

## Different Control Algorithms for a Platoon of Autonomous Vehicles

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### ABSTRACT

This paper presents a concept of platoon movement of autonomous vehicles (smart cars). These vehicles have Adaptive or Advanced cruise control (ACC) system also called Intelligent cruise control (ICC) or Adaptive Intelligent cruise control (AICC) system. The vehicles are suitable to follow other vehicles on desired distance and to be organized in platoons. To perform a research on the control and stability of an AGV (Automated Guided Vehicles) string, we have developed a car-following model. To do this, first a single vehicle is modeled and since all cars in the platoon have the same dynamics, the single vehicle model is copied ten times to form model of platoon (string) with ten vehicles. To control this string, we have applied equal PID controllers to all vehicles, except the leading vehicle. These controllers try to keep the headway distance as constant as possible and the velocity error between subsequent vehicles - small. For control of vehicle with nonlinear dynamics combination of feedforward control and feedback control approach is used. Feedforward control is based on the inverse model of nominal dynamics of the vehicle, and feedback PID control is designed based on the linearized model of the vehicle. For simulation and analysis of vehicle and platoon of vehicles - we have developed Matlab/Simulink models. Simulation results, discussions and conclusions are given at the end of the paper.

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## 1. INTRODUCTION

Grouping vehicles into platoons is a method of increasing the capacity of roads. An automated highway system is a proposed technology for doing this. Platoons decrease the distances between cars using electronic and possibly mechanical coupling. This capability would allow many cars to accelerate or brake simultaneously. Instead of waiting after a traffic light changes to green for drivers ahead to react, a synchronized platoon would move as one, allowing up to a fivefold increase in traffic throughput if spacing is diminished that much. This system also allows for a closer headway between vehicles by eliminating reacting distance needed for human reaction.

Smart cars with artificial intelligence could automatically join and leave platoons. The Automated Highway System (AHS) is a proposal for one such system, where cars organize themselves into platoons of eight to twenty-five. Potential benefits from this AHS are: greater fuel economy, reduced congestion, shorter commutes during peak periods, fewer traffic collisions, and the ability for vehicles to be driven unattended.

The origin of research on AHS was done by a team from Ohio State University led by R. E. Fenton. Their first automated vehicle was built in 1962, and is believed to be the first land vehicle to contain a

computer. Steering, braking and speed were controlled through the onboard electronics, which filled the trunk, back seat and most of the front of the passenger side of the car. Today – this field is widely explored and implemented in practice. SARTRE is a European Commission FP7 co-funded project [1]. It is built on existing results and experience and analyse the feasibility of vehicle platoons (consisting of both trucks/busses and passenger cars) as a realistic future transport and mobility concept. SARTRE aims to examine the operation of platoons on unmodified public motorways with full interaction with other vehicles. Crawford et al. [2] examine the sensory combination (GPS, cameras, scanners) to fulfill the task of following. Other authors (Halle et al. [4]) consider the car platoons as collaborative multi-agent system. They propose a hierarchical architecture based on three layers (guidance layer, management layer and traffic control layer) which can be used for simulating a centralized platoon (where a head vehicle-agent coordinates other vehicle-agents by applying its coordination rule) or a decentralized platoon (where the platoon is considered as a team of vehicle-agents trying to maintain the platoon).

This paper is organized as follows. Section 2 presents deriving of dynamic vehicle model and its linearization. Section 3 is reserved for vehicle platoon modeling and control. Section 4 discusses simulation results given using Matlab/Simulink models of the vehicle and platoon of vehicles. Finally, in Section 5 we give conclusions and directions for future work.

**2. DYNAMIC VEHICLE MODEL**

In this section we present mathematical model of longitudinal motion of the vehicle which is relevant for platoon modeling and control. For modeling in this case we've used two coordinate systems (see Figure 1): vehicle-fixed or body-fixed coordinate system,  $B(C;x,z)$ , and Earth-fixed coordinate system,  $E(O;x_0,z_0)$ . Velocity of the vehicle has components along  $x$  and  $z$  axes, i.e.  $\mathbf{V}_B = [u, v]^T$ . Figure 1 shows free body diagram of a vehicle with mass  $m$ . Vehicle is inclined upon angle  $\theta$  with respect to horizontal plane (slope of the road).

