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Optimal Trajectory Planning of Industrial Robots using Geodesic

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ABSTRACT

This paper intends to propose an optimal trajectory planning technique using geodesic to achieve smooth and accurate trajectory for industrial robots. Geodesic is a distance minimizing curve between any two points on a Riemannian manifold. A Riemannian metric has been assigned to the workspace by combining its position and orientation space together in order to attain geodesic conditions for desired motion of the end-effector. Previously, trajectory has been planned by considering position and orientation separately. However, practically we cannot plan separately because the manipulator joints are interlinked. Here, trajectory is planned by combining position and orientation together. Cartesian trajectories are shown by joint trajectories in which joint variables are treated as local coordinates of position space and orientation space. Then, the obtained geodesic equations for the workspace are evaluated for initial conditions of trajectory and results are plotted. The effectiveness of the geodesic method validated through numerical computations considering a Kawasaki RS06L robot model. The simulation results confirm the accuracy, smoothness and the optimality of the end-effector motion.

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1. INTRODUCTION

Robots are being extensively operated in industries for material handling, machine loading or unloading, assembly, machining, palletizing, welding and painting etc. to achieve high productivity through automation. The automation process using industrial robots in manufacturing and assembly environment is a challenging task due to presence of several objects. Some application areas namely machining and painting etc. needs the trajectory to be the shortest, smooth and precise. The main purpose is to operate robot end-effector such that its movement is smooth, accurate and continuous and faster along a trajectory to accomplish an intended task persistently.

The traditional ways of finding a path or trajectory are simple processes like Configuration Space Obstacle (CSO) and Generalized Voronoi Diagram (GVD). Then the polynomial interpolation method is employed as trajectory planning method for manipulator and the interpolation revolves around the joint space which can also be named as the Cartesian space. Currently this rudimentary technique is vastly being employed for manipulation process. Since they considered the manipulator joints as linearly independent, this method does not give a clear view of manipulator's potential performance. These methods are essentially graphical techniques used for obstacle avoidance. The minimum time criterion i.e. shortest path findings, path accuracy, smoothness of trajectory are not well defined by the traditional methods of trajectory planning. Earlier planning methods are prolonged process of optimization due to the nature of providing multiple solutions to inverse kinematics problems. Some of the alternative solutions to the above-mentioned problems are neural network approach, polynomial approach, and genetic algorithm and fifth order B-spline methods.

The specific reason to implement geodesic approach as a trajectory planning method is its inherent optimizing characteristics, which reduces the process of optimization cycle time. It is a convenient tool to trajectory planning. Geodesic is robust i.e. minimal error trajectory can be obtained. Moreover, it results a shortest, smooth and accurate trajectory.

2. RELATED WORKS

Many researchers have worked on different aspects and possibility of robotic manipulator trajectory planning and optimization. Ning et al. [1] analyzed the dynamic movement of primitives and proposed a novel scheme for generating trajectory. They compared the position coordinates and velocity at start end points of a trajectory obtained from their method to that of the measured values and found it to be very precise and accurate. Gasparetto et al. [2] gave stress upon planning a smooth trajectory for robot manipulators. They modeled an objective function i.e. implicitly dependent upon the integral taken over the squared jerk as well as total execution time. Chiu [3] developed Asada’s inertia ellipsoid and Yoshikawa’s manipulability ellipsoid. These tools together result a performance parameter of velocity as well as static force. Borbow [4, 5] trajectory planning study was based on optimizing time and later on he also demonstrated control of his optimized path-planning result.

Eldershaw et al. [6] used polynomial interpolation along with genetic algorithm which is based upon the natural selection procedure in order to tackle the trajectory planning problem. Tian et al [7] as well as Yun and Xi [8] also performed the same task by employing genetic algorithm. This method was further developed by Zha [9, 10] who compared the trajectory to a ruled surface and incorporated interpolation using Bezier curves between different poses. A novel scheme of trajectory planning was suggested by Olabi et al. [11] which considered continuous machining. A parametric speed interpolator came into picture which results in smooth trajectories. In order to achieve more accuracy a planning scheme with higher degree polynomials was incorporated by Boryga et al. [12]. Multi degree Splines was introduced to this type of planning methods by Liu et al. [13]. Gouasmi et al. [14] implemented dual quaternion method for kinematic analysis of robot manipulator. Shah et al. [15] have taken feed forward ANN and trained the data obtained for a 3-dof manipulator in MTALAB toolbox to show that ANN is best method to find inverse kinematic solution. Jha et al. [16] have proposed a structured artificial neural network approach i.e. multi-layered perceptron neural network (MLPNN) to solve inverse kinematics problem by considering 4-dof SCARA manipulator.

