Deriving the system equations of unbalanced two-phase induction motor

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ABSTRACT
As there is no system driven especially for the two-phase induction motor fed from unbalanced two-phase supply yet, so we start for derivation the system equations for the said motor to be generally used even for the balanced or unbalanced two-phase supply. In this paper, we will derive a system equation starting from the sequence equivalent circuit for the forward and backwards equivalent circuits, then we will re-arrange the equations with some mathematical assumptions which will lead us to the new system equations. first for the voltage equations then for the current equations and finally for both power and torque equations. Moreover, we will put an example which will cover all cases with specific values and relations charts.

Keywords:
Forward and backward equivalent circuits
Performance of two-phase motor under unbalanced supply
Sequence circuits

1. INTRODUCTION
When the motor works under a general operating conditions of unbalanced stator voltages or windings, and if we start to derive the system equations, it is necessary to start from the known equivalent circuits which are obtained for both forward and back word components [1]. Using a special analysis which will be required in order to completely describe the two-phase equivalent circuit or the system of the voltage equations for the two-phase motor [2]. Using the relations between the sequence and phase values, to obtain the important system of voltage equations for the case of unbalanced system [3]. After the derivation of the system equations we can learn the performance of the unbalanced two-phase electrical motor, using the symmetrical components representation for both voltages and currents [4]. These system equations will be used for all cases for the two-phase induction motor which fed from two-phase power supply under all conditions [5].

2. RESEARCH METHOD
2.1. Equivalent circuits of forward and backward components
From the usual equivalent circuits of forward and backward components, the following equations for both components are shown in Figure 1. From the figure, we can express the voltage equations as the following:

\[ V_{sf} = (R_s + jx_{L2} + jx_m)I_{sf} + jx_m I_{rf} \]
\[ V_{rf} = jx_m I_{sf} + (jx_m + R_r/s)I_{rf} \]
\[ V_{sb} = (R_s + jx_{L2} + jx_m)I_{sb} + jx_m I_{rb} \]
\[ V_{rb} = jx_m I_{sb} + (jx_m + R_r/(2-s))I_{rb} \]
By the same manner, the phase voltages and currents of the two-phase induction machine in the general forms.

For voltage system equations using the symmetrical components, we use:

\[
V_{s1} = (V_{sf} + V_{sb}), \quad V_{s2} = -j(V_{sf} - V_{sb}),
\]

\[
V_{r1} = (V_{rf} + V_{rb}), \quad V_{r2} = -j(V_{rf} - V_{rb}),
\]

The similar relations for the currents are:

\[
I_{s1} = (I_{sf} + I_{sb}), \quad I_{s2} = -j(I_{sf} - I_{sb}),
\]

\[
I_{r1} = (I_{rf} + I_{rb}), \quad I_{r2} = -j(I_{rf} - I_{rb}),
\]

From the previous analysis, the above system of equations, can be used to obtain the following general system of equations for the phase voltages of the two-phase induction machine in the general forms.

\[
V_{s1} = (V_{sf} + V_{sb}) = (R_s + jX_s + jX_m) I_{sf} + jX_m I_{sf} + (R_s + jX_s + jX_m) I_{sb} + jX_m I_{sb}
\]

\[
= (R_s + jX_s + jX_m) (I_{sf} + I_{sb}) + jX_m (I_{sf} + I_{sb})
\]

using the previous current equations we get

\[
V_{s1} = (R_s + jX_s + jX_m) I_{s1} + jX_m I_{r1}
\]

By the same manner,

\[
V_{s2} = -j(V_{sf} - V_{sb}) = -j((R_s + jX_s + jX_m) I_{sf} + jX_m I_{sf} - (R_s + jX_s + jX_m) I_{sb} - jX_m I_{sb})
\]

\[
= ((R_s + jX_s + jX_m) - j(I_{sf} - I_{sb}) + jX_m (I_{sf} - I_{sb}))
\]

\[
V_{s2} = (R_s + jX_s + jX_m) I_{s2} + jX_m I_{r2}
\]

