Implementation of a complex fractional order proportional-integral-derivative controller for a first order plus dead time system

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ABSTRACT

This paper presents the implementation of a complex fractional order proportional integral derivative (CPID) and a real fractional order PID (RPID) controllers. The analysis and design of both controllers were carried out in a previous work done by the author, where the design specifications were classified into easy (case 1) and hard (case 2) design specifications. The main contribution of this paper is combining CRONE approximation and linear phase CRONE approximation to implement the CPID controller. The designed controllers-RPID and CPID-are implemented to control flowing water with low pressure circuit, which is a first order plus dead time system. Simulation results demonstrate that while the implemented RPID controller fails to stabilize the system in case 2, the implemented CPID controller stabilizes the system in both cases and achieves better transient response specifications.

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1. INTRODUCTION

Fractional calculus is an extension to ordinary calculus by extending the orders of differentiation and integration to noninteger numbers. This mathematical concept was utilized in system modelling and control. In control, several fractional controllers were designed for several types of systems [1], [2]. As a further extension, complex fractional calculus was introduced as an extension to the real fractional calculus, where the orders of differentiation and integration can be complex numbers [3]. In [4], definitions and theorems were presented for complex fractional calculus mathematically. Complex fractional calculus was utilized to introduce models that describe viscoelastic materials [5], to model drug resistance in human immunodeficiency virus (HIV) infection [6], [7], and to present a new mathematical model for the atrial fibrillation [8]. Cois *et al.* [9] proposed a tool to model and study state-space with complex order. A complex fractional calculus was also applied to solve certain mathematical problems, such as fractional boundary problems [10].

In control theory and applications, fractional calculus has been well utilized to design fractional order controllers. Since the conventional proportional integral derivative (PID) controller dominates other controllers [11], [12], the most common fractional order controller is the fractional order PID controller (also called $PI^{\lambda}D^{\mu}$ controller), proposed by I. Podlubny [13], [14]. It is a generalization of the conventional PID controller, where the integer order derivative and integral actions are replaced by fractional order derivative and integral actions. Some of the recent works that utilize the $PI^{\lambda}D^{\mu}$ controller are presented in [15]–[17].

This generalization was also utilized in the differentiation and integration actions of the conventional fuzzy logic controller, resulting in a fractional order fuzzy logic controller [18], [19]. For nonlinear controllers, such as sliding mode controllers (SMC), fractional calculus was introduced in the sliding surface by taking the fractional order derivative and/or integral of the state variables [20]–[27]. Hybridization between fuzzy logic and fractional order sliding mode control can be achieved, where the resultant controller is called Fuzzy fractional order sliding mode controller (FFOSMC) [28]–[32]. All the previously mentioned works and other works in the literature lack the investigation of the effect of introducing complex fractional calculus into control action. In [33], the analysis of real fractional order PID (RPID) and complex fractional order PID (CPID) controllers were carried out, and both controllers were designed to control a first order plus dead time (FOPDT) system. In the design process, the specifications were classified into two cases, easy and hard. This classification is adopted to demonstrate the need for the CPID controller to overcome the limitations of the RPID controller. This paper implements the RPID and CPID controllers designed in [33]. The RPID controller is implemented using CRONE approximation-as most works in the literature; however, the contribution of this paper is the implementation of the CPID controller; it proposes combining CRONE approximation and linear phase CRONE approximation to obtain an acceptable approximation of this controller.

The rest of the paper is organized: section 2 presents the problem statement; section 3 presents the model of the flowing water with low pressure circuit (FWLPC), which is the plant to be controlled; section 4 presents the implementation of the RPID and CPID controllers using CRONE approximation and CRONE approximation/linear CRONE approximation, respectively; the results and their discussion is presented in section 4; and the conclusions is drawn in section 5.

