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Discrete-time Inversion Model Control of a Double-damper System with Uncertain Parameters

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ABSTRACT

This paper addresses the control at discrete time of physical complex systems multi-inputs multi-outputs with variables parameters. Classified among the robust control laws the Internal Model Control (IMC) is adopted in this work to ensure the desired performances adjacent to the complexities of the system. However, the application of this control strategy requires that these different building blocks be open loop stable, which invites us, on the one hand, to apply the algebraic approach of Kharitinov for delimiting the summits stability domain's system. On the other case, the Linear Matrix Inequalities (LMI) approach is applied to determine the corrector's stability conditions obtained by a specific inversion of the chosen model. It is in this sense that we contribute by this work to execute the command by inversion the discrete-time model in order to ensure the stability and to maintain the performances the stability conditions of required for the double damper system with variable parameters.

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1. INTRODUCTION

The Complex engineering systems are frequently multivariable [1]. They have more than one control input and more than one output. In this work, we limit ourselves to the study of systems having the same number of input-outputs and functionally controllable. The objective of the command is to have an acceptable behavior of several output variables simultaneously by the manipulation of several inputs. The realization of these control laws is based on the modeling of systems.

The design of a servo control is generally carried out from a model of the real system often called nominal model. The latter is only an approximation of reality. It may have various deficiencies among which include the modeling uncertainties. It is therefore necessary to optimize the control with respect to the model ensuring against its uncertainties [2] and [3].

The Internal Model Control (IMC) introduced by Garcia and Morari in the 1970s is a robust control structure commonly exploited for its control performance [4]. It's presented as an alternative to the classic closed loop. The IMC [5] command applied simultaneously to the process and its model (in the monovariable or multivariable, linear or nonlinear case). Their behavioral gap is used to correct the error on the reference signal. The error signal includes the influence of external disturbances and the modeling errors of the controlled system. In the IMC structure, the controller is assumed to be the inverse of the model associated to the plant. From where the need to study the problems related to this reversal because it is physically impossible in most cases (delay problems, not minimum phase or non-relative non-zero degree...).

Faced with the industrial necessities and the rapid progress of electronics that have generated considerable synthesis in the field of control by computers. It was essential to develop this strategy (IMC) in the discrete case and which was the study's subject of several works [6], [7], [8], and [9].

The application of IMC based on a linear modeling of the process. Such a model doesn't, in many cases, fully describe the behavior of the process (neglected dynamics, ignorance or variations in process parameters). This leads us to study the case of a chosen model having different transfer matrix of the process. The proposed IMC control structure mustn't only impose the system response but also maintain its behavior in the face of parametric uncertainties and external perturbations, despite the imperfections of the model. It is at this level that the use of a study strategy to verify stability conditions is required, namely the Kharitinov theorem [7], applied for systems with bounded parametric uncertainties.

In this paper, we intend to check the robustness of the control with respect to parametric uncertainties, which may be due to the sensors's precision, the frictional forces and the unpredictable external factors. This work includes the application of the LMI approach [8], in the synthesis phase of the IMC regulator obtained by specific inversion of the multivariable model will be used to ensure its stability.

It's in this sense, we approach this work by modeling the uncertain parameter systems and presentation of Kharitinov's theorem in the discrete case then we develop the IMC control structure in the MIMO case, where we will focus on the establishment of its regulator whose synthesis leads us to apply the LMI method. The aim of this work contribute to the regulation by inversion model MIMO [8] and [9] of the double damper system with uncertain parameters

2. IMC STRUCTURE PROPOSED FOR MIMO LINEAR SYSTEMS

In the IMC, the synthesis of a corrector that is equal to the direct inverse despite of the physical system's complexities of the transfer matrix in the multivariate case is principal in order to ensure perfect Instructions. Yet, directly seversal is virtually impossible particularly. We propose to develop the method of realization of an approximate inverse, inspired by the work of [5], in the case of multivariable linear systems.

