IAES International Journal of Robotics and Automation (IJRA)
Vol. 11, No. 4, December 2022, pp. 333~342
ISSN: 2722-2586, DOI: 10.11591/ijra.v11i4.pp333-342

Synthesis of control laws for magnetic levitation systems based on serial invariant manifolds

Nguyen Xuan Chiem, Thuy Xuan Pham
Department of Automation and Computing Techniques, Le Quy Don Technical University, Hanoi, Vietnam

Article Info

Article history:

Received Aug 2, 2021
Revised Sep 24, 2022
Accepted Oct 3, 2022

Keywords:

Invariant manifold
Magnetic levitation system
Nonlinear systems
Sliding mode control
Synergetic control theory

ABSTRACT

In this paper, a nonlinear controller is designed for a magnetic levitation system (MLS) based on serial invariant manifolds. Synthesized controller based on the method of synergetic control theory (SCT) through invariant manifolds, asymptotically stable. In this method, the control law is synthesized to ensure the motion of the closed-loop control object from an arbitrary initial state into the vicinity of the desired invariant manifold. Thereby, the control system not only ensures the necessary control quality but also ensures the asymptotic stability of the entire system. The quality and efficiency of the control law are proven through simulation results and comparison with the sliding mode controller (SMC).

Corresponding Author:

Chiem Nguyen Xuan
Department of Automation and Computing Techniques, Le Quy Don Technical University
236 Hoang Quoc Viet, Bac Tu Liem, Hanoi, Viet Nam
Email: chiemnx@mta.edu.vn

1. INTRODUCTION

A magnetic levitation system (MLS) is an electromagnetic device that lifts and suspends ferromagnetic objects using electromagnetic principles. MLS technology eliminates mechanical contact between moving and stationary parts thereby reducing friction. MLS brings many advantages such as low noise, the ability to work in a high vacuum environment, high-precision positioning platform. MLS typically operates on three types of forces: lift, thrust, and drag [1]. Therefore, MLS technology is used in many drivetrains: high-speed trains, magnetic bearings, semiconductor technology, precision positioning, magnetic suspension and non-contact haptic interactions, non-pollution, and multi-directional degrees of freedom (DOF) [2]–[4]. Due to the high nonlinearity and existence of many model parameters of MLS, MLS is a standard object to study and test control methods and control laws.

Currently, there is a lot of research on MLS. In order to improve the quality of control. Liu *et al*. [5] and Pradhan and Singh [6] used the classical control algorithms proportional-integral-derivative (PID) and proportional-derivative (PD) to stabilize MLS. However, the PID controller is based on the error, so the control quality for high nonlinear systems is not good. In [7], a serial multilayer neural network is used to model the system in which learning and control are performed concurrently. In addition, the adaptive controllers studied in [8] have good results. Pradhan and Singh [6] presented the optimization of the PD controller using a bat-swarm algorithm for the MLS system. However, tuning the control parameters of these algorithms is not simple to apply in real-time systems. Besides, the uncertain components from the mathematical model of the system as well as the deviation of the measuring tools affect the quality of the controller. A fuzzy logic controller for MLS is presented in [9]. In [10], quasi-time optimal controllers based on dynamic adaptation are proposed and tested. However, these controllers suffer from the limitation of being very sensitive to the initial conditions, as well as to noise effects and changes in the parameters of the system. In [11]–[14], the sliding control method and its variations are used to stabilize MLS. The obvious disadvantage of a sliding mode controller (SMC) is that the control signals are discontinuous and have a similar shape to a bipolar square wave with high bias switching frequency. These studies also try to overcome these disadvantages by changing the switching signal form and adapting to the parameters of the sliding surface.

In this paper, a model and controller are designed based on synergetic control theory (SCT) with sequential asymptotic stable serial invariant manifolds presented by Kolecnikov *et al*. [15]–[21]. The quality of the proposed control law is illustrated by simulation results. The responses of the system are compared with the sliding mode controller to prove the effectiveness of the proposed control law.

2. MATHEMATICAL MODELS OF THE MLS

The model of MLS is shown in Figure 1, where the gravity P has a downward direction, U is the control input, changed to control the electromagnetic force F to raise or lower the ball a xb distance from the electromagnet. The x distance between the ball and the electromagnet is the output control. The distance between the ball and the electromagnet is determined by the Hall effect sensor.

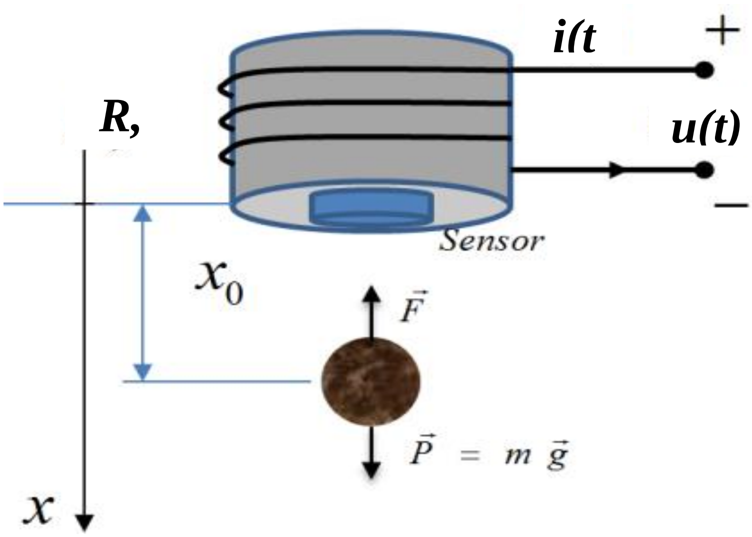

Figure 1. Model of magnetic levitation system

Based on [10, 11], the mathematic model of MLS has form as in (1),

$$\begin{cases} \dfrac{dx}{dt} = v \\ \dfrac{mdv}{dt} = mg - C\left(\dfrac{i}{x}\right)^2 \\ Ri + \dfrac{d(L(x)i)}{dt} = u \end{cases} \tag{1}$$