2. PROBLEM STATEMENT

This paper addresses the utilization of complex fractional calculus in control theory. It is concerned with implementing the CPID controller that was designed in [33]. The suggested method is to combine CRONE approximation and linear phase CRONE approximation to approximate the real differentiator/integrator and imaginary differentiator/integrator factors of the controller, respectively.

3. MATHEMATICAL MODEL OF THE FWLPC

The LPRWC-the process to be controlled-is a FOPDT system; its transfer function is

$$P(s) = \frac{k}{\tau s+1} e^{-Ls} \tag{1}$$

where $L = 50s$ (time delay), $k = 3.13$ (gain), and $\tau = 433.33s$ (time constant) [33], [34].

4. CONTROL SYSTEM IMPLEMENTATION

The RPID and CPID controllers that are implemented in this paper were designed in [33], where the design specifications (desired gain crossover frequency ω_c and desired phase margin ϕ_m) were classified into two cases; in case 1, the design specifications are easy, while in case 2, the design specifications were made hard by increasing both ω_c and ϕ_m.

4.1. Implementation of the RPID controller

The RPID control law $u(t)$ is

$$u(t) = K_P e(t) + K_I D^{-\lambda} e(t) + K_D D^{\mu} e(t) \tag{2}$$

and the corresponding transfer function is

$$C_R(s) = \frac{U(s)}{E(s)} = K_P + K_I \frac{1}{s^{\lambda}} + K_D s^{\mu} = K_P + K_I s^{-\lambda} + K_D s^{\mu} \tag{3}$$

where K_p, K_I, $K_D \in R$ and λ and $\mu \in R^+$ are the parameters of the controller were designed in [33] and are shown in Table 1. To implement $C_R(s)$, it is needed to implement the fractional-order integrating action $s^{-\lambda}$ and fractional-order differentiating action s^{μ}. In this paper, CRONE approximation (sometimes called Oustaloup filter) is used. It is given by [35].

$$s^{\gamma} \approx K \prod_{k=1}^{N} \frac{s+\omega_{z,k}}{s+\omega_{p,k}} \tag{4a}$$

$$\omega_{z,k} = \omega_l (\omega_u)^{\frac{2k-1-\gamma}{N}} \tag{4b}$$

$$\omega_{p,k} = \omega_l (\omega_u)^{\frac{2k-1+\gamma}{N}} \tag{4c}$$

$$K = \omega_h^{\gamma} \tag{4d}$$

$$\omega_u = \sqrt{\frac{\omega_h}{\omega_l}} \tag{4e}$$

where $[\omega_l, \omega_h]$ are the frequency range on which the approximation is valid. The frequency range is taken as $[\omega_l, \omega_h] = [0.01, 100]$ rad/s. Using the values of the RPID controller parameters given in Table 1 and (4a)-(4e) to approximate $s^{-\lambda}$ and s^{μ} in (3), $C_{\mathrm{R}}(s)$ becomes

$$C_{\mathrm{R}}(s) \approx \frac{690.2s^8+2.012\times10^4s^7+1.737\times10^5s^6+3.978\times10^5s^5+3.211\times10^5s^4+7.689\times10^4s^3+6925s^2+215s+2.446}{s^8+99.94s^7+1927s^6+1.182\times10^4s^5+2.042\times10^4s^4+1.193\times10^4s^3+1965s^2+102.9s+1.039} \tag{5}$$

for case 1, and

$$C_{\mathrm{R}}(s) \approx \frac{6.567\times10^4s^8+2.616\times10^6s^7+1.979\times10^7s^6+4.785\times10^7s^5+3.26\times10^7s^4+7.816\times10^6s^3+6.349\times10^5s^2+3.029\times10^4s+545.7}{s^8+254.1s^7+1.202\times10^4s^6+1.824\times10^5s^5+7.707\times10^5s^4+1.112\times10^6s^3+4.47\times10^5s^2+5.76\times10^4s+1382} \tag{6}$$

for case 2.