When the motion actually is performed in the joint space by the end-effector through interpolating sequence of via points, it is not easily predictable. The nonlinearity of the inverse kinematics may indicate an uneven joint motion in joint space in order to achieve a smooth motion in Cartesian space, when data obtained from interpolation scheme are mapped into joint space. Subsequently, an uneven and inaccurate motion in the Cartesian space could be acquired. Rodnay et al. [17] analyzed the dynamic characteristics of 2DOFs robot in 3D space. They signified trajectories by allowing the manipulator to perform free movement by introducing geodesics on the dynamic surface. Zefran et al. [18] took a Lie group approach to generate trajectories resulting from nonlinear motion. They chose Lie group and consequently defined a left invariant Riemannian metric on it to generate smooth trajectories. Selig et al. [19] steered the robot manipulators along helical trajectories by robotic control and trajectory planning. They used a Lie group method which made the planning complex, abstract, and not easy to put into real practice. Chen et al. [20] have demonstrated the effectiveness of geodesic planning by conducting simulation experiments. They specified a Riemannian metric for each of the position space and orientation space separately to accomplish the geodesic motion. The geodesic equalities are solved numerically and the results are used to manipulate the robot. Since the links of the robot manipulator are interlinked, we can’t plan trajectories separately for each space. In order to get a shortest and accurate path while keeping the kinetic energy invariant, Zhang et al. [21] chose the parameters as arc length and kinetic energy and constructed Riemannian metric accordingly. But his method had some shortcomings which decreased the credibility of that method. The authors did not take more than 3 DOFs robot to plan trajectories and also not tackled the orientation problem.

In previous works the authors did consider the use of geodesic, but they propose the trajectory planning separately for position and orientation. Practically the robot consists of the position and orientation joints on a single arm. Therefore, the trajectories need to be determined by considering the combination of position and orientation joints in an integrated manner. In order to outrun the abovementioned drawbacks, this paper presents a geodesic trajectory planning method by combining the position and orientation of joint. Geodesic is intrinsic in nature and has no relation with the coordinates. Unlike the polynomial method which is a mere approximating interpolation scheme, this geodesic method provides an exact and accurate solution. The work volume of end-effector i.e. position and orientation space are combined together to define the Riemannian metric to attain geodesic motions. First, joint variables are considered as local coordinates of position and orientation space. Then geodesics are obtained from mathematical formulation followed by obtaining joint trajectories and characterizing Cartesian trajectories by joint trajectories. The nature of geodesic implicitly makes both equivalent trajectories (Cartesian and Joint) smooth and relatively less erroneous. This method implicitly filters multiple solutions resulting from inverse kinematics and results the optimal one.

3. MATHEMATICAL MODELLING USING GEODESIC FOR SHORTEST PATH

The shortest path connecting any two points on a Riemannian manifold along itself is called as the geodesic. It has another property that velocity along this geodesic curve remains invariant. The background of geodesic has been discussed in brief in the later section.

3.1. Riemannian Manifold

A manifold ‘M^n’ is described as a Hausdroff topological space for which any point ‘p’ has a neighborhood $U \subset M^n$ homomorphic to an open subset of the Euclidian space ‘R^n’. If we define a function:

$$\Phi: U \rightarrow \Phi\,(U) \subset R^n \tag{1}$$

A Riemannian manifold is defined by (M^n, g), where ‘M^n’ is an n-dimensional differentiable manifold and ‘g’ is a Riemannian metric. Every Riemannian metric has a unique property that it is symmetric, positive definite quadratic form. Basically distance along the manifold is regarded as the metric. In any neighborhood U of a point in manifold we define local co-ordinates (Θ_1, Θ_2, Θ_3,….., Θ_n), then the metric can be written as:

$$g = \sum_{i,j=1}^{n} g_{ij}\, d\theta_i\, d\theta_j \tag{2}$$

where, $g_{ij} = g(\frac{\partial}{\partial \theta_i}, \frac{\partial}{\partial \theta_j})$

If we take a curve on a Riemannian manifold:

$$\theta_i = \theta_i(t) \tag{3}$$

then, its tangent vector can be defined by:

$$T_i = \sum_{i=1}^{n} d\theta_i \frac{\partial}{\partial \theta_i} \tag{4}$$

Geodesic is described as the shortest path along the Riemannian manifold connecting any two points belonging to it. Hence, if arc length is considered as a variable and taken as the covariant derivative of equation (4) and makes it zero then equation (4) makes out the geodesic equation (5), i.e.