where

x : ball position (m)
v : speed of ball (m/s)
i : current in the coil (A)
u : the supply voltage for coil (V)
R, L : resistor and inductor of the electromagnet coil (Ω, H)
C : magnetic force constant (Nm^2/A^2)
m : mass of ball (kg)
g : gravitational acceleration (m/s^2)

The inductance of the coil is a function of the position of the ball, determined as (2) [10],

$$L(x) = L_1 + \frac{2C}{x} \tag{2}$$

where L_1 is a parameter of the system.

Set the state variable $x_1 = x,\ x_2 = v,\ x_3 = i$, the state equation of system (1) is written as (3).

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = g - \dfrac{C}{m}\left(\dfrac{x_3}{x_1}\right)^2 \\ \dot{x}_3 = -\dfrac{R}{L}x_3 + \dfrac{2C}{L}\left(\dfrac{x_2 x_3}{x_1^2}\right) + \dfrac{1}{L}u \end{cases} \tag{3}$$

The control objective is to ensure the position of the ball to the desired position when the preset value x_{sp} is changed.

3. SYNTHESIS OF SYNERGETIC CONTROL LAW FOR MLS USING SERIAL INVARIANT MANIFOLD

3.1. Diagram of control structure for MLS

To control the balance of the ball at a given position, the diagram of the control structure of MLS is shown in Figure 2. The controller gives a voltage u acting on the inductor in MLS from signals about the position, velocity, and current in the electromagnet.

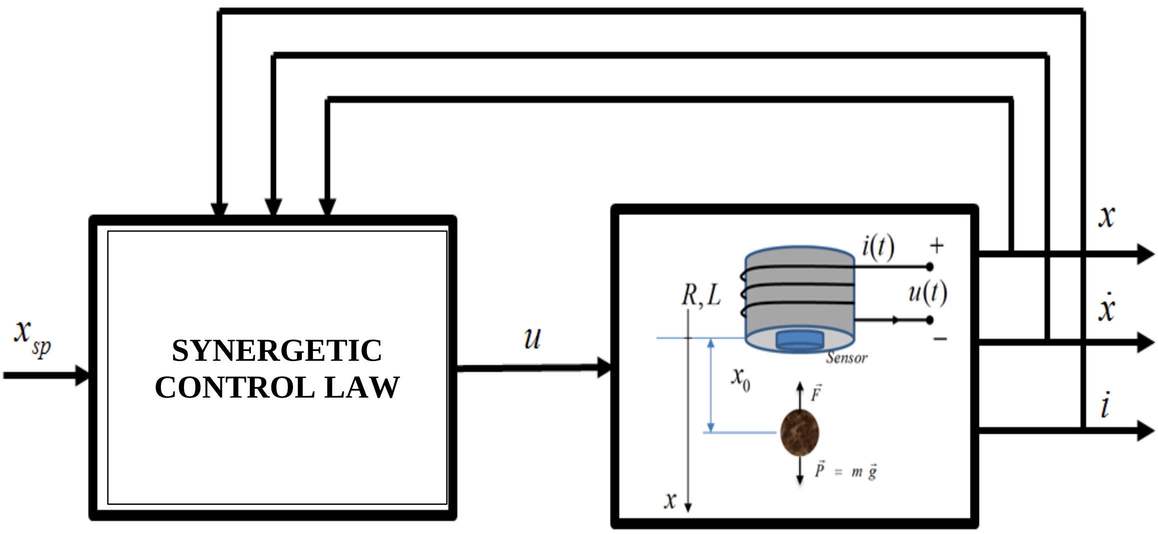

Figure 2. Diagram of control structure of MLS

3.2. Synthesis of synergetic control law based on serial invariant manifold

The purpose of the control problem is to keep the magnetized object stable at a specified position x_{sp} by varying the voltage supplied to the magnetic coil. From the point of view of SCT, this means that it is necessary to synthesize the control signal $u(x_1, x_2, x_3)$, a function that depends on the phase coordinates. The control signal will move the magnetized object from the initial position (within the controllable value range) to the final state according to the set value, when it satisfies the required quality criteria [15]–[21].

Since the control problem of MLS is to stabilize the magnetized object in the desired position, we introduce the first invariant technology corresponding to the control goal:

$$x_1 = x_d \tag{4}$$

In the first step, based on reality and mathematical model, when the control signal u changes, it will affect the change of the current x_3, so the first manifold is selected in the form of (5).

$$\psi_1 = x_3 - \varphi_1(x_1, x_2) = 0 \tag{5}$$

In the manifold (7) containing the function $\varphi_1(x_1, x_2)$, this function determines the desired characteristics of the change in current x_3 at the intersection with the invariant manifold $\psi_1 = 0$. The function $\varphi_1(x_1, x_2)$ is determined in the process of synthesizing the control law, derived from the invariant condition (4).