Table 1. RPID and CPID controller parameters

Case	Controller	Parameters						
		K_P	K_I	K_D	α	β	θ	ϕ
1	RPID	0.09	0.02	5.22	0.88		0.31	
	CPID	0.76	0.16	21.54	0.04	1.19	0.21	1.56
2	RPID	17.40	0.07	29.30	1.76		0.64	
	CPID	0.94	20.65	31.92	0.48	-1.42	0.85	0.73

4.2. Implementation of the CPID controller

The CPID control law $u(t)$ is

$$u(t) = K_P e(t) + K_I D^{-(\alpha+j\beta)} e(t) + K_D D^{\theta+j\phi} e(t) \tag{7}$$

and the corresponding transfer function is

$$C_{\mathrm{C}}(s) = \frac{U(s)}{E(s)} = K_P + K_I \frac{1}{s^{\alpha+j\beta}} + K_D s^{\theta+j\phi} = C(s) = K_P + K_I \frac{1}{s^{\alpha+j\beta}} + K_D s^{\theta+j\phi} = K_P + K_I \frac{1}{s^{\alpha} s^{j\beta}} + K_D s^{\theta} s^{j\phi} = K_P + K_I s^{-\alpha} s^{-j\beta} + K_D s^{\theta} s^{j\phi} \tag{8}$$

where β, $\phi \in R$ are the extra parameters that introduces the imaginary integrator $s^{-j\beta}$ and imaginary differentiator $s^{j\phi}$. The seven parameters of the CPID controller were designed in [33] and are shown in Table 1. The fractional integrator $s^{-\alpha}$ and fractional differentiator s^{θ} are approximated using (4a)-(4e). However, to implement (8), it is needed to find a valid approximation of the imaginary integrator $s^{-j\beta}$ and imaginary differentiator $s^{j\phi}$. The sinusoidal transfer function of $s^{j\gamma}$ is

$$s^{j\gamma} = (j\omega)^{j\gamma} = j^{j\gamma} \times \omega^{j\gamma} = e^{-\gamma\frac{\pi}{2}} \times (\cos(\gamma \ln \omega) + j \sin(\gamma \ln \omega)) \tag{9}$$

$$\left|(j\omega)^{j\gamma}\right| = e^{-\gamma\frac{\pi}{2}} \tag{10}$$

$$\angle(j\omega)^{j\gamma} = \gamma \ln \omega = \gamma \frac{\ln \omega}{\log_{10} \omega} \log_{10} \omega = \gamma \ln 10 \log_{10} \omega \tag{11}$$

From (11), it can be seen that the phase is a linear function of $\log_{10} \omega$, i.e., if the phase is plotted in a logarithmic axis, it is a straight line with slope equals to $\gamma \ln 10$. This is a significant feature of the imaginary differentiator/integrator, since the real fractional differentiator/integrator has constant phase angle (i.e., its slope equals zero). In this paper, the imaginary differentiator/integrator are approximated by linear CRONE approximation, which relies on locating the transfer function poles and zeros of the approximating transfer function such as to give a linear phase curve. Unlike the CRONE approximation, in linear CRONE approximation, the spacing distance between the poles is not equal to the spacing distance between the zeros; this different spacing distance is responsible of giving the nonzero slope of the phase line. The linear CRONE approximation is [4],

$$s^{j\gamma} \approx \prod_{k=1}^{2N} \frac{1+\frac{s}{\omega_{z,k}}}{1+\frac{s}{\omega_{p,k}}} \tag{12a}$$

$$\omega_{z,k} = \omega_c (b)^{k-N-\frac{1}{2}} \tag{12b}$$

$$\omega_{p,k} = \omega_c (a)^{k-N-\frac{1}{2}} \tag{12c}$$

$$\omega_c = \sqrt{\omega_l \omega_h} \tag{12d}$$

where $[\omega_l, \omega_h]$ are the frequency range on which the approximation is valid, a and b are the recursive coefficients of the poles and zeros, respectively. The relation between the imaginary order γ and the recursive coefficients a and b is

$$\gamma \ln 10 = \frac{\pi/2}{\log_{10} b} - \frac{\pi/2}{\log_{10} a} \tag{13}$$

Algorithm 1 shows a pseudo code to set the values of a, b, and N given the values of ω_l, ω_h, and γ. The frequency range of the linear CRONE approximation is also taken as $[\omega_l, \omega_h] = [0.01, 100]$ rad/s (to give a fair comparison with the CRONE approximation of the RPID controller). Applying the algorithm given in Figure 1, the resultant values of a, b, and N are given in Table 2.