Figure 1. Single-phase equivalent circuits for a two-phase motor under unbalanced condition
(a) forward field and (b) backward field
For the other voltage equations

\[
V_{r1} = (V_{rf} + V_{rb}) \\
V_{rf} = jx_{m} I_{rf} + (jx_{m} + (R_{r} / s) I_{rf}) \\
V_{rb} = jx_{m} I_{rb} + (jx_{m} + (R_{r} / (2 - s)) I_{rb}) \\
V_{r1} = jx_{m} I_{r1} + (jx_{m} + (R_{r} / (2 - s)) I_{r1} + jx_{m} I_{ab} + (jx_{m} + (R_{r} / (2 - s)) I_{rb})
\]

We will divide the previous equation into two parts to solve them separately then we will collect them again:

\[
V_{r1} = [jx_{m} I_{rf} + jx_{m} I_{rb}] + [(jx_{m} + (R_{r} / s)) I_{rf} + (jx_{m} + (R_{r} / (2 - s)) I_{rb}]
\]

For the first part

\[
[jx_{m} I_{rf} + jx_{m} I_{rb}] = jx_{m} (I_{rf} + I_{rb}) = jx_{m} I_{r1}
\]

For the second part

\[
[(jx_{m} + (R_{r} / s)) I_{rf} + (jx_{m} + (R_{r} / (2 - s)) I_{rb}] = \\
= jx_{m} (I_{rf} + I_{rb}) + (R_{r} / s) I_{rf} + (R_{r} / (2 - s)) I_{rb} \\
= jx_{m} I_{r1} + ((2 - s) R_{r} / (s (2 - s))) I_{rf} + (sR_{r} / (s (2 - s))) I_{rb} \\
= jx_{m} I_{r1} + [(R_{r} / (s (2 - s))) . ((2 - s) I_{rf} + s I_{rb})] \\
= jx_{m} I_{r1} + (R_{r} / (s (2 - s))). [(1 + 1 - s) I_{rf} - (1 + 1 - s) I_{rb}] \\
= jx_{m} I_{r1} + (R_{r} / (s (2 - s))) [I_{rf} + I_{rb}] \\
= jx_{m} I_{r1} + (R_{r} / (s (2 - s))) [I_{r1} + j (1 - s)]
\]

Then for both parts

\[
V_{r1} = jx_{m} I_{r1} + [jx_{m} + (R_{r} / (s (2 - s)))] I_{rf} + j (1 - s) [(R_{r} / (s (2 - s)))] I_{rb}
\]

for the following equation

\[
V_{r2} = -j (V_{rf} - V_{rb}) \\
V_{rf} = jx_{m} I_{rf} + (jx_{m} + R_{r} / s) I_{rf} \\
V_{rb} = jx_{m} I_{rb} + (jx_{m} + (R_{r} / (2 - s))) I_{rb}
\]

as the same for the previous equation of the V_{r1}

\[
V_{r2} = -j [jx_{m} I_{rf} + (jx_{m} + R_{r} / s) I_{rf} - j x_{m} I_{rb} - (jx_{m} + (R_{r} / (2 - s)) I_{rb}] \\
= -j [jx_{m} (I_{rf} - I_{rb}) + j x_{m} (I_{rf} - I_{rb}) + R_{r} / s] . I_{rf} - (R_{r} / (2 - s)) I_{rb}] \\
= -j [(1 - x_{m}) I_{r2} - (x_{m} I_{r2}) + [R_{r} / s . I_{rf} - (R_{r} / (2 - s)) I_{rb}]
\]

for the second part of the equation

\[
R_{r} / s . I_{rf} - (R_{r} / (2 - s)) I_{rb} \\
= ((2 - s) R_{r} / (s (2 - s))) I_{rf} - (sR_{r} / (s (2 - s))) I_{rb} \\
= (R_{r} / (s (2 - s))) . ((2 - s) I_{rf} - s I_{rb}) \\
= (R_{r} / (s (2 - s))). [(1 + 1 - s) I_{rf} - (1 + 1 - s) I_{rb}] \\
= (R_{r} / (s (2 - s))) . [j (1 - s) (I_{rf} - I_{rb}) + (1 - s) . (I_{rf} + I_{rb})] \\
= (R_{r} / (s (2 - s))) . [j I_{r2} + j (1 - s) . I_{r1}]
\]