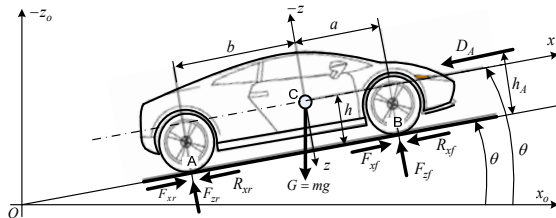


Figure 1. Forces acting on a vehicle

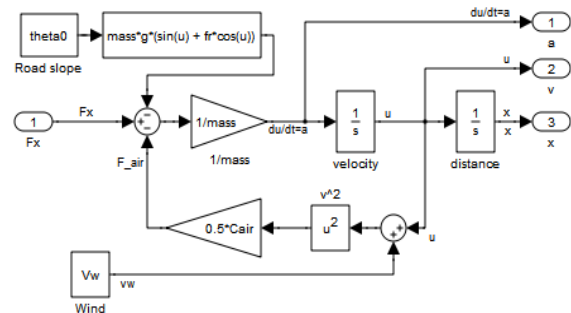


Figure 2. Simulink model of the vehicle

The diagram includes the significant forces acting on the vehicle:  $g$  is the gravitational constant;  $D_A$  is the aerodynamic force;  $G=mg$  is the weight of the vehicle;  $F_x$  is the tractive force;  $R_x$  is the rolling-resistance force; and  $ma_x$ , an equivalent inertial force, acts at the center of mass,  $C$ . The subscripts  $f$  and  $r$  refer to the front (at B) and rear (at A) tire-reaction forces, respectively.

Application of Newton's second law for the  $x$  and  $z$  directions gives [12]:

$$m\dot{u} = F_{xr} + F_{xf} - G \sin \theta - R_{xr} - R_{xf} - D_A \tag{1}$$

$$m\dot{v} = 0 = G \cos \theta - F_{zf} - F_{zr} \tag{2}$$

The aerodynamic-drag force depends on the relative velocity between the vehicle and the surrounding air and is given by the semi-empirical relationship:

$$D_A = \frac{1}{2} \rho C_d A_f (u + u_w)^2 = \frac{1}{2} C_{air} (u + u_w)^2 \quad (3)$$

where  $\rho$  is the air density ( $= 1.202 \text{ kg/m}^3$  at an altitude of 200 m),  $C_d$  is the drag coefficient,  $A_f$  is the frontal area of the vehicle,  $u$  is the vehicle-forward velocity, and  $u_w$  is the wind velocity (i.e., positive for a headwind and negative for a tailwind). The drag coefficient for vehicles ranges from about 0.2 (i.e., streamlined passenger vehicles with underbody cover) to 1.5 (i.e., trucks); 0.4 is a typical value for passenger cars [12].

The rolling resistance arises due to the deformation on the tire and the road surface, and it is roughly proportional to the normal force on the tire:

$$R_x = R_{xf} + R_{xr} = f_r (F_{zf} + F_{zr}) = f_r mg \cos \theta \quad (4)$$

where  $f_r$  is the rolling-resistance coefficient in the range of about 0.01 to 0.4, with 0.015 as a typical value for passenger vehicles.

For further consideration we use equation (1). Equation (1) is nonlinear in the forward velocity,  $u(t)$  but otherwise is a simple dynamic system: it only has one state variable. So, what are the main challenges in cruise-control design problems? The difficulties arise mainly from two factors: (1) plant uncertainty due to change of vehicle weight, and (2) external disturbances due to road grade. Thus, a good cruise-control algorithm must work well under these uncertainties.

Equation (1), using (3) and (4) can be rewritten:

$$m\dot{u} = F_x - mg \sin \theta - f_r mg \cos \theta - \frac{1}{2} C_{air} (u + u_w)^2 \quad (5)$$

where  $C_{air} = \rho A_f C_d$  is a constant.