$$\frac{d^2\theta_i}{dt^2} + \tau_{kj}^{i} \frac{d\theta_k}{dt} \frac{d\theta_j}{dt} = 0 \tag{5}$$

where, τ_{kj}^{i} is the Christoffel symbol and is defined by:

$$\tau_{kj}^{i} = \frac{1}{2} g^{mi} \left(\frac{\partial g_{km}}{\partial \theta_j} + \frac{\partial g_{jm}}{\partial \theta_k} - \frac{\partial g_{kj}}{\partial \theta_m}\right) \tag{6}$$

Where, g^{mi} is a general element of inverse matrix of Riemannian metric coefficient matrix.

4. GEODESIC APPROACH FOR TRAJECTORY PLANNING

The link frames of robot manipulators have been attached and the corresponding link parameters have been found out from the D-H representation of the link frames. The general transformation matrix $^{i-1}T_i$ for a single link can be defined as follows:

$$^{i-1}T_i = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\propto_{i-1} & c\theta_i c\propto_{i-1} & -s\propto_{i-1} & -d_i s\propto_{i-1} \\ s\theta_i s\propto_{i-1} & c\theta_i s\propto_{i-1} & c\propto_{i-1} & d_i c\propto_{i-1} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (7)$$

Where,

$s\theta_i = \sin\theta_i$, $c\theta_i = \cos\theta_i$, θ_i is the i^{th} joint rotation angle,

$s\alpha_i = \sin\alpha_i$, $c\alpha_i = \cos\alpha_i$, α_i is twist angle,

a_i is length of link, d_i is offset distance at joint i.

The forward kinematics of the end-effector with respect to the base frame is determined by multiplying all of the $^{i-1}T_i$ matrices of link frames.

$$^{base}T_{end\text{-}effector} = {}^0T_1 * {}^1T_2 * {}^2T_3 \ldots\ldots {}^{n-1}T_n \quad (8)$$

Assuming the final transformation, the T matrix of a robot end-effector as:

$$T = \begin{pmatrix} R & P \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (9)$$

Where, R and P represent the orientation (n, o, a) and position $(p_x, p_y, p_z)$of the end-effector respectively.

By combining together, R and P as orientation space and position space respectively, a Riemannian metric for the workspace can be constructed for the T matrix.

4.1. Selection of Local Coordinates

In order to obtain a solution to the inverse kinematic problem of the robot manipulator, an explicit scheme should be modeled to inter-relate joint space and Cartesian space. If all joint variables are chosen as a coordinate system of Cartesian space, then both the spaces can be easily inter-related. However this process has some shortcomings. If the joint space is to be mapped to Cartesian space, then forward kinematics has to be used. The intention is to map Cartesian space to joint space. Multiple solutions may be obtained instead of a single solution. Hence, joint variables should be treated as local coordinates instead of general coordinates to tackle especially the inverse kinematics problems.

4.2. Application of Geodesic for trajectory planning

Considering an Euclidean space with distance metric, the geodesic becomes a straight line. As Euclidean space is taken to be the position space, hence a Riemannian metric has been taken as distance metric. The Riemannian metric is given by:

$$g_p = (dP)^2 + (dn)^2 + (do)^2 + (da)^2 \quad (10)$$

where, dP is the derivative of the position vector $P = (p_x, p_y, p_z)$ and dn, do, and $da$are the derivatives of the orientation vector,$n = (n_x, n_y, n_z)$, $o = (o_x, o_y, o_z)$, $a = (a_x, a_y, a_z)$ respectively. A flow chart of the robotic manipulator trajectory planning using geodesic method is represented in Figure 1.

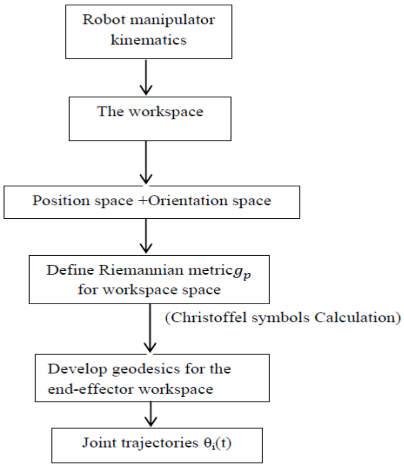


Figure 1. Flowchart showing the sequential process of manipulator trajectory planning by geodesic method.