Algorithm 1: A flow chart to set the values of a, b, and N given the values of ω_l, ω_h, and γ.
1- Input ω_l, ω_h, and γ.
2- Set a equals to a certain value >1 and set $N = 0$.
3- Calculate b from equation (13).
4- $N = N + 1$.
5- Calculate $\omega_{z,k}$ from (12b).
6- Calculate $\omega_{p,k}$ from (12c).
7- If $b < a$ and $\omega_{z,k} > \omega_h$, go to 10.
8- If $b > a$ and $\omega_{p,k} > \omega_h$, go to 10.
9- Go to 4.
10- Output the values of a, b, poles $\omega_{p,k}$, zeros $\omega_{z,k}$, and N.

Using the parameter values of the CPID controller in Table 1, (12a)-(12d), and Algorithm 1 to approximate $s^{-j\beta}$ and $s^{j\phi}$ in equation (8), $C_{\mathrm{C}}(s)$, for case 1 and case 2, becomes

$$C_{\mathrm{C}1}(s) \approx \frac{\sum_0^{24} a_i s^i}{\sum_0^{24} b_i s^i} \tag{14}$$

for case 1, and

$$C_{\mathrm{C}2}(s) \approx \frac{\sum_0^{24} c_i s^i}{\sum_0^{24} d_i s^i} \tag{15}$$

for case 2, where a_i and b_i are given in Table 3 and c_i and d_i are given in Table 4.

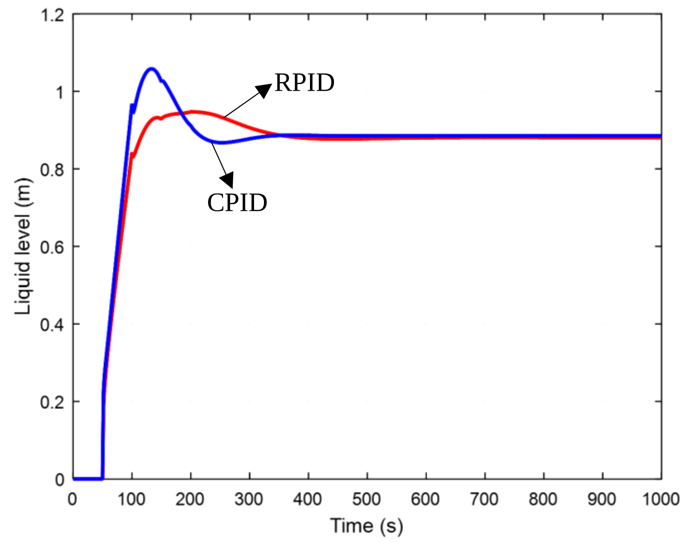

Figure 1. Unit step response of the RPID and CPID control systems: case 1

Table 2. Linear CRONE parameter values a, b and N and the resultant slope of phase line

Case	Imaginary differentiator/integrator	a	b	N	Slope of phase line
1	$s^{-j1.19}$	2	1.9280	4	0.2916
	$s^{j1.56}$	2	2.1346	4	-0.4482
2	$s^{-j(-1.42)}$	2	1.5487	4	3.0505
	$s^{j0.73}$	2	1.8550	4	0.6358

Table 3. Values of the coefficients a_i and b_i in (14)