According to the method of analytical design of aggregated regulators (ADAR) [16]–[18], the macro variable ψ_1 must satisfy the root of the basic functional equation:

$$T_1\dot{\psi}_1 + \psi_1 = 0 \tag{6}$$

where $T_1 > 0$ ensures the asymptotic stability of the system motion.

Substitute (5) into (6):

$$T_1\left(\dot{x}_3 - \sum_{i=1}^{2}\frac{\partial\varphi_1}{\partial x_i}\dot{x}_i\right) + \psi_1 = 0 \tag{7}$$

Substitute $\dot{x}_3$ in the state (3):

$$T_1\left(-\frac{R}{L}x_3 + \frac{2C}{L}\left(\frac{x_2x_3}{x_1^2}\right) + \frac{1}{L}u\right) - T_1\sum_{i=1}^{2}\frac{\partial\varphi_1}{\partial x_i}\dot{x}_i + \psi_1 = 0 \tag{8}$$

From (8) we find the control law u of the form:

$$u = Rx_3 - 2C\left(\frac{x_2x_3}{x_1^2}\right) + L\sum_{i=1}^{2}\frac{\partial\varphi_1}{\partial x_i}\dot{x}_i - \frac{L}{T_1}(x_3 - \varphi_1(x_1, x_2)) \tag{9}$$

When the system enters the manifold, the performance point of the system touches the intersection of the $\psi_1 = 0$ manifold, then the system will have a dynamic decomposition of the system (3) and the dynamics of the closed loop system are described by (10).

$$\begin{cases}\dot{x}_1 = x_2 \\ \dot{x}_2 = g - \dfrac{C}{m}\dfrac{\varphi_1(x_1, x_2)^2}{x_1^2}\end{cases} \tag{10}$$

The function $\varphi_1(x_1, x_2)$ in the decomposition system (7) can be thought of as an internal control signal.

In the second step of the synthesis, to search for the control and to determine the function $\varphi_1(x_1, x_2)$, an additional invariant manifold is introduced, which will ensure the stability of the closed-loop system and the response of invariant technology (4). We choose a second manifold:

$$\psi_2 = x_2 - k(x_1 - x_d) = 0 \tag{11}$$

The system dynamics on this manifold can be rewritten as:

$$\dot{x}_1 = k(x_1 - x_d) \tag{12}$$

From the dynamics equation (9) the relative stability condition of the system at $x_1 = x_d$ is $k < 0$.

To satisfy the condition $\psi_2 = 0$, macro variable ψ_2 must satisfy the root of the equation:

$$T_2\dot{\psi}_2 + \psi_2 = 0 \tag{13}$$

where $T_2>0$ is the condition for asymptotic stability of the motion of the system with the invariant manifold.

Substitute (11) into (13) to find the internal control signal $\varphi_1(x_1, x_2)$.

$$T_2(\dot{x}_2 - k(\dot{x}_1 - \dot{x}_d)) + x_2 - k(x_1 - x_d) = 0 \tag{14}$$

Furthermore, the equations of the decomposition system (10) are substituted into (14), giving the expression as in (15).

$$T_2\left(g - \frac{C}{m}\frac{\varphi_1(x_1, x_2)^2}{x_1^2} - k(x_2 - \dot{x}_d)\right) + x_2 - k(x_1 - x_d) = 0 \tag{15}$$

From (15), we find the internal control signal $\varphi_1(x_1, x_2)$:

$$\varphi_1(x_1, x_2) = \pm\sqrt{\frac{mx_1^2}{T_2C}(T_2g - T_2k(x_2 - \dot{x}_d) + x_2 - k(x_1 - x_d))} \tag{16}$$

Gravity pulls the magnetic object down, so to keep the object in the desired position it is necessary to supply the electromagnet with a voltage of a fixed direction. In this paper, a positive control signal value is selected and creates upward force. The desired control law is found as a common root of the (9) and (16) and has the following form:

$$\begin{aligned} u = Rx_3 - 2C\left(\frac{x_2x_3}{x_1^2}\right) + \frac{L}{2T_2}\sqrt{\frac{m}{C}}\frac{(2T_2g - k(3x_1 - 2x_d))x_1 + 2(1 - T_2k)x_1x_2}{\varphi_1(x_1, x_2)}x_2 \\ + \frac{L}{2T_2}\sqrt{\frac{m}{C}}\frac{(1 - kT_2)x_1^2}{\varphi_1(x_1, x_2)}\left(g - \frac{C}{m}\left(\frac{x_3}{x_1}\right)^2\right) - \frac{L}{T_1}(x_3 - \varphi_1(x_1, x_2)) \end{aligned} \tag{17}$$

3.3. Design of sliding mode controller for MLS

Represent the (3) in the form (18):

$$\ddot{y} = a(x) + b(x)u \tag{18}$$

where $y = x$.

$$a(x) = \frac{2C}{m}\left(\left(1 - \frac{2C}{Lx_1}\right)\frac{x_2x_3^2}{x_1^3} + \frac{R}{L}\frac{x_3^2}{x_1^2}\right), \quad b(x) = -\frac{2Cx_3}{Lmx_1^2}$$

Then, it is necessary to determine the control voltage so that $y \to x_d, \dot{y}, \ddot{y} \to 0$ when $t \to \infty$ to $x \to x_d$ as the initial control target.