Then for both parts

\[
j V_{r2} = [(1 - s) . R_{r} / (s (2 - s))]. I_{r1} + (R_{r} / (s (2 - s))) . I_{r1} \\
+ j (R_{r} / (1 - s) / (s (2 - s))) . I_{r2}
\]

Then we drive the following system equations for voltages

\[
V_{r1} = (R_{r} + jx_{m} + j x_{m}). I_{r1} + jx_{m}. I_{r1}
\]

\[
V_{r2} = (R_{r} + jx_{m} + j x_{m}). j I_{r2} + jx_{m} j I_{r2}
\]

\[
V_{r1} = jx_{m} I_{r1} + [(jx_{m} + (R_{r} / (s (2 - s))) I_{r1} + j (1 - s) [(R_{r} / (s (2 - s))) . I_{r2}
\]

\[
V_{r2} = [(1 - s) . R_{r} / (s (2 - s))]. I_{r1} + (R_{r} / (s (2 - s))) + jx_{m}). I_{r1} \\
+ j (R_{r} / (1 - s) / (s (2 - s))) . I_{r2}
\]

Deriving the system equations of unbalanced 2-phase induction motor (Hany Ibrahim Shousha)
2.2.2. For current system equations

Using the previous equations for the single-phase equivalent circuits will give as the following:

\[ V_{sf} = Z_{sm} \cdot I_{sf} - Z_{mr} \cdot I_{rf} \]
\[ V_{rf} = Z_{mr} \cdot I_{rf} - Z_{sm} \cdot I_{sf} \]
\[ I_{rf} = \frac{ (Z_{sm} \cdot I_{sf} )}{ Z_{m} - V_{sf} / Z_{m} } \]

Also, according to the relation between current, impedance and voltage

\[ I_{rf} = \left( \frac{Z_{m}}{Z_{mr}} \right) \cdot I_{sf} \]

Then,

\[ I_{rf} = \left( \frac{Z_{m}}{Z_{mr}} \right) \cdot I_{sf} = \left( \frac{Z_{sm} \cdot I_{sf} )}{ Z_{m} - V_{sf} / Z_{m} } \right) \cdot I_{sf} \]
\[ V_{sf} / Z_{m} = \left( \frac{Z_{sm} / Z_{mr} - Z_{m} \cdot I_{sf} )}{ Z_{m} - V_{sf} / Z_{m} } \right) \cdot I_{sf} \]
\[ I_{rf} = \left[ \frac{Z_{mr} / ((Z_{sm} \cdot Z_{mr}) - Z_{m} \cdot I_{sf} )}{ Z_{m} - V_{sf} / Z_{m} } \right] \cdot V_{sf} \]

Now we get the first current equation as following

\[ I_{rf} = \left[ \frac{Z_{mr} / ((Z_{sm} \cdot Z_{mr}) - Z_{m} \cdot I_{sf} )}{ Z_{m} - V_{sf} / Z_{m} } \right] \cdot V_{sf} \]
\[ I_{rf} = \left[ (Z_{m} + j X_{m} + (R_f / S_f)) / ((Z_{m} + j X_{m} + (R_f / S_f)) - Z_{m}^2 \right] \cdot V_{sf} \]

Using the following equation

\[ V_{rf} = Z_{mr} \cdot I_{rf} - Z_{m} \cdot I_{sf} = 0 \]

Then we get

\[ I_{rf} = \left[ \frac{Z_{m} / ((Z_{sm} \cdot Z_{mr}) - Z_{m} \cdot I_{sf} )}{ Z_{m} - V_{sf} / Z_{m} } \right] \cdot V_{sf} \]
\[ I_{rf} = \left[ \frac{Z_{mr} / ((Z_{sm} + Z_{m}) \cdot Z_{m} + j X_{m} + (R_f / S_f)) - Z_{m}^2 \right] \cdot V_{sf} \]