4.1. Riemannian Metric Selection Procedure

The main idea of geodesic method is to assign an appropriate metric for chosen trajectory. The metric should be symmetric, positive definite quadratic form and it has to be a representation of distance and other variables which relates to the preferred motions.

The general form of Riemannian metric, ‘g’ can be defined as:

$$g = (d\theta_1 \quad d\theta_2 \quad d\theta_3 \; \ldots \ldots d\theta_n) G \begin{pmatrix} d\theta_1 \\ d\theta_2 \\ d\theta_3 \\ . \\ . \\ d\theta_n \end{pmatrix} \qquad (11)$$

Here, $G = (g_{ij})_{nxn}$ is the coefficient matrix conforming to g and θ_i (i = 1. . .n) are joint positions which can be linear or angular. After knowing coefficient matrix G, we can compute the Christoffel symbols τ^i_{kj} by equation (6). The geodesic equations are obtained in the form as given in equation (5). Regarding the Riemannian metric, a set of geodesic equations in the form of equation (5) i.e. simultaneous differential equations are established for the given initial conditions. The solutions to simultaneous differential equations provide the joint trajectories for the desired motion.

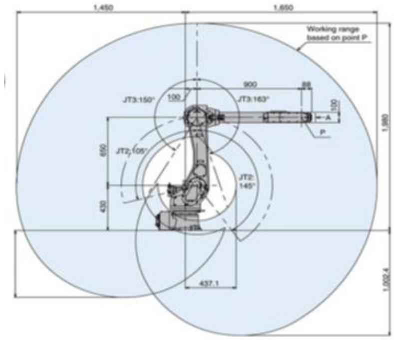
4.1. Mathematical Formulation and Numerical Computation for RS06L Robot

Kawasaki RS06L robot has been considered to demonstrate the entire trajectory planning process by the help of geodesic. The Kawasaki RS06L robot and its link diagram are shown in the Fig 2 (a) and (b) respectively. The obtained link parameters from the D-H representation of the robot manipulator are tabulated in Table 1. The calculated final transformation matrix, T of the robot end-effector is with the form as equation (9),
where,

$n_x = c_1[c_{23}(c_4c_5c_6 - s_4s_6) - s_{23}s_5c_6] - s_1(s_4c_5c_6 + c_4s_6)$
$n_y = c_1[c_{23}(c_4c_5c_6 - s_4s_6) - s_{23}s_5c_6] + c_1(s_4c_5c_6 + c_4s_6)$
$n_z = s_{23}(c_4c_5c_6 - s_4s_6) + c_{23}s_5c_6$
$o_x = c_1[c_{23}(-c_4c_5c_6 - s_4c_6) + s_{23}s_5c_6] + s_1(s_4c_5s_6 - c_4c_6)$
$o_y = s_1[c_{23}(-c_4c_5c_6 - s_4c_6) + s_{23}s_5c_6] + c_1(s_4c_5s_6 - c_4c_6)$
$o_z = s_{23}(-c_4c_5s_6 - s_4c_6) - c_{23}s_5s_6$
$a_x = c_1(c_{23}c_4s_5 + s_{23}c_5) + s_1s_4s_5$
$a_y = s_1(c_{23}c_4s_5 + s_{23}c_5) - c_1s_4s_5$
$a_z = c_{23}c_5 - s_{23}c_4s_5$

2(a)

Figure 2a. Kawasaki RS06L robot model and



2(b)

Figure 2b. link diagram of RS06L [Kawasaki Robotics Pvt. Ltd.]

$$p_x = c_1(a_2c_2 - d_4s_{23})$$
$$p_y = s_1(a_2c_2 - d_4s_{23})$$
$$p_z = a_2s_2 + d_4c_{23})$$

Table 1. Link parameters of Kawasaki RS06L robot

i	α_{i-1}	a_{i-1}	d_i	θ_i	Parameter value
1	0	0	0	θ_1	a_2=650mm
2	90	0	0	θ_2	d_4=900mm
3	0	a_2	0	θ_3	
4	-90	0	d_4	θ_4	
5	90	0	0	θ_5	
6	90	0	0	θ_6	