2.1. Structure of the proposed IMC regulator

The structure of the regulator proposed in the case of monovariable systems [5] and [10] is extended to multivariate linear systems having the same number of input-outputs [11]. It is presented in Figure 1.
There are:

- m: the number of system inputs, outputs;
- A_1: a square inversion matrix, to choose of dimension (m×m)
- M(z): the multivariate system transfer matrix of dimension (m×m)
- e: the input vector of the dimension regulator (m×1)
- u: the dimension control vector (m×1)

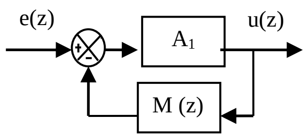

Figure 1. Generalized controller structure C(z)

According to the diagram in Figure 1, the controller transfer matrix can be expressed by the next equation (1):

$$C(z) = ue^{-1} = (I_m + A_1 M(z))^{-1} A_1 = (A_1^{-1} + M(z))^{-1} \quad (1)$$

With I_m is the identity matrix of dimension m.

The inversion matrix K_1 is an invertible square matrix. It must ensure the regulator's stability discussed later.To simplify our study, we can choose A_1 of the form $A_{1,} = \alpha \times I_m$ with $\alpha \in \mathbb{R}^{+}$. For A_1 (chosen such that α took sufficiently high thus to approximate $\left(A_1^{-1} + M(z)\right)^{-1}$ into $M(z)^{-1}$. In this case C(z) can be considered as an approximate inverse matrix (2) of the transfer matrix M(z) (2):

$$C(z) \approx M(z)^{-1} \tag{2}$$

2.2. The stability's study of the proposed regulator

The regulator C(z) have the following form (3):

$$C(z) = \frac{t_{com}\left(I_m + A_1 M(z)\right)A_1}{\det\left(I_m + A_1 M(z)\right)} \tag{3}$$

The chosen model M (z) must be stable, to garantee the stability of the regulator C(z), the matrix and the sampling period T applied must ensure the stability of the regulator C(z).

2.3. The regulator's precision

The matrix of the static gains of the regulator C (1) is defined by the following equation (4):

$$C(1) = \left(I_m + A_1 M(1)\right)^{-1} A_1 \tag{4}$$

The precision is ensured for (5)

$$K_1 \approx 1 : C(1) \approx M(1)^{-1} \tag{5}$$

2.4. The IMC's structure proposed

The IMC structure use explicitly the model as a controller algorithm of the plant that is stable in open loop. In this case, the inverse model can obtain the controller (6-9). The IMC structure for multivariable discrete-time system is shown in Figure 2.

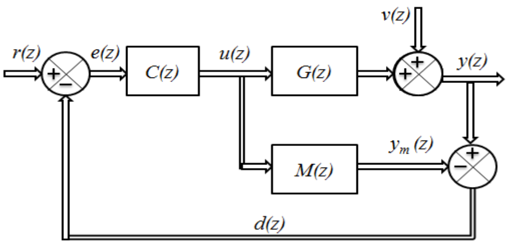

Figure 2. Internal Model Control design

G(z): the process
y(z): the Output vector of the process
v(z): the disturbance vector
y_m (z): the model output vector
r(z): the reference vector

e(z): the reference vector
u(z): the control vector
d(z) : the difference between the outputs of the model and the process one

$$u(z) = \left(I_m + \left(I_m + A_1 M(z)\right)^{-1} A_1 \left(G(z) - M(z)\right)\right)^{-1} \left(I_m + A_1 M(z)\right)^{-1} A_1 \left(r(z) - v(z)\right) \tag{6}$$

$$y(z) = y_r(z) r(z) + y_v(z) v(z) \tag{7}$$

$$y_r(z) = G(z)\left(I_m + \left(I_m + A_1 M(z)\right)^{-1} A_1 \left(G(z) - M(z)\right)\right)^{-1} \left(I_m + A_1 M(z)\right)^{-1} A_1 \tag{8}$$

$$y_v(z) = I_m - G(z)\left(I_m + \left(I_m + A_1 M(z)\right)^{-1} A_1 \left(G(z) - M(z)\right)\right)^{-1} \left(I_m + A_1 M(z)\right)^{-1} A_1 \tag{9}$$