Set the output of the system:

$$e = x_d - x = x_d - y \tag{19}$$

From (18) and (19), the relationship between the output and input of the system in the initial coordinates is established as (20).

$$\ddot{e} = \ddot{x}_d - a(x) - b(x)u \tag{20}$$

Select the sliding surface of the controller as (21).

$$\sigma = \ddot{e} + a_1\dot{e} + a_0e \tag{21}$$

With the coefficients a_1, a_0 is chosen so that the characteristic equation $s^2 + a_1s + a_0 = 0$ is a Hurwitz polynomial. Then, the sliding mode control law is defined as follows:

$$u = \frac{1}{b(x)}\left[\ddot{x}_d - a(x) + a_1\left[\ddot{x}_d - g_c + \frac{C}{m}\left(\frac{x_3}{x_1}\right)^2\right] + a_0[\dot{x}_d - x_2] - Ksign(\sigma)\right] \tag{22}$$

where K is a positive constant. To avoid chattering, use the *sat* function instead of the *sign* function in the control law (22).

4. RESULTS AND DISCUSSION

Simulations were performed using MATLAB/Simulink software. The parameters of the model are set as follows: mass of ball $m = 0.06$ kg; coil resistance $R = 11{,}4\ \Omega$; inductance $L_1 = 0{,}6$ H; magnetic force constant $C = 1{,}4 \times 10 - 4\ \mathrm{Nm^2/A^2}$; and the gravitational acceleration $g = 9{,}8\ \mathrm{m/s^2}$, the maximum voltage supplied to the system from 0 V to 24 V. Parameters of control law based on method of synergetic control theory: $k = -40$, $T_1 = 0.001$, $T_2 = 0.03$; and parameters sliding mode controller: $a_0 = 400$, $a_1 = 20$, $K = 100$.

The simulation results of the sliding mode controller and the synergetic controller with the ladder signal are shown in Figures 3 to 6. The efficiency of the position response of the magnetized object is shown in Figure 3. It can be seen that the response of SMC gives a poorer response. The position response of STC is no oscillation occurs and no overshoot when changing the set value. The SMC gives good results, but the overshoots when the set value changes are 6%, 3.5%, and the oscillations occur in the first stage. The velocity response of STC is better when the maximum amplitude is smaller and no oscillation occurs as shown in Figure 4. The variation of the control input u and the current intensity is shown in Figure 5 and Figure 6, where control input u is always within the limits and the time response is faster when using STC.