Coefficient	Value	Coefficient	Value
a_0	43.1	b_0	17.5
a_1	4664	b_1	1776
a_2	2.188×10^5	b_2	7.297×10^4
a_3	5.8×10^6	b_3	12.9×10^6
a_4	1×10^8	b_4	2.3×10^7
a_5	1.1×10^9	b_5	2.2×10^8
a_6	9.8×10^9	b_6	1.5×10^9
a_7	5.8×10^{10}	b_7	7.1×10^9
a_8	2.5×10^{11}	b_8	2.4×10^{10}
a_9	8.1×10^{11}	b_9	6.3×10^{10}
a_{10}	1.9×10^{12}	b_{10}	1.2×10^{11}
a_{11}	3.4×10^{12}	b_{11}	1.7×10^{11}
a_{12}	1.9×10^{12}	b_{12}	1.8×10^{11}
a_{13}	4.3×10^{12}	b_{13}	1.4×10^{11}
a_{14}	3.16×10^{12}	b_{14}	8.4×10^{10}
a_{15}	1.71×10^{12}	b_{15}	3.7×10^{10}
a_{16}	6.8×10^{11}	b_{16}	1.2×10^{10}
a_{17}	1.9×10^{11}	b_{17}	2.8×10^9
a_{18}	4.2×10^{10}	b_{18}	4.9×10^8
a_{19}	6.4×10^9	b_{19}	6×10^7
a_{20}	6.7×10^8	b_{20}	5.1×10^6
a_{21}	4.7×10^7	b_{21}	2.8×10^5
a_{22}	2.1×10^6	b_{22}	9316
a_{23}	5.4×10^4	b_{23}	160.4
a_{24}	582.8	b_{24}	1

Table 4. Values of the coefficients c_i and d_i in (15)

Coefficient	Value	Coefficient	Value
c_0	1.8	d_0	0.9
c_1	194.1	d_1	106.5
c_2	8113	d_2	5079
c_3	1.8×10^5	d_3	1.3×10^5
c_4	2.4×10^6	d_4	2.1×10^6
c_5	2.2×10^7	d_5	2.2×10^7
c_6	1.4×10^8	d_6	1.6×10^8
c_7	6.7×10^8	d_7	8.7×10^8
c_8	2.3×10^9	d_8	3.3×10^9
c_9	5.9×10^9	d_9	9.4×10^9
c_{10}	1.1×10^{10}	d_{10}	1.9×10^{10}
c_{11}	1.7×10^{10}	d_{11}	3×10^{10}
c_{12}	1.9×10^{10}	d_{12}	3.5×10^{10}
c_{13}	1.7×10^{10}	d_{13}	3×10^{10}
c_{14}	1.1×10^{10}	d_{14}	1.9×10^{10}
c_{15}	5.7×10^9	d_{15}	9.5×10^9
c_{16}	2.2×10^9	d_{16}	3.4×10^9
c_{17}	6.3×10^8	d_{17}	9×10^8
c_{18}	1.3×10^8	d_{18}	1.7×10^8
c_{19}	2×10^7	d_{19}	2.3×10^7
c_{20}	2.2×10^6	d_{20}	2.2×10^6
c_{21}	1.5×10^5	d_{21}	1.3×10^5
c_{22}	6943	d_{22}	5408
c_{23}	160.4	d_{23}	114.6
c_{24}	1.4	d_{24}	114.6

5. SIMULATION RESULTS AND DISCUSSIONS

The implemented RPID and CPID controllers were used to control the FWLPC system given by (1), where the systems were simulated using MATLAB Simulink. The unit step responses for both systems were obtained for comparison purposes. Figure 1 shows these responses for case 1 and Figures 2-3 show these responses for case 2, and Table 5 shows the performance of both systems in terms of transient response specifications. The obtained results demonstrated that in case 1, the performance of the implemented CPID controller is better than that of the implemented RPID controller by reducing both the rise time and steady state error, while in case 2 the RPID controller failed to stabilize the system while the CPID controller stabilized it. The significance of these results is that it insured the results that were obtained when the controllers were designed [33], where the RPID controller stabilized the system only in case1, while the CPID controller stabilized it in both cases and gave better design specifications. For both systems, the steady-state error is not zero; this is due to approximating the pure integrator in both the RPID and CPID controllers.