Using (5) we create nonlinear SIMULINK model for vehicles in the platoon, Figure 2. For analysis of dynamics and stability of the vehicle and string stability of the platoon we need linearized model of the vehicle. Linearization of (5) around the specified operating (i.e., equilibrium) state is made using a Taylor series expansion. Variables and functions in the equation (5) are presented in form

$$u = u^0 + \Delta u; F_x = F_x^0 + \Delta F; \theta = \theta^0 + \Delta \theta \quad (6)$$

where  $u^0$  is the nominal velocity of the vehicle,  $F_x^0$  is the nominal tractive force, and  $\theta^0$  is the nominal slope of the road. Substituting (6) in (5) and performing mathematical operations, using approximations  $\sin \Delta \theta = \Delta \theta$ ,  $\cos \Delta \theta = 1$ , and neglecting the small quantities like  $\Delta u^2 = 0$ , we obtain two equations:

$$m\dot{u}^0 = F_x^0 - mg \sin \theta^0 - f_r mg \cos \theta^0 - \frac{1}{2} C_{air} (u^0 + u_w)^2 \quad (7)$$

$$m\Delta \dot{u} = -C_{air} (u^0 + u_w) \Delta u + \Delta F_x + d \quad (8)$$

$$d = (mg f_r \sin \theta^0 - mg \cos \theta^0) \Delta \theta \quad (9)$$

where  $d$  is the disturbance. Equation (7) describes nominal motion of the vehicle and it has the same form like (5), and (8) describes perturbed motion around nominal trajectory.

If nominal velocity  $u^0$  is constant then from (7) we can find nominal tractive force which is needed for movement near to nominal state:

$$F_x^0 = mg \sin \theta^0 + f_r mg \cos \theta^0 + \frac{1}{2} C_{air} (u^0 + u_w)^2 \quad (10)$$

Linearized equation (9) is of first order in which  $\Delta u$  is state-velocity perturbation and  $\Delta F_x$  is perturbation of the tractive force and we can use it for stabilization and control of the vehicle by obtaining it from suitable linear controller.

Taking perturbation in position,  $\Delta x$ , using (8) we can write next state-space equation for the vehicle:

$$\begin{aligned}\Delta \dot{x} &= \Delta u \\ \Delta \dot{u} &= -\frac{1}{Km} \Delta u + \frac{1}{m} \Delta F_x + \frac{1}{m} d\end{aligned}\quad (11)$$

where  $K = 1/[C_{air}(u^0 + u_w)]$ .

In vector-matrix form (11) becomes:

$$\begin{bmatrix} \Delta \dot{x} \\ \Delta \dot{u} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{1}{Km} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta u \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} \Delta F_x + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} d\quad (12)$$

We can find the transfer function of the vehicle from (11):

$$\tau \Delta \dot{u} + \Delta u = K(\Delta F_x + d),\quad (13)$$

where  $\tau = Km = m/[C_{air}(u^0 + u_w)]$  is time constant.

Applying Laplace transformation to (13), and neglecting disturbance  $d$ , we can compute transfer function:

$$G(s) = \frac{\Delta u(s)}{\Delta F_x(s)} = \frac{K}{\tau s + 1}\quad (14)$$

Using the numerical values:

$$\begin{aligned}u^0 &= 20 \text{ m/s}, \theta^0 = 0, m = 1000 \text{ kg}, \rho = 1.2 \text{ kg/m}^3, \\ A_f &= 1.2 \text{ m}^2, C_d = 0.5, f_r = 0.01, g = 9.81 \text{ m/s}^2, u_w = 0\end{aligned}\quad (15)$$

we can compute the parameters in above equations (10), (11), (12) and (13):

$$F_x^o = 242.1 \text{ N}, K = 0.0694 \text{ (m/s)/N}, \tau = 69.44 \text{ s},\quad (16)$$

$$\begin{bmatrix} \Delta \dot{x} \\ \Delta \dot{u} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -0.0144 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta u \end{bmatrix} + \begin{bmatrix} 0 \\ 0.001 \end{bmatrix} \Delta F_x + \begin{bmatrix} 0 \\ 0.001 \end{bmatrix} d\quad (17)$$

$$G(s) = \frac{\Delta u(s)}{\Delta F_x(s)} = \frac{0.0694}{69.44s + 1}\quad (18)$$