Then we can express for the forward as the following

\[ I_{rf} = \left[ (Z_{m} + j X_{m} + (R_f / S_f)) / ((Z_{m} + j X_{m} + (R_f / S_f)) - Z_{m}^2 \right] \cdot V_{sf} \]
\[ I_{rf} = \left[ \frac{Z_{mr} / ((Z_{sm} + Z_{m}) \cdot Z_{m} + j X_{m} + (R_f / S_f)) - Z_{m}^2 \right] \cdot V_{sf} \]

For the backwards,

\[ I_{rb} = \left[ (Z_{m} + j X_{m} + (R_f / S_f)) / ((Z_{m} + j X_{m} + (R_f / S_f)) - Z_{m}^2 \right] \cdot V_{sb} \]
\[ I_{rb} = \left[ \frac{Z_{mr} / ((Z_{sm} + Z_{m}) \cdot Z_{m} + j X_{m} + (R_f / S_f)) - Z_{m}^2 \right] \cdot V_{sb} \]

To obtain the air gap power equation

\[ I_{rf} = (I_{r1} + j \cdot I_{r2}) / 2, \quad I_{rb} = (I_{r1} - j \cdot I_{r2}) / 2 \]

For the \( P_{gf} \) we use the expression

\[ P_{gf} = m_i I_{rf} (I^*_{rf} R_f / S_f) \]
\[ 2 \cdot I_{rf} = I_{r1} + j \cdot I_{r2} + I_{r1} - j \cdot I_{r2} \]
\[ = (I_{r1} - I_{r2}) + j (I_{r1} + I_{r2}) \]

Then

\[ 2 \cdot I_{rf} = (I_{r1} - I_{r2}) + j (I_{r1} + I_{r2}), \quad 2 \cdot I^{*}_{rf} = (I_{r1} - I_{r2}) - j (I_{r1} + I_{r2}) \]
\[ P_{gf} = m_i I_{rf} (I^*_{rf} R_f / S_f) \]
\[ = [(m_i \cdot R_f) / 4 s) \cdot (((I_{r1} - I_{r2}) + j (I_{r1} + I_{r2})) \cdot (I_{r1} - I_{r2}) - j (I_{r1} + I_{r2})) \]
\[ = [(m_i \cdot R_f) / 4 s) \cdot ((I_{r1} - I_{r2})^2 + (I_{r1} + I_{r2})^2) \]
\[ = [(m_i \cdot R_f) / 4 s) \cdot (I_{r1} - 2 \cdot I_{r1} \cdot I_{r2} \cdot I_{r2} + I_{r1}^2 + 2 \cdot I_{r1} \cdot I_{r2} + I_{r2}^2)] \]

And for, \( I_{rb} \)

\[ P_{gb} = m_i I_{rb} (I^*_{rb} R_f / S_b) \]
\[ I_{rb} = (I_{r1} - j \cdot I_{r2}) / 2. \]
With the same manner
\[
2 I_{th} = I_{th} + j I_{th1} - j I_{th2} + I_{th2} \\
= (I_{th1} + I_{th2}) + j (I_{th1} - I_{th2})
\]
then:
\[
2 I_{th} = (I_{th1} + I_{th2}) + j (I_{th1} - I_{th2}) \quad 2 I_{th}^- = (I_{th1} + I_{th2}) - j (I_{th1} - I_{th2})
\]
\[
P_{gb} = m_i I_{th} I_{th}^* R_t / s_b
\]
where
\[
P_g = P_{gf} - P_{gb}
\]
\[
= \left[\left(\frac{m_i}{2} R_t / 4 s_0\right) . \left(\left(I_{th1}^2 + 2 I_{th1} I_{th2} + I_{th2}^2 + 2 I_{th1} I_{th2} + I_{th2}^2\right)\right)\right] - \\
\left[\left(\frac{m_i}{2} R_t / 4 s_0\right) . \left(\left(I_{th1}^2 + 2 I_{th1} I_{th2} + I_{th2}^2 + 2 I_{th1} I_{th2} + I_{th2}^2\right)\right)\right]
\]
\[
= \left[\left(\frac{m_i}{2} R_t / 2 s_0\right) . \left(I_{th1}^2 + I_{th2}^2 + I_{th2}^2 + 2 \left(I_{th1} I_{th2} + I_{th2}^2\right) + (1-\sigma) \right) . \left(2 I_{th1} I_{th2} + 2 I_{th1} I_{th2} + I_{th2}^2\right)\right]
\]
For the power equations
\[
P_{gf} = m_i I_{th} I_{th}^* R_t / s_b
\]
\[
P_{gb} = m_i I_{th} I_{th}^* R_t / s_b
\]
where, \(s_i = s\) and \(s_b = 2-s\)
\[
P_g = P_{gf} - P_{gb}
\]
\[
P_m = P_g \cdot (1-\sigma)
\]
\[
T = P_g / o_s
\]
where \(o_s = o / p\), with \(p = \) number of pole pair.