If we assume that the process is not subjected to any perturbation and in the case of a perfect modelization, then the expression of the output (7) is reduced to the following equation (10):

$$y(z)=G(z)\left(I_m+A_1G(z)\right)^{-1}A_1r(z) \tag{10}$$

2.5. The precision

For the perfect modeling and after the output vector of the process, there can be defined the matrix B such that (11):

$$B=G(1)\left(I_m+A_1G(1)\right)^{-1}A_1 \tag{11}$$

with G (1) is the matrix of the static gains of the process G. For high values of α, we obtain $B \approx I_m$, which allows for a gap asymptotically zero between the vector of outputs and references.

2.6. The rejection of external disturbances

The attached output vector of an external disturbance for a perfect modelling is written in the following form (12):

$$y_v(z)=I_m-G(z)\left(I_m+A_1G(z)\right)^{-1}A_1 \tag{12}$$

Which generates an output vector $y_v \approx 0$ for sufficiently high values of α.

3. THE SYSTEM'S PARAMETRIC UNCERTAINTY AND ROBUST CONTROL STUDY

It's envisaged to check the robustness of the control with respect to the parametric uncertainties, which may be due to the precision of the sensors, the frictional forces and the unpredictable external factors which were not taken into account during the modeling of the system.

3.1. The precision

In the case of a continuous system, and taking into account the presence of parametric uncertainties at the level of the element $G_{ij}(p)$ of the transfer matrix G of the process, $G_{ij}(p)$ is written in the following form (13):

$$G_{ij}(p)=\left(\sum_{i=0}^{m}(b_k\pm\Delta_k)p^i\right)\backslash\left(\sum_{j=0}^{n}\left(a_l\pm\Delta_l^{'}\right)p^j\right),\quad m\le n \tag{13}$$

with Δ_k and $\Delta_l^{'}$ are the parametric uncertainties respectively b_k and a_l.

3.2. Stability of the uncertain process/ apply of the Kharitinov'

From the expression of the output of the system (14):

$$y(p)=\frac{A_1N(p)r(p)+D(p)v(p)}{\Gamma(p)+\lambda(p)} \tag{14}$$

$$N(p)=\sum_{i=0}^{m}\left(b_i\pm\Delta_i\right)p^i \tag{14.1}$$

$$D(p)=\sum_{j=0}^{m}\left(a_j\pm\Delta_j^{'}\right)p^j \tag{14.2}$$

$$\Gamma(p)=\sum_{i=0}^{m}\left[A_1\left(b_i\pm\Delta_i\right)+\left(a_j\pm\Delta_j^{'}\right)\right]p^i \tag{14.3}$$

$$\lambda(p)=\sum_{j=m+1}^{n}\left(a_j \pm \Delta_j^{'}\right)p^j \tag{14.4}$$

From the last equations, the stability of the controlled process depends on the controller structure and the value's gain A_1. To ensure the process's stability we are interested in the determination of the non-localized extreme models using the indirect method based on the algebraic Kharitonov's approach [2], [7] and [10] which has nooperating points or predetermined areas of validity. Let consider the case of continuous-time process, whose evolution is described by a differential equation of the form (15):

$$b_0(.)y+b_1(.)y^{(1)}+...+b_n y^{(n)}=a_0(.)u+a_1(.)u^{(1)}+...+a_m(.)u^{(m)} \tag{15}$$

The symbol (.) represents the set of variables, uncertainties, noise or disturbances affecting the coefficients of the process such as (16):

$$\begin{cases}\overline{a}_i=\max(a_i);\underline{a}_i=\min(a_i)\\ \overline{b}_i=\max(b_i);\underline{b}_i=\min(b_i)\end{cases} \tag{16}$$