### 3. VEHICLE CONTROL SYSTEM

In cases when the real vehicle is with nonlinear dynamics (in our case equation (5) for longitudinal dynamics) it is very useful to implement combination of feed-forward control and feedback control approach, presented on Figure 3. The feed-forward control is formed on the *inverse model* of the object and on the generator of *nominal trajectories* which generates the desired trajectory  $\mathbf{x}^o(t)$ . This desired trajectory is based on the previously prepared data or from the system operation based on the measured data. For realization of this trajectory it is necessary that regulator in feedback is present, which will generate the needed control  $\Delta \mathbf{u}(t)$  for elimination trajectory error of the object from the desired trajectory. This provides stabilisation of the control process of the object. The sum control  $\mathbf{u}(t)$  of the moving object from Figure 3, when the linear regulator is formed by the matrix  $\mathbf{K}(t)$ , is given with the following relation:

$$\begin{aligned} \mathbf{u}(t) &= \mathbf{u}^o(t) + \Delta\mathbf{u}(t) = \mathbf{u}^o(t) - \mathbf{K}(t)\Delta\mathbf{x}(t) = \\ &= \mathbf{u}^o(t) - \mathbf{K}(t)[\mathbf{x}(t) - \mathbf{x}^o(t)] \end{aligned} \quad (19)$$

The synthesis of the control law given by equation (19) is performed in two steps. In the first step the nominal control  $\mathbf{u}^o(t)$  is determined under assumption of ideal conditions i.e. when no disturbances are present.

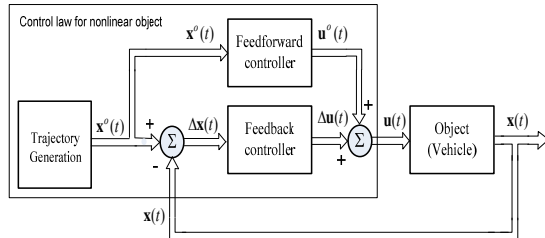


Figure 3. Concept of feed-forward and feedback control system of nonlinear object

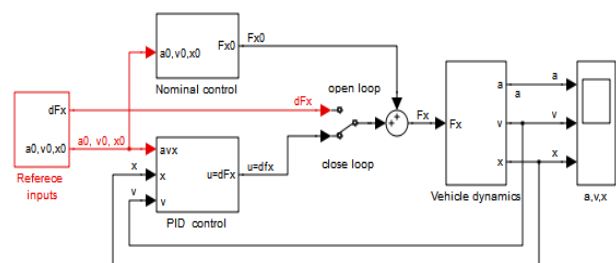


Figure 4. Simulink diagram for the vehicle control

According to the described concept (Figure 3), the control laws for vehicles can be developed. In this paper feed-forward control is determined based on the (7), i.e. (10) for nominal tractive force which is a nominal control.

Feedback controller, which provides stabilization of the object around the nominal trajectory, can be designed using linearized model. Under assumption that the dynamic behavior of the object with respect to the nominal trajectory is linear, as described with (8), or (12) to (14), for the control  $\Delta\mathbf{u}(t)$ , we can apply methods for synthesis developed for linear systems: PID controller design, Linear Quadratic Regulator (LQR), methods for pole placement, adaptive optimal control etc.[3].

In this paper PID control design approach is used and PID feedback controller is obtained based on the linear model of the vehicle derived above with parameters determined using numerical values (15). For simulation and testing of vehicle dynamics and vehicle control system Simulink model is developed which is shown on Figure 4.

Module reference inputs, generate reference acceleration  $a_o$ , velocity  $v_o$ , and position  $x_o$ , similar like the leader of the platoon. These signals go to the PID controller where are processed according to:

$$u = \Delta F_x = K_p(x_o - x) + \frac{K_I}{s}(x_o - x) + K_D(v_o - v) \quad (20)$$

where  $K_p$ ,  $K_I$ , and  $K_D$  are proportional, integral and derivative gains of the controller,  $a$ ,  $v$  and  $x$  are real acceleration, velocity and position of the vehicle.

Module *Nominal control*, Figure 4, consists of equation (10) and module *Vehicle dynamics*, which is based on full nonlinear model, equation (5).

Simulink model in Figure 4 can be used for open loop, and closed loop simulation of the controlled vehicle. (i.e., its own motion and headway to the vehicle in front). In this paper we discuss the vehicle-following control approach, which is the focus of most current research and development work in the area [12].

