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# Kinematic modelling of three link robot manipulator and joint torque optimization using genetic algorithm in MATLAB

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**ABSTRACT**

This research article presents the non-linear dynamic of a three-link robotic manipulator formulated by the Newton-Euler method. The planar manipulator is composed of three links and three revolute joints rotating about the z-axis. The three nonlinear non-homogeneous dynamic equations have been solved graphically with the help of MATLAB by phase variable method. The work represents the graphical solution of the transient response of angular position, and angular velocity of each link member for a predetermined interval of time. With the help of simulated value from MATLAB, torque characteristics have been determined for different torque ratios and optimum torque has been derived using a genetic algorithm to move the manipulator in a proper direction.



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## 1. INTRODUCTION

Robot manipulators can be assumed to be a chain of link mechanisms with highly nonlinear dynamics. The forward and inverse kinematics are presented using the D-H convention and the deduced mathematical model has been inferred by Newton-Euler approach. To obtain the optimal performance of the robot manipulator, a precise dynamic model of the robot manipulator is required. Garg *et al*. [1] determined an optimal path using a genetic algorithm and optimization was achieved using simulated annealing (SA). Exhaustive simulation was conducted for different types of manipulators namely MELFA RV-1A having six degrees of freedom and three planar revolute joints to find the distance between the end effector and the object using an artificial neural network [2]. The payload of the mobile robotic arm is determined by a nonlinear control law using optimal feedback [3] designed for a given trajectory task. This law is given by the solution by the iterative method of a sequence of nonlinear Hamilton-Jacobi-Bellman equations. Aghanouri *et al.* [4] developed a manipulator actuated by DC motors. The optimum path is derived by optimizing system parameters to minimize performance indices including energy [5]. Talezadeh *et al.* [6] presented the nonlinear dynamics of two-link manipulators for dynamic modeling of the system by the Lagrange equation of motion where optimum control was adopted to analyse the motion of the manipulator. Some literature has presented the whole-body dynamics of nonlinear equations of the hybrid cable-driven robots (HCDRs) where new methods were developed to solve redundancy. Compared to the existing methods, torque optimization for actuated and un-actuated joints can solve resolution problems as well as active satisfactory disturbance [7].

Baressi *et al*. [8] describe the kinematic modeling using the Denavit Hartenberg (DH) convention and the dynamic modeling robotic arm has been presented by Lagrange-Euler (LE) and Newton-Euler algorithms. Different algorithms such as artificial intelligence, genetic algorithm, simulated annealing, and differential evolution have been adopted and compared to provide the best results for minimization of torque. Televnoy *et al*. [9] provides Lagrange equations of motion of a nonlinear matrix equation. The experiment has been performed to find out the solution of a moving object between two points. The angular displacement, speed, and acceleration for six links of the manipulator have been presented. Agustian *et al*. [10] present the vision-based robotic manipulator where inverse kinematics using pseudo-inverse Jacobian (PIJ) and DH forward kinematics have been adopted and controlled by a proportional derivative controller. The sorting task depending on color was made to evaluate the error to implement the manipulator on a real system. The reviews was given [11] on dynamic analysis and intelligent control techniques for flexible robot manipulators. A comparative study of dynamic analysis and control strategies was presented for flexible manipulators. Korayem *et al*. [12] have determined the non-linear dynamics and control of flexible mobile manipulators focusing on the determination of maximum payload. Wu *et al*. [13] present the minimum actuator torque range by torque optimization of a 3-DOF parallel manipulator. Two approaches, Lagrangian and primal-dual neural network, were presented together to make real-time optimization of joint torque for kinematically redundant manipulators [14], [15]. Gao *et al*. [16] proposed joint torque optimization of flexible manipulators with redundant space and vibration suppression where the Lagrange method has been adopted to represent the dynamics of the robot arm. Naghshineh and Keshmiri [17] investigated an over actuated system for dynamic cost function by applying real time optimization on a cooperative robot. Wolniakowski *et al*. [18] presented a method required for joint torque minimization of a serial manipulator to determine the optimal task placement. Woolfrey *et al*. [19] described a control strategy for minimizing joint torque using null space control of a redundant manipulator where the dynamic torque has been reduced by applying an external force to the gripper element. Agbaraji *et al*. [20] presented the design of the manipulator by calculating and analyzing the joint torques by evaluating performance in terms of speed and displacement of the arm based on the predetermined values of the torques. Singh *et al*. [21] find the position vectors of the six-arm robot by forward kinematics and joint angles by inverse kinematics in MATLAB with the help of a robotic toolbox. Brandstotter *et al*. [22] and Petrenko *et al*. [23] presented a generalized closed-form solution of the dynamic models of parallel robots using some simple Jacobian matrices where the dynamics of the legs were expressed in the joint coordinates and that of the platform in the form of cartesian variables. Sun *et al*. [24] proposed the methodology for deriving the closed-form inverse kinematic solutions of the 6-DOF robot on the position level where the analytical inverse solution of all the joints was given out and compared with that of the forward kinematic solution. The mathematical formulation and simulation of the two-link planar robot manipulator with forward kinematics and dynamics were presented with the help of the D-H convention and Newton-Euler method where the results in terms of joint angles and angular velocities have been presented graphically [25].

