox, the standard CRONE toolbox and controller implementation is considered. To handle the time delay in the system, the second-generation CRONE controller along with Smith predictor is used. The frequency domain specifications used for the CRONE controller design are gain crossover frequency of 0.08 rad/sec and phase margin of 45 degrees. The FOC obtained using CRONE is given by (38).

\[ C_{\text{CRONE}}(s) = \frac{661.1s + 678}{s^{4.479}} \]

For getting the simulation results, Figure 1 is implemented in the MATLAB/SIMULINK with the transfer function \( C(s) \) given by (31) and \( G(s) \) given by (30). The setpoint \( R \) for the tank 2 height, of the coupled tank system is chosen as 10 cm. FOMCON toolbox provides SIMULINK blocks for fractional transfer function, FOPID controller transfer functions. Using these blocks, the step responses of the closed loop system for a setpoint of 10 cm are obtained with different controllers and are shown in Figure 6. Upper portion of Figure 6 shows the step responses, whereas bottom portion shows the corresponding control efforts of each controller in a scale of (0-100) %. From Figure 6, it can be observed that the proposed FOC has good transient and steady state performance. Novelty of these results is that, they are obtained from the time domain specifications directly. Figure 7 shows the regulatory responses to a disturbance of 5 cm at 500 sec. Figure 7 shows that the disturbance applied at 500 sec settles at 600 sec. While the controller, FOIMC settled at 610 sec and IOIMC settled at 620 sec. Hence it can be concluded that that the proposed controller has good disturbance rejection.

![Figure 6. Step responses and control efforts](image)

![Figure 7. Regulatory responses and control efforts](image)

Figure 8 shows the servo responses with time varying step input. Figure 8 shows that the proposed controller gave good step response for changes in the step input. The Table 2 shows the time domain specifications and integral performance indices of the controllers. Control effort indicates the power consumed in obtaining the desired control action. From Table 2, it can be observed that the CRONE controller has the lowest integral absolute value. The CRONE controller toolbox consumes the highest control energy. The peak overshoot of the CRONE toolbox is also the highest among all the controllers. But the CRONE controller has the least settling time and the least rise time.

From the Table 2, it can be seen that the proposed controller satisfies the controller objectives, \( M_p \leq 10 \% \) and \( t_s \leq 150 \text{ sec} \), which proves the validity of the proposed design method. This suggests that it is not necessary to convert the time domain specifications to frequency domain specifications or use only frequency domain specifications for the controller design. Hardships faced during the design of the proposed FOC are
difficulty in choosing proper weighting function $W(s)$, proper time domain specifications. The time domain specifications must not be too much ambitious and the weighting function should have proper bandwidth. The proposed controller has the least time domain specifications like rise time, settling time, steady state error and peak overshoot.

Figure 9 shows the robustness of the controllers to a change in the operating point from 10 cm to 20 cm. The proposed controller showed an increase of 10% to 4% in peak overshoot. For the FOIMC, peak overshoot has increased by from 25% to 75%. The rise time reduced from 900 sec to 500 sec and settling time reduced from 3600 sec to 2600 sec. From this discussion, the proposed controller showed better robustness. The robustness to change in the system parameters is verified by observing the step responses of the system with the gain varied by ±20%. The corresponding responses are shown in Figure 10 and Figure 11, it is seen that the change in gain is not adversely affecting the proposed controller performance.

### Table 2. Time domain performance specifications of the controllers

<table>
<thead>
<tr>
<th>Parameter</th>
<th>FOC (Proposed)</th>
<th>FOIMC</th>
<th>IOIMC</th>
<th>CRONE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$IAE$</td>
<td>628</td>
<td>269.3</td>
<td>3034</td>
<td>62.36</td>
</tr>
<tr>
<td>$ISE$</td>
<td>$3.562 \times 10^3$</td>
<td>$1.7 \times 10^4$</td>
<td>$1.612 \times 10^4$</td>
<td>273.5</td>
</tr>
<tr>
<td>$ITAE$</td>
<td>$5.9 \times 10^3$</td>
<td>$9.7 \times 10^3$</td>
<td>$8.547 \times 10^3$</td>
<td>1300</td>
</tr>
<tr>
<td>$ITSE$</td>
<td>$4.15 \times 10^3$</td>
<td>$2.45 \times 10^5$</td>
<td>$2.257 \times 10^5$</td>
<td>$5.469 \times 10^7$</td>
</tr>
<tr>
<td>$t_r$ (sec)</td>
<td>35</td>
<td>25</td>
<td>150</td>
<td>15</td>
</tr>
<tr>
<td>$t_s$ (sec)</td>
<td>400</td>
<td>100</td>
<td>1200</td>
<td>35</td>
</tr>
<tr>
<td>$M_p$ (%)</td>
<td>10</td>
<td>25</td>
<td>0</td>
<td>27</td>
</tr>
<tr>
<td>$\int</td>
<td>u</td>
<td>^2$</td>
<td>$1.759 \times 10^7$</td>
<td>$7.868 \times 10^7$</td>
</tr>
</tbody>
</table>