We observe the movement of vehicles in the inertial (or absolute) coordinate system  $G(O; x_o, y_o)$  which is fixed to the road with origin in the starting point,  $O$ . Positions,  $x_i$ , velocities,  $v_i = \dot{x}_i$ , and accelerations,  $a_i = \dot{v}_i$ ,  $i = L, 1, 2, 3, 4$ , measured with respect to  $G(O; x_o, y_o)$ , are absolute quantities. Coordinate system  $L(L; x_L, y_L)$  is fixed to the vehicle-leader with origin in the center of its mass. Relative position, velocity and acceleration of the vehicles with respect to  $L(L; x_L, y_L)$  are denoted as:  $l_i = x_L - x_i$ ,  $v_{ri} = v_L - v_i$ ,  $a_{ri} = a_L - a_i$ ,  $i = 1, 2, 3, 4$  respectively. Distances between vehicles are denoted as  $dx_i = x_{i-1} - x_i$ ,  $i = L, 1, 2, 3, 4$ , and relative velocities and accelerations of the vehicles with respect to vehicle in front of them are respectively:

$$dv_i = v_{i-1} - v_i = \dot{x}_{i-1} - \dot{x}_i,$$

$$da_i = a_{i-1} - a_i = \ddot{x}_{i-1} - \ddot{x}_i, \quad i = L, 1, 2, 3, 4.$$

The main Simulink diagram of our model is shown on Figure 5. In this model each vehicle gets information about acceleration, velocity and position of the previous vehicle, and also gets the same information about the vehicle-leader.

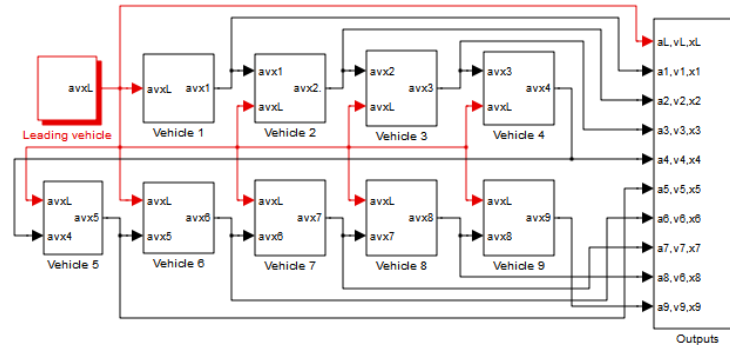


Figure 5. Matlab/Simulink model of the platoon of 10 vehicles.

Using vehicle model (5), or Figure 2, if  $\theta = 0$  and  $V_w = 0$ , we can find acceleration of the vehicle in this form:

$$\dot{u} = a = \frac{1}{m}(F_x - f_r mg - \frac{1}{2}C_{air}u^2), \quad (21)$$

$$F_x = \Delta F_x + F_{x0}$$

Control force  $\Delta F_x$  is determined by a PID controller, i.e. with equation (20). Substituting (20) in (21) we can find acceleration for the  $i$ -th vehicle:

$$a_i = \frac{1}{m}[K_{pi}(x_{i-1} - x_i - hd_i) + \frac{K_{li}}{s}(x_{i-1} - x_i - hd_i) + K_{Di}(v_{i-1} - v_i) + F_{x0} - f_r mg - \frac{1}{2}C_{air}u_i^2], \quad (22)$$

where  $hd_i$  is constant distance between  $i-1$ -th and  $i$ -th vehicle. Deriving (21) we can get jerk which acts on the  $i$ -th vehicle ( $F_{x0}$  and  $f_r mg$  are constant), and using relations:

$$\dot{x}_i = v_i \quad (23)$$

$$\dot{v}_i = a_i \quad (24)$$

we can find:

$$\dot{a}_i = \frac{1}{m}[K_{li}(x_{i-1} - x_i - hd_i) + K_{pi}(v_{i-1} - v_i) + K_{Di}(a_{i-1} - a_i) - C_{air}u_i^o a_i] \quad (25)$$

Equations (23), (24) and (25) represent linear state-space model of the  $i$ -th vehicle in the platoon. Variables  $x_{i-1}$ ,  $v_{i-1}$ , and  $a_{i-1} - a_i$  in equation (25) are input variables for the  $i$ -th vehicle and they are position, velocity and acceleration of the previous, or  $i-1$ -th, vehicle.

Equations (23), (24) and (25) can be used for obtaining the state space model of string of several vehicles. This model is useful for stability analysis of the string using techniques of linear control theory.