In this paper, the Newton-Euler formulation has been adopted to present the dynamics of the three-link manipulator. The solution relating torque and angular displacement is obtained analytically by expressing the three second-order non-linear non-homogeneous differential equations into six first-order differential equations by phase variable method. The angular positions and speed of each link have been presented graphically for constant torques of fixed duration using MATLAB. Then optimization is done using a genetic algorithm to obtain minimum torque that can be applied to each link-joint actuator to obtain optimum results.

## 2. THREE LINK ROBOT MANIPULATOR

The length of the three links of the robotic manipulator is given by $l_1, l_2$ *and* $l_3$. Three joints are named $J_1$,$J_2$,$J_3$, respectively, as in Figure 1. $m_1$, $m_2$, $m_3$ are the mass of the first link, second link and third link, respectively. Link parameters are considered to be $\alpha_1 = \alpha_2 = \alpha_3 = 0$. All the three joints are revolute joints rotating about the z-axis, and all the links are rigid links. The angular rotations of each link-joint combination are denoted as $\theta_1$,$\theta_2$, $\theta_3$.

The initial conditions are assumed to be

$$\omega_0 = 0 \ , \ \dot{\omega}_0 = 0 \ , \ v_0 = 0$$

where $\omega_0$, initial angular speed, $\dot{\omega}_0$ , initial angular acceleration, $v_0$, initial linear velocity and $\dot{v}_0$,initial linear acceleration. Therefore,

$\dot{v}_0 = [0 \quad 9.81 \quad 0]^T$
$q_i$= [$\theta_1$,$\theta_2$,$\theta_3$];
$\dot{q}_i$= [$\dot{\theta}_1$, $\dot{\theta}_2$, $\dot{\theta}_3$]
$\ddot{q}_i$ = [$\ddot{\theta}_1$, $\ddot{\theta}_2$, $\ddot{\theta}_3$];
Link variable: $F_i, f_i, n_i, \tau_i$

where $F_i$ is the total external force exerted at the center of ith link, $f_i$ is the force exerted on ith link by i-1st link, $n_i$ is the moment acting on ith link by i-1st link, $\tau_i$ is the torque on ith joint.