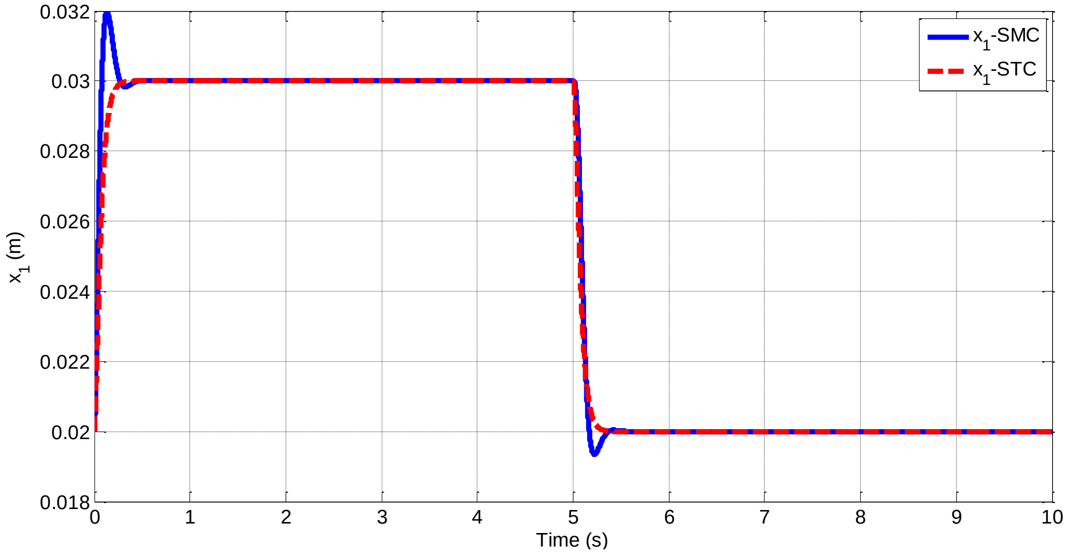

Figure 3. The position response of SMC and STC

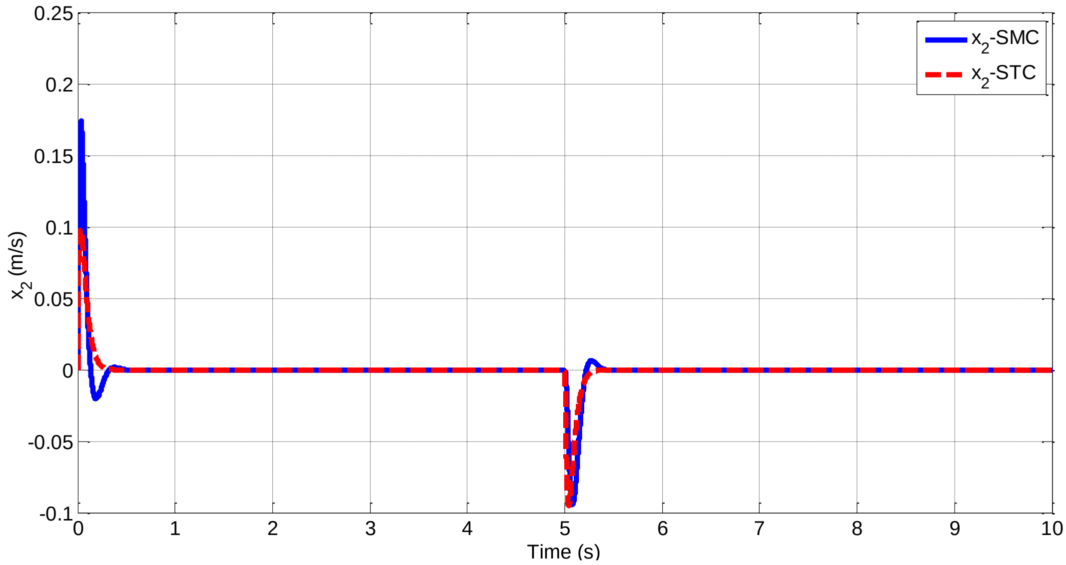

Figure 4. The velocity response of SMC and STC

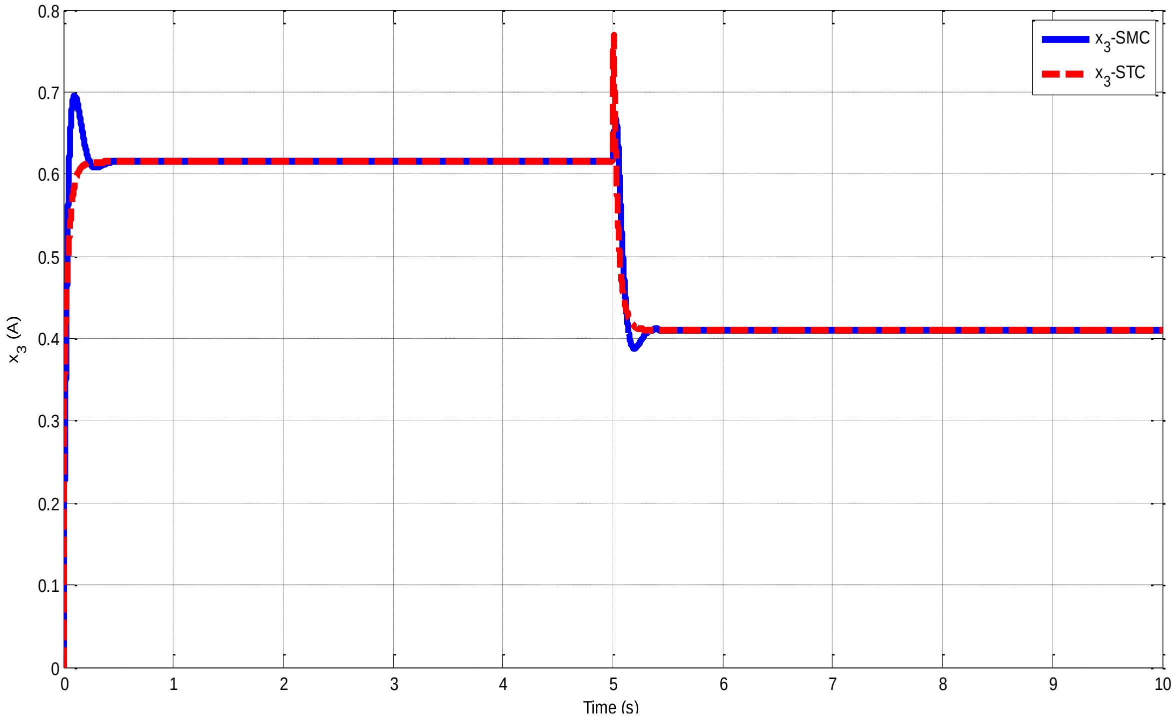

Figure 5. The current intensity response of SMC and STC

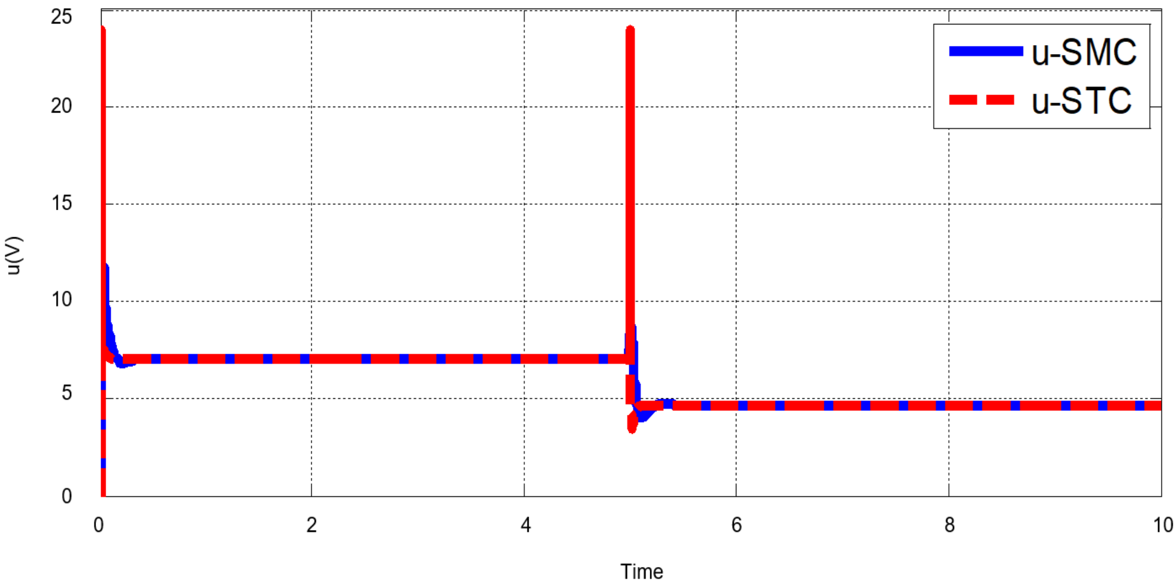

Figure 6. The control signal of SMC and STC

When the set value for the position of the ball is: $x_{sp}(t)= 0.03+0.01\sin(t)$, the simulation results of SMC and STC are shown in Figures 7 to 10. In Figure 7, the position response of STC law is better than SMC at the amplitude of oscillation, the time to reach and the position error at the peaks of the set value. Figure 8 shows the results of the velocity of the magnetized object. It is clear that the amplitude of oscillation when using STC is much smaller when using SMC and the stabilization time is also smaller. The current intensity and control voltage of both control laws are within the original technical limitation. Therefore, the simulation results show that the system can have better control quality by using the STC and partial sequential invariant manifolds with fast response frequency (electrical part) to stabilize the slower response frequency part (mechanical part).