Here we form model of string with three vehicles: vehicle-leader, and two vehicles-followers. Outputs of the vehicle-leader generate input variables,  $x_L$ ,  $v_L$ , and  $a_L$ , for the first vehicle in the string. Other two vehicles are described with these equations - for the first vehicle:

$$\begin{aligned}\dot{x}_1 &= v_1 \\ \dot{v}_1 &= a_1 \\ \dot{a}_1 &= \frac{1}{m}[K_{I1}(x_L - x_1 - hd_1) + K_{P1}(v_L - v_1) + \\ &\quad + K_{D1}(a_L - a_1) - C_{air}u^o a_1],\end{aligned}\quad (26)$$

and for the second vehicle:

$$\begin{aligned}\dot{x}_2 &= v_2 \\ \dot{v}_2 &= a_2 \\ \dot{a}_2 &= \frac{1}{m}[K_{I2}(x_1 - x_2 - hd_2) + K_{P2}(v_1 - v_2) + \\ &\quad + K_{D2}(a_1 - a_2) - C_{air}u^o a_2]\end{aligned}\quad (27)$$

Now we form state vector:  $\mathbf{x} = [x_1 \ dxx_2 \ v_1 \ v_2 \ a_1 \ a_2]^T$  where state  $dxx_2$  is the distance between the first and the second vehicle, and

$$d\dot{x}_2 = \dot{x}_1 - \dot{x}_2 = v_1 - v_2 = dv_2 \quad (28)$$

Now we can write state-space equation of the string in vector-matrix (29):

$$\begin{aligned}\begin{bmatrix} \dot{x}_1 \\ d\dot{x}_2 \\ \dot{v}_1 \\ \dot{v}_2 \\ \dot{a}_1 \\ \dot{a}_2 \end{bmatrix} &= \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{-K_{I1}}{m} & 0 & \frac{-K_{P1}}{m} & 0 & \frac{-K_{D1} - C_{air}u^o}{m} & 0 \\ 0 & \frac{K_{I2}}{m} & \frac{K_{P2}}{m} & \frac{-K_{P2}}{m} & \frac{K_{D2}}{m} & \frac{-K_{D2} - C_{air}u^o}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ dxx_2 \\ v_1 \\ v_2 \\ a_1 \\ a_2 \end{bmatrix} + \\ &+ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{K_{I1}}{m} & \frac{K_{P1}}{m} & \frac{K_{D1}}{m} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_L \\ v_L \\ a_L \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{-K_{I1}}{m} & 0 \\ 0 & \frac{-K_{I2}}{m} \end{bmatrix} \begin{bmatrix} hd_1 \\ hd_2 \end{bmatrix}\end{aligned}\quad (29)$$

If we choose outputs as - distance between vehicles,  $dxx_2$ , and velocities  $v_1$  and  $v_2$ , we can form output vector,  $\mathbf{y} = [dxx_2 \ v_1 \ v_2]^T$ , as:

$$\mathbf{y} = \begin{bmatrix} dxx_2 \\ v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_L \\ v_L \\ a_L \end{bmatrix}\quad (30)$$

Stability analysis of the individual vehicle and platoon of vehicles can be made in Matlab using their linear models and computing poles of the system or finding gain and phase margins with help of Nyquist plot. For example, for string of two vehicles-followers, using model (29) and parameters (15), we can find eigenvalues or poles,  $p_1, \dots, p_6$ :

$$-1.2690, -1.2690, -0.5306, -0.5306, -0.0149, -0.0149$$

which are real and negative, and system is stable.

For a platoon of vehicles, besides individual vehicle stability, it is defined *string stability* of the platoon [17, 18]. If the preceding vehicle is accelerating or decelerating, then the spacing error could be nonzero; we must ensure that the spacing error attenuates as it propagates along the string of vehicles because it propagates upstream toward the tail of the string. Taking (23) in Laplace domain, and using transfer function  $G_i = v_i / v_{i-1}$  and relation for range error between  $i$ -th and  $i-1$ th vehicles:

$$x_i = \frac{1}{s}v_i, v_i = G_i(s)v_{i-1}, \varepsilon_i = x_{i-1} - x_i - D_i \tag{31}$$

where  $D_i = h_i v_i$  denotes the desired range for the  $i$ th vehicle,  $h_i$  is a constant time-headway policy adopted for all vehicles, we can find transfer function  $G_{i,k}$  from the range error of  $i$ -th vehicle to the range error of  $i+k$ th vehicle:

$$G_{i,k}(s) = \frac{\varepsilon_{i+k}}{\varepsilon_i} = G_i G_{i+1} \dots G_{i+k-1} \frac{1 - G_{i+k} - s h_{i+k} G_{i+k}}{1 - G_i - s h_i G_i} \tag{32}$$