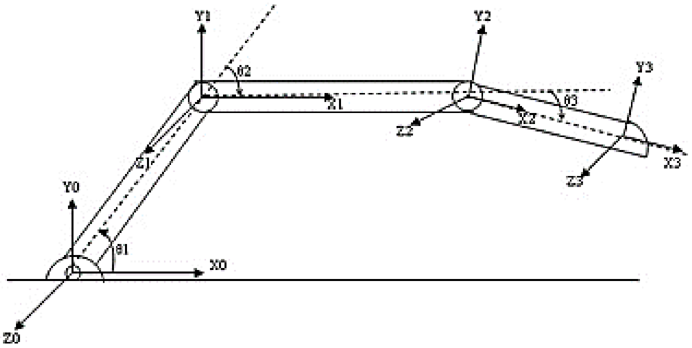


Figure 1. Coordinate assignment of the robot manipulator

## 3. NEWTON-EULER FORMULATION FOR COMPUTATION OF JOINT TORQUE

Computational work has been carried out using Newton – Euler methodology for transformations of three link coordinates to formulate the speeds of three-link manipulator. Initial speed of each link is assumed to be zero. For the kinematic model, computation of each joint and link variables is required for computation of each joint torque to move the manipulator in a desired direction. Forward kinematics and backward kinematics of motion has been applied in the Newton – Euler approach as presented in (1) to (3). $\tau_1$,$\tau_2$,$\tau_3$are the joint torque applied to the joint actuator for link i = 1, 2, 3, respectively.

$$\tau_3 = \frac{1}{2}m_3 l_1 l_3[cos(\theta_2+\theta_3)\ddot{\theta}_1 + sin(\theta_2+\theta_3)\dot{\theta}_1{}^2] + \frac{1}{2}m_3 l_2 l_3[sin\,\theta_3\,(\dot{\theta}_1+\dot{\theta}_2)^2 + cos\,\theta_3\,(\ddot{\theta}_1+\ddot{\theta}_2)] + \frac{1}{3}m_3 l_3{}^2(\ddot{\theta}_1+\ddot{\theta}_2+\ddot{\theta}_3) + \frac{1}{2}m_3 l_3 g\,cos(\theta_1+\theta_2+\theta_3) \quad (1)$$

$$\tau_2 = \frac{1}{2}m_3 l_1 l_3[cos(\theta_2+\theta_3)\ddot{\theta}_1 + sin(\theta_2+\theta_3)\dot{\theta}_1^2] + \frac{1}{2}m_3 l_2 l_3[sin\,\theta_3\,(\dot{\theta}_1+\dot{\theta}_2)^2 + cos\,\theta_3\,(\ddot{\theta}_1+\ddot{\theta}_2)] + \frac{1}{3}m_3 l_3^2(\ddot{\theta}_1+\ddot{\theta}_2+\ddot{\theta}_3) + \frac{1}{2}m_3 l_3 g\,cos(\theta_1+\theta_2+\theta_3) + (m_3 + \frac{1}{2}m_2)l_1 l_2[cos\,\theta_2\,\ddot{\theta}_1 + sin\,\theta_2\,\dot{\theta}_1^2] + (m_3+\frac{1}{3}m_2)l_2^2(\ddot{\theta}_1+\ddot{\theta}_2) + \frac{1}{2}m_3 l_2 l_3[cos\,\theta_3\,(\ddot{\theta}_1 + \ddot{\theta}_2+\ddot{\theta}_3) - sin\,\theta_3\,(\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3)^2] + (m_3+\frac{1}{2}m_2)l_2 g\,cos(\theta_1+\theta_2) \quad (2)$$