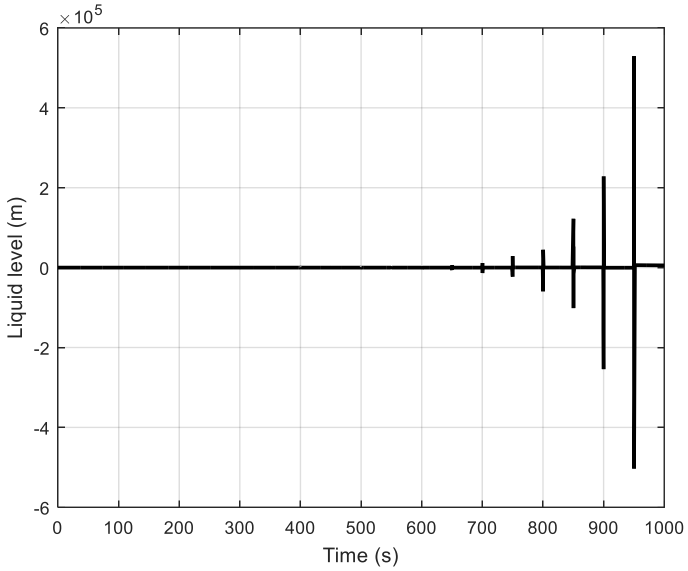

Figure 2. Unit step response of the RPID control system: case 2

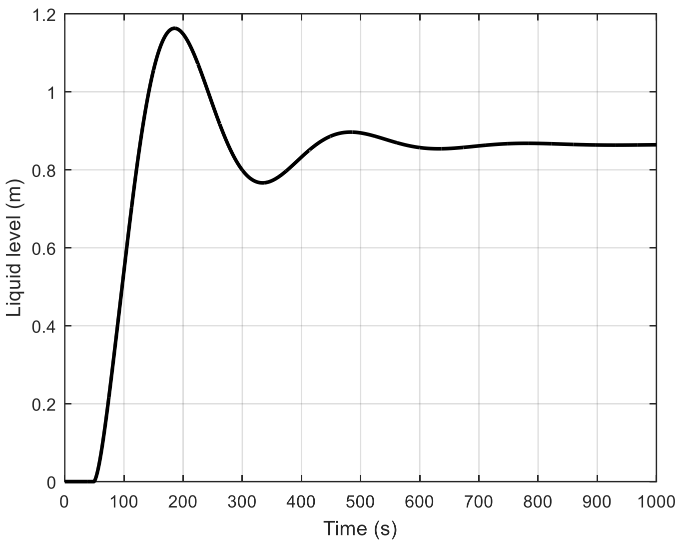

Figure 3. Unit step response of the CPID control system: case 2

Table 5. Transient response specifications of the RPID and CPID control systems

	RPID		CPID	
	Case 1	Case 2	Case 1	Case 2
Rise time (s)	70	unstable	62	90
Percentage overshoot			7.5%	16%
Steady-state error (m)	0.12		0.11	0.14

6. CONCLUSIONS

In this paper, RPID and CPID controllers have been implemented to control a FOPDT system. The RPID controller has been approximated using CRONE approximation, and the CPID controller has been approximated by combining CRONE approximation and linear phase CRONE approximation. The following two conclusions are draw: i) adding extra implementation parameters has a significant impact on the order of the controller; ii) the CPID controller outperforms the RPID controller from the stability and performance (transient response specifications) point of views. As a suggestion for future work, the steady-state error of both the RPID and CPID control systems can be investigated by identifying its cause and suggesting proper solutions to reduce or eliminate it.

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