This method envisages considering the four extreme models defined by the following transfer functions:

$$H_1(p)=\frac{\underline{a}_0+\underline{a}_1p+\overline{a}_2p^2+\overline{a}_3p^3+...}{\underline{b}_0+\underline{b}_1p+\overline{b}_2p^2+\overline{b}_3p^3+...} \tag{16.1}$$

$$H_2(p)=\frac{\underline{a}_0+\overline{a}_1p+\overline{a}_2p^2+\underline{a}_3p^3+...}{\underline{b}_0+\overline{b}_1p+\overline{b}_2p^2+\underline{b}_3p^3+...} \tag{16.2}$$

$$H_3(p)=\frac{\overline{a}_0+\overline{a}_1p+\underline{a}_2p^2+\underline{a}_3p^3+...}{\overline{b}_0+\overline{b}_1p+\underline{b}_2p^2+\underline{b}_3p^3+...} \tag{16.3}$$

$$H_4(p)=\frac{\overline{a}_0+\underline{a}_1p+\underline{a}_2p^2+\overline{a}_3p^3+...}{\overline{b}_0+\underline{b}_1p+\underline{b}_2p^2+\overline{b}_3p^3+...} \tag{16.4}$$

$H_5(p)$ is often useful to add the average model, denoted as a fifth model in the library [7]. The latter. The transfer function of the fifth sample is given by (17):

$$H_5=\frac{a_{5.0}+a_{5.1}p+a_{5.2}p^2+a_{5.3}p^3+...}{b_{5.0}+b_{5.1}p+b_{5.2}p^2+b_{5.3}p^3+...} \tag{17}$$

Such as:

$$a_{5.i}=\frac{\underline{a}_i+\overline{a}_i}{2} \text{ and } b_{5.i}=\frac{\underline{b}_i+\overline{b}_i}{2}$$

To the discrete case and to check the domain of stability, we adopt the geometric method [13] [14] and [15] which have to calculate the distance (18) between the output vector $y_{s,k}$ of the system and r partial exits $y_{i,k}$ base model for (18)

$$i=1,2,....,r: D_{i,k}=\left\|y_{i,k}-y_{s,k}\right\| \tag{18}$$

The normalized distances $\left|D_{i,k}\right|$ are given by (19):

$$\left|D_{i,k}\right| = \frac{D_{i,k}}{\sum_{j=1}^{r} D_{j,k}} \tag{19}$$

The Validities are given by (20):

$$V_{i,k} = \frac{t_{i,k}}{\sum_{j=1}^{r} t_{j,k}} \quad \text{Satisfying the conditions of convexity:} \begin{cases} 0 \le V_{i,k} \le 1 \\ \sum_{i=1}^{r} V_{i,k} = 1 \end{cases}$$
$$t_{j,k} = \left(1 - \sum_{j=1}^{r} \left|D_{i,k}\right|\right) \prod_{j=1}^{r} \left[1 - \exp\left(-\left(\frac{\left|D_{i,k}\right|}{\eta}\right)^2\right)\right] \tag{20}$$

η is a variable parameter set between 0 and 0.99

3.3. The controller's stabilization condition: the LMI approach

In this section we apply the LMI [8], [16], in the synthesis phase of the regulator to check its stability in open loop. Consider a discrete-time MIMO system represented by its state space (21):

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) \\ y(k) = Cx(k) \end{cases} \tag{21}$$

x(k), u(k) and y(k), are respectively state, input and output vectors such that $x(k) \in \mathbb{R}^n, u(k) \in \mathbb{R}^m$ and $y(k) \in \mathbb{R}^p$ and matrices A, B and C are known constant matrices. The system is represented by equation (21), is asymptotically stable if: $\lim_{k\to\infty} x(k) = 0,\ \forall x_0 \neq 0$.

The system (21) is stable in the Lyapunov sense [17], if there exists a quadratic Lyapunov function represented by equation (22):

$$V(k) = x^T(k)Px(k) > 0 \tag{22}$$

it comes back to get $\max_i \left\|\lambda_i(A)\right\| < 0$ if and only if there exists a symmetric matrix $P = P^T > 0$. $A \in \mathbb{R}^{n \times n}$ is constant. After derivating, the quadratic Lyapunov function V of the system in (21) her form become:

$$\Delta V(x(k)) < 0 \Leftrightarrow V(x(k+1)) - V(x(k)) < 0 \tag{23}$$

which leads us to:

$$\Delta V(x(k)) = V(x(k+1)) - V(x(k)) < 0 \tag{24}$$

If and only if:

$$\left(A^T PA - P\right) < 0 \tag{25}$$

A is given matrices of appropriate sizes and P is the variable. The system (21) is stable if there exists a matrix $P \in \mathbb{R}^n$ such that the following LMI (Linear Matrix Inequality) is feasible: $P > 0$, $\left(A^T PA - P\right) < 0$ (26). In this work we extend the study developed in [11], to the discretization of multivariable systems witn uncartain parameters. The plant G(s) and the model M(s) are discretized bye the bilinear method .The LMI approach is used in this work to guarantee the quadratic stability of the controller C(z).

4. RESULTS AND ANALYSIS

In order to validate the proposed internal model control for multivariable uncertain linear systems, let us consider the example of the double-damper system of a car. Designing an automotive suspension system is an interesting and challenging control problem. The suspension system is designed by 1/4 model (one of the four wheels). We used to simplify the problem to a 1D multiple spring-damper system. A diagram of this system is shown in Figure 3.

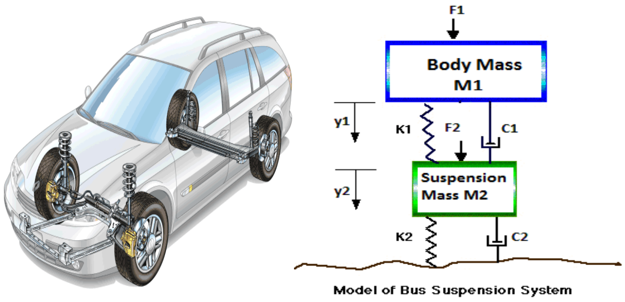

Figure 3. Structure of a double-damper system of a car (1/4 model)

The different parameters of the process are presented as follows.

M1:	body mass	kg
M2:	suspension mass	kg
K1:	spring constant of suspension system	N/m
K2:	spring constant of the wheel and tire	N/m
C1:	damping constant of suspension	N.s/m
C2:	damping constant of wheel and	N.s/m
F1, F2 :	external forces	N
y1, y2 :	system outputs (displacements)	m

This system is a two inputs two outputs system and it is represented by the following equations

$$\begin{cases} M_1\ddot{y}_1 = F_1 - K_1(y_1 - y_2) - C_1(\dot{y}_1 - \dot{y}_2) \\ M_2\ddot{y}_2 = F_2 + K_1(y_1 - y_2) - C_1(\dot{y}_1 - \dot{y}_2) - K_2\, y_2 - C_2\dot{y}_2 \end{cases} \tag{27}$$

This system will be modeled by calculating the forces acting on both masses (body and suspension).Then; we applied the Newton's law to each mass. The transfer matrix of the outputs of the system is expressed by:

$$\begin{bmatrix} y_1 \; y_2 \end{bmatrix} = [G(s)]\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \tag{28}$$

The system can be arranged in the following state-space model and represented as

$$G(s) = \begin{pmatrix} \dfrac{C_1 s + K_1}{DEN} & \dfrac{M_1 s^2 + C_1 s + K_1}{DEN} \\ \dfrac{M_2 s^2 + (C_1 + C_2)s + K_1 + K_2}{DEN} & \dfrac{C_1 s + K_1}{DEN} \end{pmatrix} \tag{29}$$

$$DEN = M_1M_2s^4 + M_1(C_1 + C_2) + C_1M_2)s^3 + (M_1(K_1 + K_2) + C_1(C_1 + C_2)$$
$$+ K_1M_2 - C_1^2)s^2 + (C_1(K_1 + K_2) + K_1(C_1 + C_2) - 2C_1K_1)s + K_1(K_1 + K_2) - K_1^2$$

The uncertain parameters are:

M1: 10±50% kg ; M2: 500±50% kg

K1: 2000±50% N/m; K2: 2000±50% N/m
C1: 500±50% N.s/m ; C2: 500±50% N.s/m

The reference signals r_1, r_2 are chosen as vector of steps of amplitude equal to 10^{-5}.