$$\tau_1 = \frac{1}{2}m_3 l_1 l_3[cos(\theta_2+\theta_3)\ddot{\theta}_1 + sin(\theta_2+\theta_3)\dot{\theta}_1{}^2] + \frac{1}{2}m_3 l_2 l_3[sin\,\theta_3\,(\dot{\theta}_1+\dot{\theta}_2)^2 + cos\,\theta_3\,(\ddot{\theta}_1+\ddot{\theta}_2)] + \frac{1}{3}m_3 l_3^2(\ddot{\theta}_1+\ddot{\theta}_2+\ddot{\theta}_3) + \frac{1}{2}m_3 l_3 g\,cos(\theta_1+\theta_2+\theta_3) + (m_3 + \frac{1}{2}m_2)l_1 l_2[cos\,\theta_2\,\ddot{\theta}_1 + sin\,\theta_2\,\dot{\theta}_1^2] + (m_3+\frac{1}{3}m_2)l_2^2(\ddot{\theta}_1+\ddot{\theta}_2) + \frac{1}{2}m_3 l_2 l_3[cos\,\theta_3\,(\ddot{\theta}_1 + \ddot{\theta}_2+\ddot{\theta}_3) - sin\,\theta_3\,(\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3)^2] + (m_3+\frac{1}{2}m_2)l_2 g\,cos(\theta_1+\theta_2) + (m_3+m_2+\frac{1}{3}m_1)l_1^2\ddot{\theta}_1 + (m_3+\frac{1}{2}m_2)l_1 l_2[cos\,\theta_2\,(\ddot{\theta}_1+\ddot{\theta}_2) - sin\,\theta_2\,(\dot{\theta}_1+\dot{\theta}_2)^2] - \frac{1}{2}m_3 l_1 l_3[sin(\theta_2+\theta_3)(\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3)^2 - cos(\theta_2+\theta_3)(\ddot{\theta}_1+\ddot{\theta}_2+\ddot{\theta}_3)] + (m_3 + m_2 + \frac{1}{2}m_1)l_1 g\,cos\,\theta_1 \quad (3)$$

## 4. COMPUTED JOINT TORQUE CHARACTERISTICS OF EACH LINK-JOINT PAIR

The dynamic equations of motion of each link-joint coupling are highly non-linear in nature as shown in (1) to (3). To solve the above equations of motion relating $\tau_i$and $\theta_i$ (for ith link), phase variable model has been adopted. The three non-linear non-homogeneous 2nd order differential equations are converted into six first-order differential equations by phase variable method. Assuming $z_1, z_2, z_3, z_4, z_5\ and\ z_6$ are six phase variables where:

$$z_1 = \dot{\theta}_1 \quad (4)$$

$$z_2 = \dot{\theta}_2 \quad (5)$$

$$z_3 = \dot{\theta}_3 \quad (6)$$

$$z_4 = \theta_1 \quad (7)$$

$$z_5 = \theta_2 \quad (8)$$

$$z_6 = \theta_3 \quad (9)$$

Therefore:

$$\dot{z}_1 = \ddot{\theta}_1 \quad (10)$$

$$\dot{z}_2 = \ddot{\theta}_2 \quad (11)$$

$$\dot{z}_3 = \ddot{\theta}_3 \quad (12)$$

$$\dot{z}_4 = \dot{\theta}_1 = z_1 \quad (13)$$

$$\dot{z}_5 = \dot{\theta}_2 = z_2 \quad (14)$$

$$\dot{z}_6 = \dot{\theta}_3 = z_3 \quad (15)$$

Non-dimensionalizing (1), (2) and (3), we have

$$\tau_3 = \frac{1}{2}cos(\theta_2 + \theta_3)\ddot{\theta}_1 + \frac{1}{2}sin(\theta_2 + \theta_3)\dot{\theta}_1{}^2 + \frac{1}{2}sin\,\theta_3\,(\dot{\theta}_1 + \dot{\theta}_2)^2 + \frac{1}{2}cos\,\theta_3\,(\ddot{\theta}_1 + \ddot{\theta}_2) + \frac{1}{3}(\ddot{\theta}_1 + \ddot{\theta}_2 + \ddot{\theta}_3) + \frac{1}{2}cos(\theta_1 + \theta_2 + \theta_3) \quad (16)$$

$$\tau_2 = \frac{1}{2}cos(\theta_2 + \theta_3)\ddot{\theta}_1 + \frac{1}{2}sin(\theta_2 + \theta_3)\dot{\theta}_1^2 + \frac{1}{2}sin\,\theta_3\,(\dot{\theta}_1 + \dot{\theta}_2)^2 + \frac{1}{2}cos\,\theta_3\,(\ddot{\theta}_1 + \ddot{\theta}_2) + \frac{1}{3}(\ddot{\theta}_1 + \ddot{\theta}_2 + \ddot{\theta}_3) + \frac{1}{2}cos(\theta_1 + \theta_2 + \theta_3) + \frac{3}{2}cos\,\theta_2\,\ddot{\theta}_1 + \frac{3}{2}sin\,\theta_2\,\dot{\theta}_1^2 + \frac{4}{3}(\ddot{\theta}_1 + \ddot{\theta}_2) + \frac{1}{2}cos\,\theta_3\,(\ddot{\theta}_1 + \ddot{\theta}_2 + \ddot{\theta}_3) - \frac{1}{2}sin\,\theta_3\,(\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3)^2 + \frac{3}{2}cos(\theta_1 + \theta_2) \quad (17)$$