2.3. Performance of two-phase motor under unbalanced supply
Case study for different values of the phase angle of the second voltage source uses Matlab software for simulation the results according to different values of the voltage angle values (90° 60° 30° 15° 0°) and other assumptions as the following

\[
V=220 \, v \, ac, \, f=50 \, Hz, \, p=2, \, w=\pi*\pi, \, w=w. / p, m=2, V_i=v, \\
x_o=40, \, L_o=1, \, t_o=2, \, D_x=2, \, D_x=x, \, \tau=2, \, D_x=2, \, D_x=x, \, \tau=2, \\
no=1.0, \, Bv=20 \, (pi.z/2), \, .1/1.0, \, vs=2, \, v, \, ((cos(Bv2))-(j*sin(Bv2))), \\
s=0.00:0.001:1, \, s_i=s, \, s_o=2-s
\]

From the previous values and using Matlab we can get the result as in Figures 2–5.
3. **RESULTS AND DISCUSSION**

The target achieved, as we derive a system equations of unbalanced two-phase induction motor could be applied for any running conditions of the motor, even it runs in balanced or un-balanced conditions according to the following equations:
\[ V_{s1} = (R_1 + j x_{L1} + j x_m) \cdot I_{s1} + j x_m \cdot I_{s2} \]
\[ V_{s2} = (R_2 + j x_{L2} + j x_m) \cdot I_{s2} + j x_m \cdot I_{s1} \]
\[ V_{rl} = j x_m I_{s1} + [ j x_m + (R_r / (s (2 - s)))] I_{s1} + j (1 - s) [(R_r / (s (2 - s))) \cdot I_{s2}] \]
\[ j V_{rl} = [(1 - s) \cdot R_r / (s (2 - s))] \cdot I_{s1} + (R_r / (s (2 - s))) \cdot x_m \cdot I_{s1} + (R_r / (s (2 - s))) \cdot I_{s2} \]
\[ I_f = [(Z_m + j X_m + (R_r / S_l)) / ((Z_m + Z_s) \cdot (Z_m + j X_m + (R_r / S_l))] - Z_m^2 \} \cdot V_{df} \]
\[ I_{rb} = [Z_m / ((Z_m + Z_s) \cdot (Z_m + j X_m + (R_r / S_l))] - Z_m^2 \} \cdot V_{db} \]
\[ I_f = (I_{s1} + j I_{s2}) / 2 \quad I_{rb} = (I_{s1} - j I_{s2}) / 2 \]
\[ P_{gf} = m_i I_{s1}^2 R_r / S_l \]
\[ P_{gb} = m_i I_{s2}^2 R_r / S_l \]
\[ P_g = [(m_i R_r) / (2 s \cdot (2 - s))] \cdot (I_{s1}^2 + I_{s2}^2 + I_{s1} \cdot I_{s2} + 2 I_{s1} \cdot I_{s2} + 2 I_{s1} \cdot I_{s2} + I_{s1}^2) \]
\[ T = P_g \cdot (1 - s) \]

4. CONCLUSION

As there is no system driven especially for the two-phase induction motor fed from unbalanced two-phase supply, so we start for derivation the system equations for the said motor to be generally used even for the balanced or unbalanced two-phase supply. In this paper, we derive a system equation starting from the sequence equivalent circuit for the forward and backwards equivalent circuits, then we rearrange the equations with some mathematical assumptions which lead us to the new system equations. First for the voltage equations then for the current equations and finally for both power and torque equations. Moreover, we put an example which cover all cases with specific values and relations charts.

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