4.1. Case of Imperfect modelling without disturbances

Let's consider the imperfect modeling characterized by the absence of disturbances, such that v(z) =0 where the model is chosen diffrent to the plant $M(z)\neq G(z)$ and the sampling time is equal to T=0.2 s. The chosen matrix A_1 is equal to $A_1=50\times I$. The two outputs y_1 and y_2 are shown in Figure 4 and Figure 5.

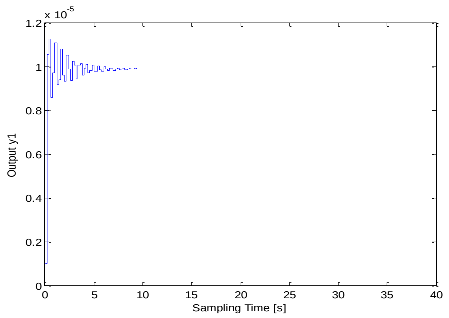

Figure 4. Output y_1 for non disturbed IMC control

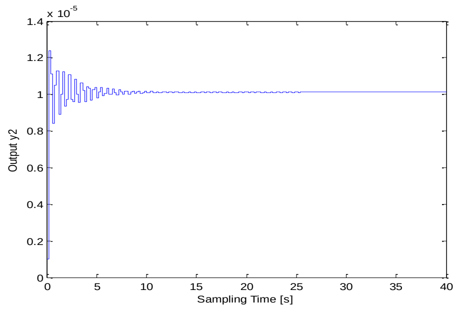

Figure 5 Output y_2 for non disturbed IMC control

It is clear that the system outputs reach perfectly the input reference. The IMC applied of the double-damp system is maintaining the stability of the chosen discrete model despite the presence of uncertainty parameters.

4.2. Case of disturbed system

Now let's consider the presence of a disturbance vector and let's show its effect in the case of the IMC proposed for the double-damp system control. The disturbances are applied at the time T=15s. A_1 is considered as the same at last method. simulations results are shown in Figure 6 and Figure 7.

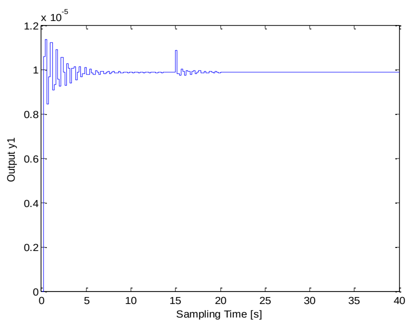

Figure 6. Output y_1 of disturbed system

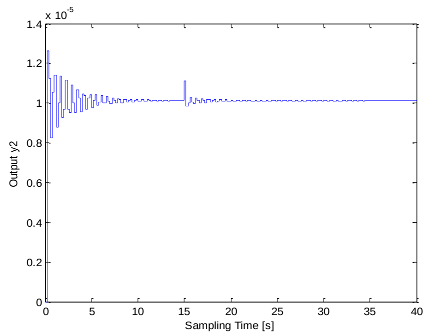

Figure 7. Output y_2 of disturbed system

The simulations show a robust behavior even on the presence of disturbances affecting directly the process outputs. We conclude that the proposed IMC for the multivariable uncertain double-damp system rejects disturbances and ensure again its robustness. The LMI approach is used in this work to guarantee the quadratic stability of the controller. LMIs has been performed in MATLAB environment. Solving the LMI in equation (25), we obtain a matrix P of dimension (24×24), this matrix ensures the stability of our system for $50<A_1<700$.

Then we adopt the algebraic Kharitonov's approach, which is based on the calculation of the distance (18) between the output vector y of the system and r partial exits y_i base model.The Table (1) presents the different values between the outputs of the process G(z) and the model ones taken during the time of simulation.