$$\tau_1 = \frac{1}{2}cos(\theta_2 + \theta_3)\ddot{\theta}_1 + \frac{1}{2}sin(\theta_2 + \theta_3)\dot{\theta}_1{}^2 + \frac{1}{2}sin\,\theta_3\,(\dot{\theta}_1 + \dot{\theta}_2)^2 + \frac{1}{2}cos\,\theta_3\,(\ddot{\theta}_1 + \ddot{\theta}_2) + \frac{1}{3}(\ddot{\theta}_1 + \ddot{\theta}_2 + \ddot{\theta}_3) + \frac{1}{2}cos(\theta_1 + \theta_2 + \theta_3) + \frac{3}{2}cos\,\theta_2\,\ddot{\theta}_1 + \frac{3}{2}sin\,\theta_2\,\dot{\theta}_1^2 + \frac{4}{3}(\ddot{\theta}_1 + \ddot{\theta}_2) + \frac{1}{2}cos\,\theta_3\,(\ddot{\theta}_1 + \ddot{\theta}_2 + \ddot{\theta}_3) - \frac{1}{2}sin\,\theta_3\,(\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3)^2 + \frac{3}{2}cos(\theta_1 + \theta_2) + \frac{7}{3}\ddot{\theta}_1 + \frac{3}{2}cos\,\theta_2\,(\ddot{\theta}_1 + \ddot{\theta}_2) - \frac{3}{2}sin\,\theta_2\,(\dot{\theta}_1 + \dot{\theta}_2)^2 - \frac{1}{2}sin(\theta_2 + \theta_3)(\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3)^2 + \frac{1}{2}cos(\theta_2 + \theta_3)(\ddot{\theta}_1 + \ddot{\theta}_2 + \ddot{\theta}_3) + \frac{5}{2}cos\,\theta_1 \quad (18)$$

Using (16) to (18), $\ddot{\theta}_1$, $\ddot{\theta}_2$ and $\ddot{\theta}_3$ can be expressed as a function of $\theta_1$, $\theta_2$, $\theta_3$ and their derivatives. Therefore, six first order equations can be formed from (10) to (15) and can be solved using MATLAB which gives the relationship of angular displacement and applied torque for various torque ratios as shown in Figures 2 to 8.

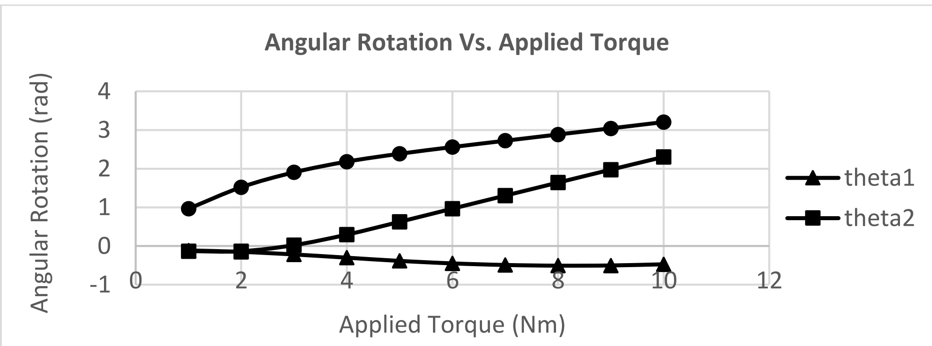


Figure 2. Angular position versus applied torque (τ1: τ2: τ3: 1:1:1 (in Nm); time: 0.5sec)