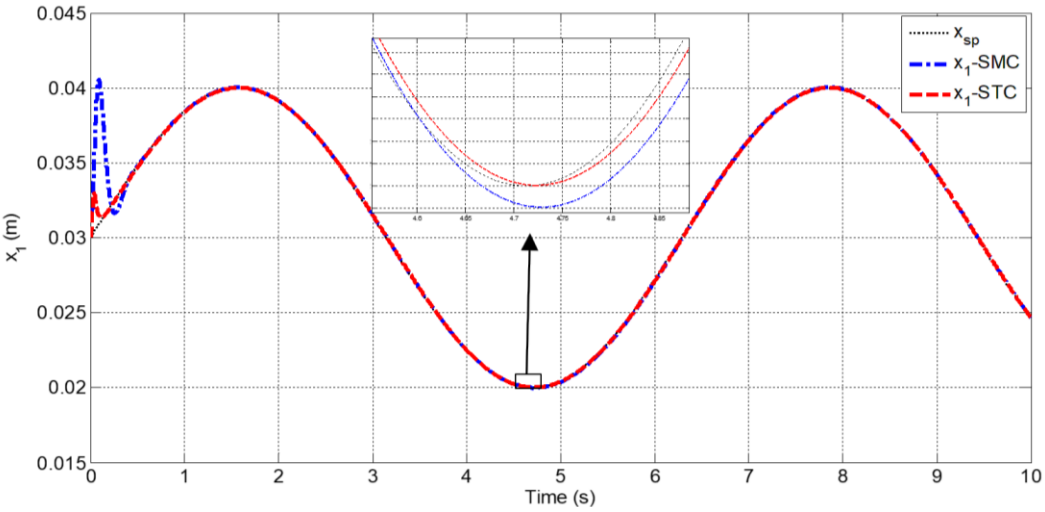

Figure 7. The position response of SMC and STC

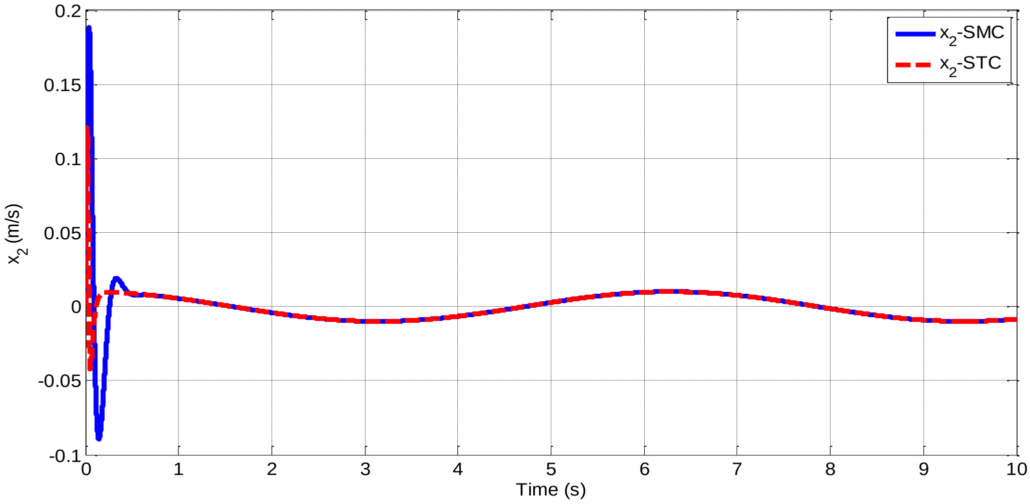

Figure 8. The velocity response of SMC and STC

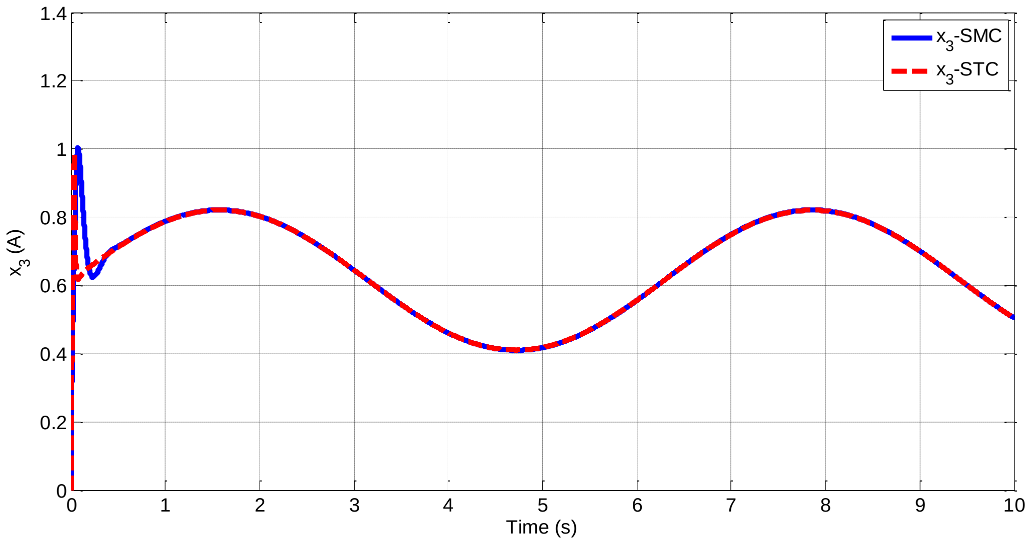

Figure 9. The current intensity response of SMC and STC

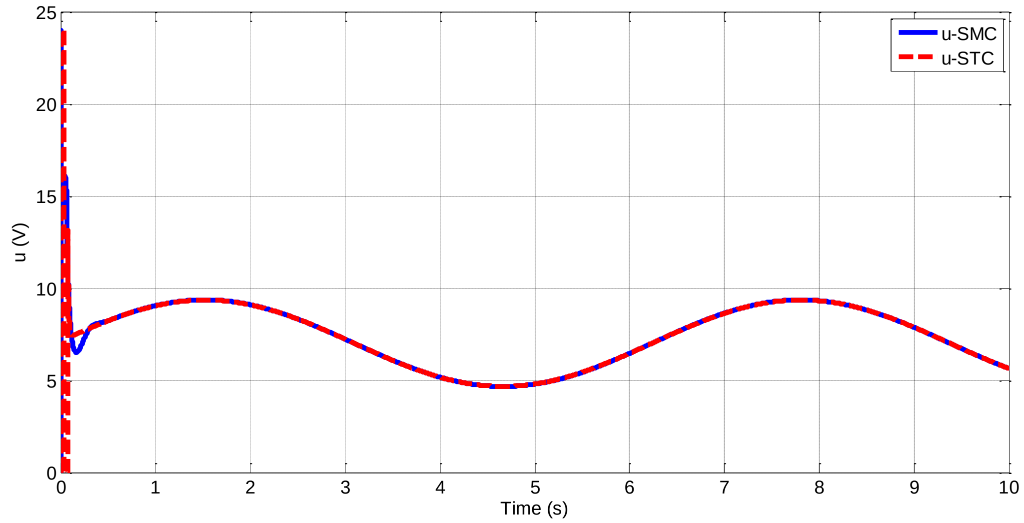

Figure 10. The control signal of SMC and STC

5. CONCLUSION

On the basis of the nonlinear model of the magnetic levitation system, using the method of synergetic control theory with serial invariant manifolds, the control law has been synthesized to ensure the motion of the object to the desired position for ahead or follow a desired trajectory. An important advantage of synergetic nonlinear control laws is the ability to obtain control laws in analytical form, by solving macro variables and functional equations to ensure their asymptotic stability. Analysis of transient characteristics of the nonlinear controller, synthesized by SCT and SMC, allows the authors to conclude that SCT is performed more efficiently, with no unwanted output overshoot, better response time, fewer oscillations, and smaller tracking error. Further studies will consider the dynamics of the manifolds to choose the law of the optimal manifold for different objects.

REFERENCES

[1] U. Hasirci, A. Balikci, Z. Zabar, and L. Birenbaum, “A novel magnetic-levitation system: design, implementation, and nonlinear control,” *IEEE Transactions on Plasma Science*, vol. 39, no. 1, pp. 492–497, Jan. 2011, doi: 10.1109/TPS.2010.2053389.

[2] D. M. Rote and Yigang Cai, “Review of dynamic stability of repulsive-force maglev suspension systems,” *IEEE Transactions on Magnetics*, vol. 38, no. 2, pp. 1383–1390, Mar. 2002, doi: 10.1109/20.996030.

[3] D. Cho, Y. Kato, and D. Spilman, “Sliding mode and classical controllers in magnetic levitation systems,” *IEEE Control Systems*, vol. 13, no. 1, pp. 42–48, Feb. 1993, doi: 10.1109/37.184792.