Table 1. The Performance of the Kharitinov's method for verifing stability

Variable	t=0s	t=5s	t =10s	t =15s	t =20s	t =25s	t = 30s	t =35s	t =40s
y-y1	0.22	0.086	0.04	0.034	0.028	0.004	0.0034	0.0013	0.0001
y-y2	0.5	0.1	0.3	0.43	0.31	0.04	0.054	0.014	0.0021
y-y3	0.44	0.38	0.321	0.33	0.24	0.071	0.002	0.0017	0.001
y-y4	0.399	0.291	0.21	0.351	0.314	0.004	0.003	0.0015	0.002

5. CONCLUSION

In this work, a new approach for IMC of linear multivariable uncertain systems is developed in discrete-time. The realized research is an extension of the IMC concept defined for discrete multivariable uncertain systems. An application of a double-damp system with uncertain parameters is proposed to test the effectiveness of the control despite the presence of disturbances and uncertainties. The chosen system is a two-input-two-output linear system. The simulation results show the proposed approach capability to preserve the system stability and performances on preserving the rejection of the external disturbances.

REFERENCES

[1] H. Trebiber, J. 1984. Multivariable Control of Non-square Systems. Industrial & Engineering Chemistry. Process Design and Development, vol. 23, no. 4, pp. 854-857.

[2] Kardous Khaldi Z "Sur la modélisation et la commande multimodèle des processus complexes et/ou incertains", PhD Thesis, USTL, Décembre 2006 (in French).

[3] Saeed Salvati, Mongi Behrouz Ebrahimi., Karlos Grigoriadis and Mathew Franchek, "*internal model control for a class of uncertain time-delay systems"*, ACC 2016, July 6-8, 2016. Boston, MA, USA.

[4] R. Arulmozhiyal and K. Baskaran, "Implementation of a Fuzzy PI Controller for Speed Control of Induction Motors Using FPGA," *Journal of Power Electronics*, vol. 10, pp. 65-71, 2010.

[5] Mohamed Benrejeb, Mongi Naceur., and Dhaou Soudani., "*On an internal mode controller based on the use of a specific inverse model"*, ACIDCA 2005, pp. 623-626, Tozeur, 2005.

[6] Ikbel Ben Cheikh Ahmed, Dhaou Soudani, Mongi Naceur and Mohamed Benrejeb, "*Sur la commande stabilisante par modèle interne de systèmes échantillonné*", *JETA 2008.*

[7] Kharitonov V.L. "Asymptotic stability of an equilibrum position of a family of system of linear differential equation", Differential, Uravnen, Vol 14, 1978.

[8] S. Boyd, L. El Ghaoui, E. Feron & V. Balakrishnan, "Linear Matrix Inequalities in System and Control Theory". Philadel-phia: SIAM Press, 1994.

[9] J. Chen, B. Zhang & X. Qi, 2011. "A new control method for MIMO first order time delay non-square systems". Journal of Process Control.21(4), pp: 538-546.

[10] M. Naceur, F. 2008. "Sur la Commande par Modèle Interne des Systèmes Dynamiques Continus et Echantillonnés". Thèse de doctorat, Ecole Nationale d'Ingénieurs de Tunis.

[11] Marwa Hannachi, Dhaou Soudani, "Internal Model Control of Multivariable Discrete-Time Systems", International Conference on Modelling, Identification and Control, ICMIC, Sousse, 2015.

[12] El Kamel A., Borne P., Ksouri-Lahmari M. and Benrejeb M. "On the stability of nonlinear multimodel systems", SACTA, Vol. 2, N° 1-2, pp. 40-52.

[13] Ghorbel C., Abdelkrim A.andt Benrejeb M. "An adaptive fuzzy control of continuous nonlinear systems". 6th International Multi- Conference on Systems, Signals and Devices, SSD, 23-26 mars2009, Djerba, Tunisie.

[14] Chapella H. and Bhattacharyya S. P. "A generalization of Kharitonov's theorem: Robust stability of interval plants". IEEE trans. on Automatic Control, vol. 34, n° 3, 306-311, 1989.

[15] Delmote F. "Analyse multimodèle". PhD Thesis, USTL, Lille, 1997.

[16] S. Dussy, J. 2000. "Robust Diagonal Stabilization: An LMI Approach". IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. 45, NO.

[17] A. Fossard, "Commande des systèmes multidimensionnels", Dunod, P.1972.

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