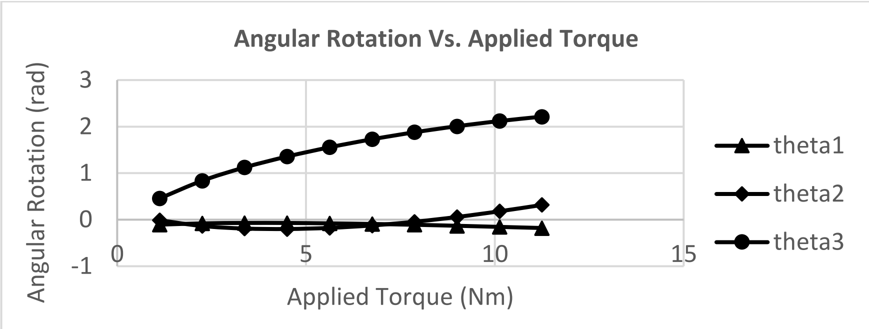


Figure 3. Angular position versus applied torque (τ1: τ2: τ3: 2.25:1.5:1 (in Nm); time: 0.5sec)

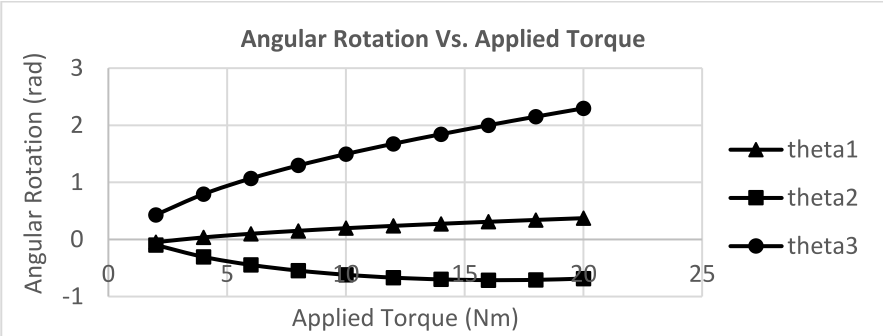


Figure 4. Angular position versus applied torque (τ1: τ2: τ3: 4:2:1 (in Nm); time: 0.5sec)

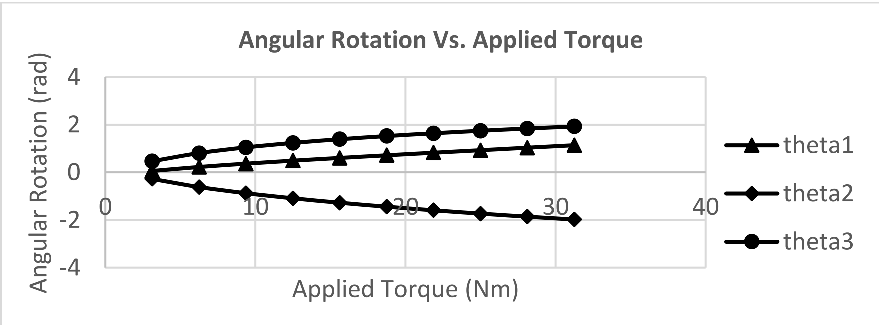


Figure 5. Angular position versus applied torque (τ1: τ2: τ3: 6.25:2.5:1 (in Nm); time: 0.5sec)