For string stability must be satisfied [13]:

$$\|\varepsilon_{i+k}\| \leq \|\varepsilon_i\|, \text{ or } \|G_{i,k}(s)\|_{\infty} \leq 1 \tag{33}$$

This discussion for string stability can be easily applied to platoon described in this work. In the next section we present some results for platoon movement.

#### 4. RESULTS AND ANALYSIS

We have simulated a platoon with 10 vehicles. All vehicles are the same with parameters (15), desired distances among vehicles are  $dx_{i0}=50$  m. Parameters of PID controllers are:  $K_{p_i}=700$ ,  $K_{i_i}=10$ , and  $K_{D_i}=1800$ . Vehicle-leader generates acceleration, velocity and position which are shown in the pictures below. Figure 6 shows velocity profile of the vehicle leader and responses of vehicles – followers.

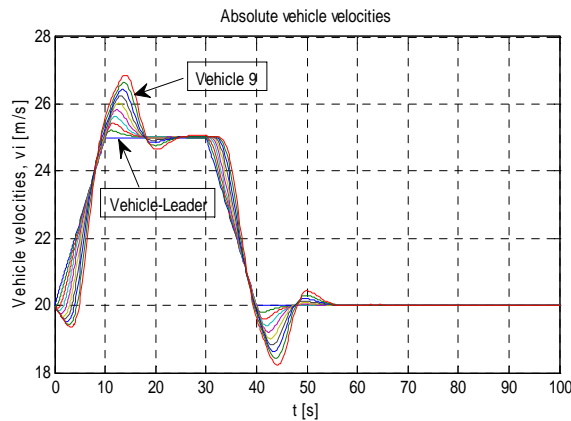


Figure 6. Trapezoidal change of vehicle-leader velocity and responses of vehicles in the platoon.

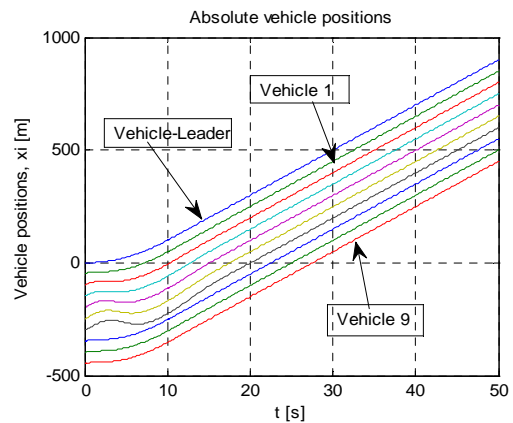


Figure 7. Positions of the vehicles.

Figure 8 shows distance errors between vehicles for the same inputs as in Figure 6. Figure 9 shows positions of the vehicles in the platoon when each vehicle gets information for acceleration, velocity and position only from previous vehicle. Figure 7 shows the situation when only last three vehicles get information for acceleration, velocity and position from the vehicle-leader. In this situation errors in positions between vehicles are smaller. It is known in the literature that the information for vehicle-leader movement and inter-vehicle communication influence to better control and string stability of the platoon.