In order to ensure the position of the end-effector, the joint variables need to be managed. For the linear motion of ‘P’, evaluation a Riemannian metric will be done for the end-effector position as well as orientation. Then by generating the Christoffel symbols from the Riemannian metric, the geodesics are computed. The Riemannian metric can be described by:

$$g = (dP)^2 + (dn)^2 + (do)^2 + (da)^2 = (d\theta_1 \quad d\theta_2 \quad d\theta_3 d\theta_4 \quad d\theta_5 \quad d\theta_6)G\begin{pmatrix} d\theta_1 \\ d\theta_2 \\ d\theta_3 \\ d\theta_4 \\ d\theta_5 \\ d\theta_6 \end{pmatrix} \tag{12}$$

Where, G=$(g_{ij})_{6x6}$ is the coefficient matrix conforming to ‘g’.

$$G = (g_{ij})_{6x6} = \left(\begin{array}{ccc|ccc} (d_4s_{23} - a_2c_2)^2 + 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_2{}^2 + d_4{}^2 - 2a_2d_4s_3 + 2 & d_4{}^2 - a_2d_4s_3 & 0 & 0 & 0 \\ 0 & d_4{}^2 - a_2d_4s_3 & d_4{}^2 + 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 2c_5 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2c_5 & 0 & 2 \end{array}\right) \tag{13}$$

The geodesic, conforming to above metric is computed by the equation (5) and (6). The geodesic equation (5) regarding to the Riemannian metric ‘g’ is a second-order differential equation group. The geodesic is solely determined by given initial conditions or boundary conditions.

Considering the linear moment of end-effector that is steered from an initial point to a final point, the coordinates of end points in Cartesian space and joint space are given in Table 2 and Table 3 respectively. The robot accomplishes a linear motion according to the planned geodesic. The trajectories of the end-effector with its position, velocity, acceleration, and jerk obtained from the geodesic solution are presented in Fig 3 and Fig 4. The trajectories for the orientation vector of the robot end-effector are represented in Fig 5. The joint trajectories of Kawasaki RS06L as obtained from geodesic method are represented in Fig 6. From the results as presented through figures, it can be observed that all the joint trajectories of the end-effector as well as position, velocity, acceleration and jerk profiles of joint motion are smooth.

Table 2. Coordinates of end points in Cartesian space

	Initial state in mm (t=0 sec)	Final state in mm (t=5 sec)
p	(-194.1970, -1196.2542, -924.989403)	(304.7006, -1247.4042, -831.73297)
n	(-8.3173297, 1.122066, -0.209403)	(-0.664937, 0.969651, -0.29735)
o	(0.90235, 0.68201, -0.10494)	(0.73957, 0.24437, -0.13394)
a	(-0.00457, -0.23418, -0.97218)	(0.10437, -0.30897, -0.94532)

Table 3. Coordinates of end points in joint space

θ_s	Initial state (t=0 sec) in rad	Final state (t=5 sec) in rad
θ_1	-1.73173	-1.33122
θ_2	-0.43816	-0.38488
θ_3	-1.93838	-1.89743
θ_4	0.06514	0.05567
θ_5	-0.53183	-0.52889
θ_6	-0.65825	-0.55279

5. RESULTS OF THE SIMULATION

The method proposed in this paper has been verified in simulations for the Kawasaki RS06L robot. The simulation results confirm the accuracy, smoothness and the optimality of the end-effector motion, which are represented in Figure 3 and Figure 4 Similarly, the accuracy as well as smoothness of end-effector orientation vector is represented in Figure 5 The joint trajectories and their derivatives are found to be smooth, which is presented in Figure 6. Hence the method can be implemented for smooth and accurate trajectory planning for end-effectors and the relevant joint trajectories of the robot manipulators.

6. CONCLUSION AND FUTURE WORK

An optimal trajectory planning with smooth and accurate movement for robot manipulators combining both position and orientation space by implementing geodesic method is presented in this paper. The key intent is to assign an appropriate metric to acquire the necessary geodesic motion. The geodesic equations are solved numerically by simultaneous Runge-Kutta 4th step method and the results are used to control the robot. Geodesic simulations of the results using the Kawasaki RS06L robot are presented. The presence of obstacle in geodesic formulation extends the process of finding geodesic from initial point to obstacle and again from obstacle to final target point. The proposed method will have a high impact on the robotic welding and machining application, where the end-effector has to perform a smooth and accurate motion in least duration considering the trueness of geodesic method to optimization, smoothness and accuracy.

The method concerns to robots with less than or equal to six DOFs and focuses exclusively on linear motions. Future work will be focusing on trajectory planning of complex robots having more than six DOFs with different types of non-linear motion.

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