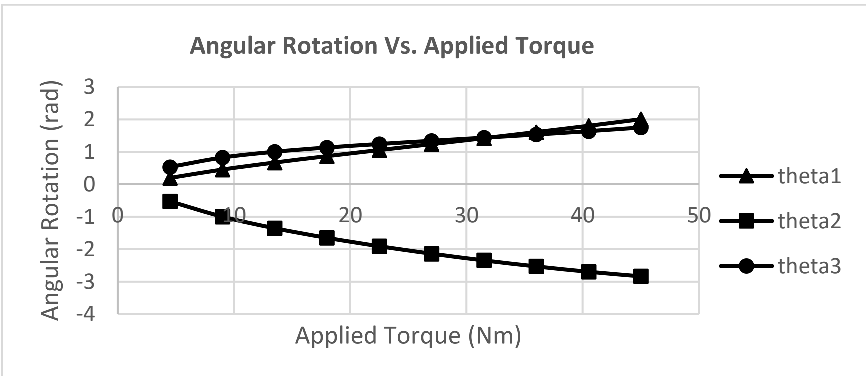


Figure 6. Angular position versus applied torque (τ1: τ2: τ3: 9:3:1 (in Nm); time: 0.5sec)

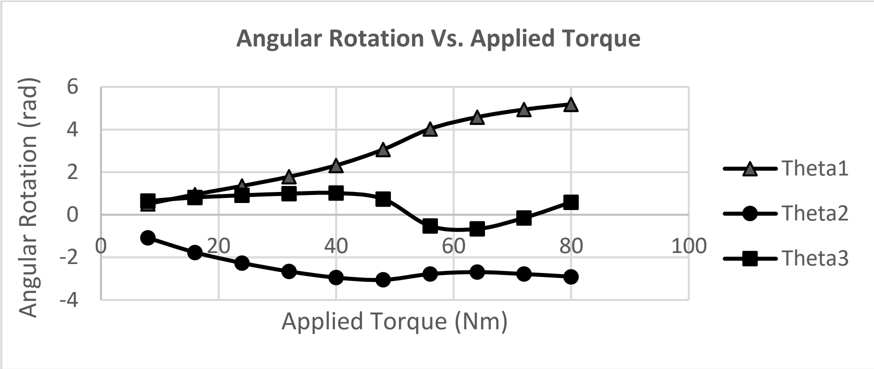


Figure 7. Angular position versus applied torque (τ1: τ2: τ3: 16:4:1 (in Nm); time: 0.5sec)

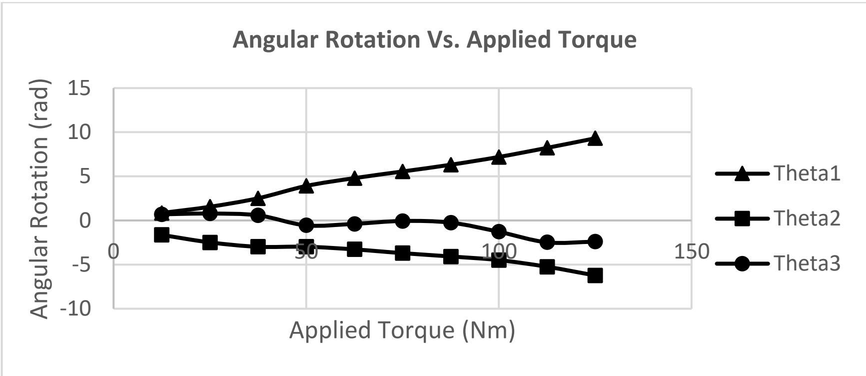


Figure 8. Angular position versus applied torque (τ1: τ2: τ3: 25:5:1 (in Nm); time: 0.5sec)

## 5. TORQUE OPTIMIZATION USING GENETIC ALGORITHM IN MATLAB

To optimize $\tau_3$, joint torque in the third joint actuator has been plotted against $\tau_2$ and $\theta_2$ as shown in Figure 9. Genetic algorithm has been applied to minimize the objective function given in (19). Optimized value of $\boldsymbol{\theta_3}$ has been evaluated by plotting $\boldsymbol{\theta_3}$ with respect to evaluated $\boldsymbol{\theta_1}$and $\boldsymbol{\theta_2}$ as shown in Figure 10. The optimized results of all the required parameters are given in Table 1.