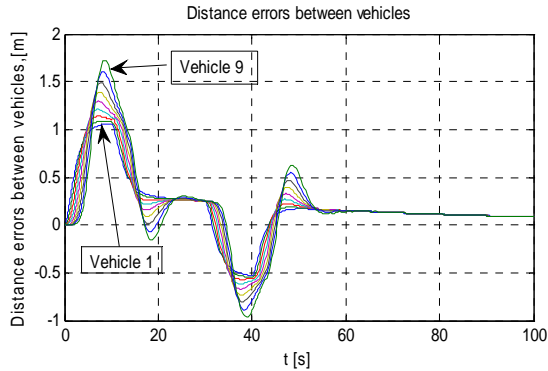


Figure 8. Distance errors between vehicles

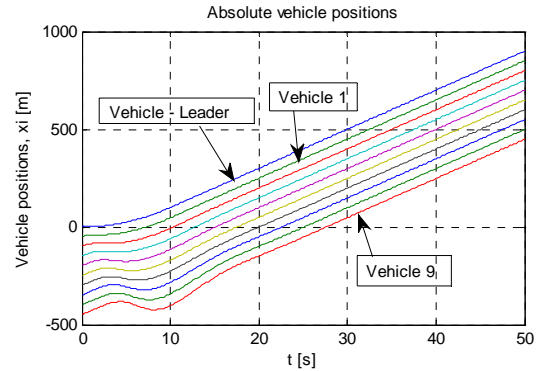


Figure 9. Positions of the vehicles

## 5. CONCLUSION

In this paper we have developed a nonlinear and linearized model of the longitudinal motion of the vehicle. Feed-forward control and feedback PID control approach is applied to design vehicle controller. Using this vehicle model with its designated control system – we've developed a model of platoon with ten vehicles. In this model, vehicles can get information for acceleration, velocity and position from previous vehicle and from movement of the vehicle –leader. String stability of the platoon is discussed and transfer function of the string useful for stability analysis is presented. Based on the developed models, Matlab/Simulink models are created which can be used for simulation and performance analysis of the vehicle dynamics and platoon's control system. This Simulink models can be useful for different experiments and testing of designed controllers. Simulation results given in the previous section show how vehicles behave in the platoon under given conditions with PID controllers applied.

In future work, we plan to develop more accurate models of the vehicles and platoons. We plan to design and test different than PID control laws, for example LQR and Fuzzy logic control. Practical realization using different sensors and wireless communication among vehicles will be our interest in the future.

## REFERENCES

- [1] Chan E., P. Gilhead, P. Jelinek, P. Krejčí; "SARTRE cooperative control of fully automated platoon vehicles", 18th World Congress on Intelligent Transport Systems, 2011, Orlando, USA.
- [2] Crawford S.A., "Performance evaluations of sensor combinations for mobile platoon control", University of Calgary, Dept. of Geomatics Engineering, Rep. No. 20213, 2005.
- [3] Franklin, G.F., J.D. Powell, A. Emami-Naeini; "Feedback control of dynamic systems", fourth edition (2002).
- [4] Güvenç, B.A., E. Kural; "Adaptive cruise control simulator. A low-cost multiple-driver-in-the-loop simulator", Istanbul Technical University, Department of Mechanical Engineering (2006).
- [5] Halle S., B. Chaib-draa; "A collaborative driving system based on multiagent modelling and simulations", Transportation Research Part C: Emerging Technologies, pp.320-345.
- [6] Hatipoğlu, C., Ü. Özgüner, M. Sommerville; "Longitudinal headway control of autonomous vehicles", Department of Electrical Engineering, Ohio State University (1996).
- [7] Higashimata A., K. Adachi, T. Hashizume, S. Tange; "Design of a headway distance control system for ACC", *JSAE Review*, 22 (2001), pp. 15 – 22.
- [8] Holzmann H., Ch. Halfmann, S. Germann, M. Wurtenberger, R. Isermann; "Longitudinal and lateral control and supervision of autonomous intelligent vehicles", *Control Eng. Practice*, Vol. 5 No. 11 (1997), pp. 1599-1605.
- [9] Jones, W.D.; "Keeping cars from crashing", *IEEE Spectrum*, Volume 38, Issue 9 (September 2001), pp. 40 – 45.
- [10] Kim, H.M., J. Dickerson, B. Kosko; "Fuzzy throttle and brake control for platoons of smart cars", *Fuzzy Sets and Systems*, vol. 84,(1996), pp.209-234.
- [11] Liang, C.Y., H. Peng; "Optimal adaptive cruise control with guaranteed string stability", *Vehicle System Dynamics*, (1999), pp. 313 – 330.
- [12] A. Galip Ulsoy, Huei Peng, Melih Cakmakci: *Automotive Control Systems*, Cambridge University Press, Cambridge, 2012.
- [13] Liang, C.Y., H. Peng; "String Stability Analysis of Adaptive Cruise Controlled Vehicles", <http://www-personal.umich.edu/~hpeng/JSME 2000.pdf>.

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