[4] B. V. Jayawant, P. K. Sinha, and D. G. Aylwin, “Feedback control systems for d.c. electromagnets in passenger-carrying vehicles,” *International Journal of Control*, vol. 24, no. 5, pp. 627–639, Nov. 1976, doi: 10.1080/00207177608932851.

[5] H. Liu, X. Zhang, and W. Chang, “PID control to maglev train system,” in *2009 International Conference on Industrial and Information Systems*, Apr. 2009, pp. 341–343., doi: 10.1109/IIS.2009.24.

[6] S. K. Pradhan and R. Singh, “Nonlinear control of a magnetic levitation system using feedback linearization,” in *2014 IEEE International Conference on Advanced Communications, Control and Computing Technologies*, May 2014, pp. 152–156., doi: 10.1109/ICACCCT.2014.7019369.

[7] D. Soloway and P. J. Haley, “Neural generalized predictive control: a newton-raphson implementation.” NASA Langley Research Center Hampton, VA United States, pp. 1–17, 1997.

[8] E. Lughofer and M. Sayed-Mouchaweh, “Adaptive and on-line learning in non-stationary environments,” *Evolving Systems*, vol. 6, no. 2, pp. 75–77, Jun. 2015, doi: 10.1007/s12530-015-9128-2.

[9] J. A. Meda-Campana, J. Rodriguez-Valdez, T. Hernandez-Cortes, R. Tapia-Herrera, and V. Nosov, “Analysis of the fuzzy controllability property and stabilization for a class of T–S fuzzy models,” *IEEE Transactions on Fuzzy Systems*, vol. 23, no. 2, pp. 291–301, Apr. 2015, doi: 10.1109/TFUZZ.2014.2312025.

[10] C. X. Nguyen, T. D. Pham, A. D. Lukynov, P. C. Tran, and Q. D. Truong, “Design embedded control system based controller of the quasi time optimization approach for a magnetic levitation system,” *IOP Conference Series: Materials Science and Engineering*, vol. 1029, no. 1, p. 012020, Jan. 2021, doi: 10.1088/1757-899X/1029/1/012020.

[11] N. F. Al-Muthairi and M. Zribi, “Sliding mode control of a magnetic levitation system,” *Mathematical Problems in Engineering*, vol. 2004, no. 2, pp. 93–107, 2004, doi: 10.1155/S1024123X04310033.

[12] S. Ahmed Al-Samarraie, B. Fateh Midhat, and R. Ahmed Bahaa Al-Deen, “Adaptive sliding mode control for magnetic levitation system,” *Al-Nahrain Journal for Engineering Sciences*, vol. 21, no. 2, pp. 266–274, Apr. 2018, doi: 10.29194/NJES21020266.

[13] Y. Eroğlu and G. Ablay, “Cascade control of magnetic levitation with sliding modes,” *MATEC Web of Conferences*, vol. 42, p. 01009, Feb. 2016, doi: 10.1051/matecconf/20164201009.

[14] V. S. A. and S. S., “Design of sliding mode controller for magnetic levitation system,” *Computers & Electrical Engineering*, vol. 78, pp. 184–203, Sep. 2019, doi: 10.1016/j.compeleceng.2019.07.007.

[15] A. A. Kolesnikov, “Synergetics control theory.” Moscow: Energoatomizdat, p. 344, 1994.

[16] A. A. Kolesnikov, "Synergetics and problems of control theory: collection of articles," Ed. 3., Scientific, 2004.

[17] A. A. Kolesnikov, "Synergetic methods of complex systems control: the theory of system synthesis." URSS, Moscow, 2006.

[18] A. A. Kolesnikov, G. E. Veselov, A. N. Popov, A. A. Kuz'menko, M. E. Pogorelov, and I. V. Kondratyev, "Synergetic methods of complex systems control: energy systems." URSS, Moscow, 2006.

[19] A. A. Kolesnikov, A. A. Kuz'menko, and G. E. Veselov, "New design technology of modern control systems for the generation of electricity." Publishing House, Moscow, 2011.

[20] A. A. Kolesnikov, G. E. Veselov, A. N. Popov, B. V. Topchiev, A. S. Mushenko, and V. A. Kobzev, "Synergetic methods of complex systems control: mechanical and electromechanical systems." URSS, Moscow, 2006.

[21] A. A. Kolesnikov, "Introduction of synergetic control," in *2014 American Control Conference*, Jun. 2014, pp. 3013–3016., doi: 10.1109/ACC.2014.6859397.

BIOGRAPHIES OF AUTHORS

Nguyen Xuan Chiem was born in 1983. He received a Ph.D. degree in Technology and equipment for mechanical and physical-technical processing from Don State Technical University, Russia in 2012. From 2013 to the present, he is a lecturer at the Department of Automation and Computing Techniques in the Faculty of Control Engineering, Le Quy Don Technical University. His research interests include nonlinear control, intelligent control, and embedded systems. He can be contacted at chiemnx@mta.edu.vn.

Thuy Xuan Pham received the M. Sc degree in industrial sciences, electronic engineering from GroupT Leuven, Belgium, in 2013, the PhD. Degree in Signal, Image and Automation from Universite Paris-Est, France, 2019. His current research interests including computational intelligence, high performance computing, and computer vision. He can be contacted at thuy.pham@univ-paris-est.fr.