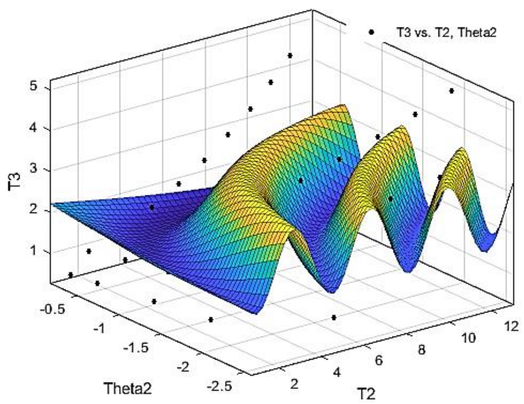


Figure 9. Surface generation of T3 for torque optimization

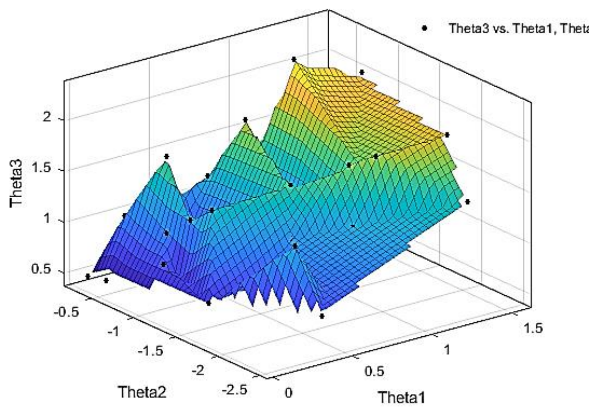


Figure 10. Surface generation of $\theta_3$ with respect to $\theta_1$ and $\theta_2$

Table 1. Optimized torque and corresponding angular displacement

| $\tau_1$(in Nm) | $\tau_2$(in Nm) | $\tau_3$(in Nm) | $\theta_1$ (rad) | $\theta_2$(rad) | $\theta_3$(rad) |
|---|---|---|---|---|---|
| 31.25 | 12.5 | 1.3658 | 1.1376 | -0.1861 | 1.3202 |

$$\tau_3 = a + b sin(m\pi x_1 x_2) + c e^{-(w x_2)^2} \quad (19)$$

where *a*, *b*, *c*, *m*, and *w* are constant coefficients of the objective function (19) and evaluated in MATLAB as

$$a = 2.683$$
$$b = 0.9224$$
$$c = -0.4006$$
$$m = 0.2138$$
$$w = -0.6463$$

$x_1 and\ x_2$ are the torque required and resulting angular displacement of the link2, respectively. Boundary condition has been placed as $1 \leq x_1 \leq 12.5$ and $-3 \leq x_2 \leq 0$.

From Table 1, it is evident that the results show a successful optimization using genetic algorithm on the problem of torque optimization. For practical implementation, assumptions can be made that $\mathbf{0 \leq \theta_1 \leq \frac{\pi}{2}}$, $\mathbf{0 \leq \theta_2 \leq \frac{\pi}{2}}$ and $\mathbf{0 \leq \theta_3 \leq \frac{\pi}{2}}$ and the condition$|\theta_2| \leq 2\theta_1$must be followed if $\theta_1$ is positive and $\theta_2$ found to be negative. Therefore, the result shown is to satisfy the above-mentioned condition and the joint torques as well as corresponding angular displacement may be evaluated as given in Table 1.

## 6. CONCLUSION

The manipulator dynamics, as stated in (2) to (4), represent the non-linearity of the robot arm system. Here the open chain of links and its corresponding behavior of the manipulator has been studied which shows the angular position vs. applied torque characteristics. The optimum value of torque with respect to the desired movement of the manipulator arm has been evaluated using a genetic algorithm with boundary conditions in MATLAB which also satisfies the relative angular position constraints. Also, the system may be assumed to be a series of three inverted pendulums exhibiting the nature of a chaotic system which is certainly nonlinear in nature. Therefore, stability of the system may be achieved with the help of a sliding mode controller as well as by behavior-based control where it is to split a complex dynamic into several simple equations which are quietly related to the problem stated in this research paper.

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