Figure 8. Servo responses and control efforts

Figure 9. Verification of robustness

Figure 10. Robustness to +20 % gain change

Figure 11. Robustness to -20 % gain change
Table 3 shows the comparison of robust stability norms and fragility indices of the controllers. From Table 3 it is observed that the proposed controller and Ref-2 are resilient, Ref-1 is nonfragile when all of the controller parameters are varied by +20%. The proposed controller and Ref-2 have robust stability norms less than 0 dB; hence the proposed controller and Ref-2 are robustly stable. The Ref-1 has robust stability norm of 0.59 dB (>0 dB); hence Ref-1 is not robustly stable. The proposed controller and Ref-2 have robust performance norms less than 0 dB; hence the proposed controller and Ref-2 will give good robust performance. The Ref-1 has robust stability norm of 0.408 dB (>0 dB), hence Ref-1 will not give good robust performance.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Controller</th>
<th>FOC (Proposed)</th>
<th>FOIMC</th>
<th>IOIMC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{FLD}$</td>
<td></td>
<td>0.087</td>
<td>0.345</td>
<td>0.00207</td>
</tr>
<tr>
<td>Stability norm (dB)</td>
<td></td>
<td>-4.05</td>
<td>0.59</td>
<td>-0.056</td>
</tr>
<tr>
<td>Performance norm (dB)</td>
<td></td>
<td>-4.3</td>
<td>0.408</td>
<td>-0.0566</td>
</tr>
</tbody>
</table>

4.3. Frequency domain analysis

Frequency domain analysis complements the time domain analysis of the system. Figure 12 shows the Bode plots of the system with and without (uncontrolled system) the proposed controller. OLTF of the system has a flat phase response around the GCF of 0.208 rad/sec. Figure 13 compares the Bode plots of OLTF with different controllers. From Figure 13, it is observed that the proposed controller has good isodamping property and low pass frequency response. Figure 14 compares Bode plots of OLTF with the different controllers at a different operating point of 20 cm and the transfer function of the new operating region is given by (34). Figure 14 indicates that at a different operating point also the proposed controller showed flat phase response around GCF, when compared to other controllers.

![Figure 12. Comparison of Bode plots of system](image)

![Figure 13. Bode plots with different controllers](image)

Peak overshoot of the proposed controller is lesser by 25% compared to FOIMC and lesser by 10% compared to IOIMC. The settling time of the proposed controller is 200% lesser than the FOIMC 285% lesser than IOIMC. Robust performance and robust stability of the proposed controller are verified and the proposed controller satisfied the robust stability infinity norm. Frequency domain analysis of the proposed controller and other controllers is done. Bode plots of the loop transfer function showed that the proposed controller has flatter phase response than other controllers. Bode plots of the controller at two different operating points established the robustness property of the proposed controller. From the results it is concluded that, the novel configuration of the FOIMC tuning based on the time domain specifications achieves the controller objectives. The transfer function of the system considered is a generalized FOPDT model, hence the results obtained with the proposed novel controller are applicable for any other FOPDT system other than the coupled tank system.
5. CONCLUSION

This paper proposes a fractional controller for the level control of a coupled tank system. A FOPDT model of the coupled tank system is considered. The FOC is designed analytically based on the specified peak overshoot and settling times. Modified Bode's ideal open loop transfer function is taken as the desired closed loop transfer function and using the direct synthesis method, the controller parameters are obtained. Performance of the proposed controller is compared with a FOIMC and IOIMC controllers. The proposed controller satisfied the peak overshoot and settling times specifications.

The FOC implementation using the FOMCON toolbox is compared with the CRONE toolbox. The CRONE toolbox based FOC consumed more control energy for achieving the control action when compared to the FOMCON toolbox based implementation.

REFERENCES


fractional order direct torque control of permanent magnet synchronous machine,”


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