Use of artificial intelligence in banknote reconstruction

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ABSTRACT

Banknotes may be damaged during various events, such as floods, fires, insect infestations, and mechanical or manual shredding. Disaster victims might need to perform banknote reconstruction when applying for currency exchange, or investigative agencies might need to conduct such reconstruction during evidence collection. When the number of banknote fragments is small, they can be manually assembled; however, when this number is large, manual assembly becomes increasingly difficult and time-consuming. Therefore, an automated and effective method is required for banknote reconstruction. The process of banknote reconstruction can be considered similar to solving a large-scale jigsaw puzzle. This study employed an artificial intelligence (AI) system to reconstruct damaged banknotes. A robotic arm was used to replace manual separation and automated digital image processing techniques, and AI image registration technology, deep learning, and logical operations were utilized. A deep convolutional neural network was used to estimate the relative homography between images, and fragmented banknotes were mapped to a reference banknote for image transformation, thereby reconstructing the damaged banknotes. Additionally, a repetitive matching method was established to optimize the matching results to achieve the best possible mapping and enhance validation efficiency.

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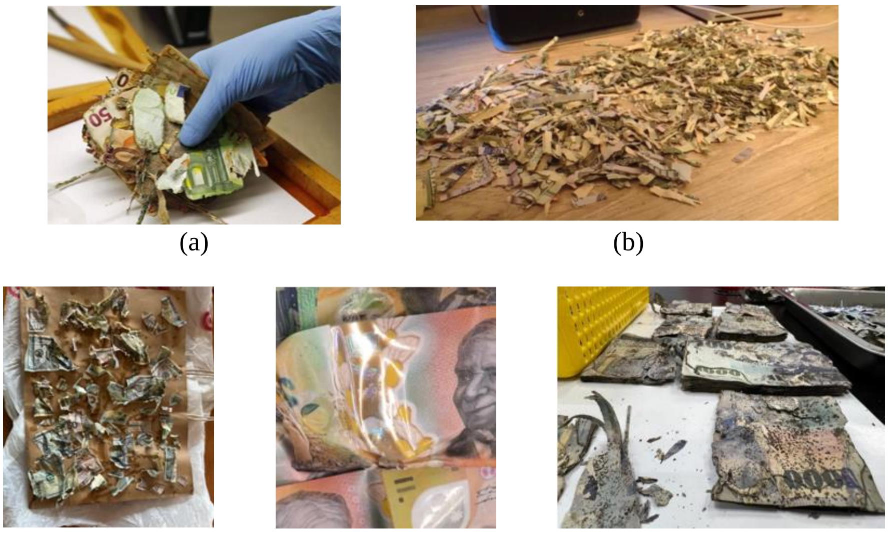
1. INTRODUCTION

Utilizing artificial intelligence (AI) to solve puzzle games is a critical research direction because of its practical application value. Two-dimensional puzzles cover a wide range of applications, and AI tools have been increasingly used to solve puzzles in fields such as finance, multimedia, forensic science, and medicine. From a macro perspective, numerous research problems across different fields can be viewed as puzzle problems. Such problems include the reconstruction and preservation of cultural heritage [1]–[3], DNA assembly [4], [5], the repair of damaged or degraded documents [6], [7], the proofreading and classification of text documents [8]–[11], the matching and creation of three-dimensional (3D) geometric models [12], [13], the reconstruction and removal of neuronal artifacts [14], the extraction and detection of biomedical signals [15]–[21], image forgery detection [22]–[26], image reconstruction [27]–[31], speech descrambling [32], drug molecule docking design [33], lung cancer detection [34], and brain decoding through magnetic resonance imaging [35]. Puzzles are a common benchmark for assessing computer vision and AI technologies. Puzzle-solving systems typically apply various techniques, including computer vision, partial boundary matching, pattern recognition, and combinatorial optimization. The methods used to solve puzzles can be used to solve problems efficiently in various fields, such as science, engineering, and business.

In criminal investigations, civil litigations, financial fraud inquiries, human accidents, and natural disasters, police, investigative bodies, or cultural institutions might need to reassemble fragments of banknotes, ancient coins, paintings, and documents to collect evidence and comprehend the full scope of incidents for maintaining historical records. In one case, a man in Taiwan accidentally dropped a bag containing over NT$200,000 in banknotes into a shredder. Subsequently, officials from Taiwan's Ministry of Justice Investigation Bureau reassembled the banknote fragments and applied them to the Central Bank of the Republic of China (Taiwan) for exchange. This bureau receives approximately 250 applications annually for the reconstruction of damaged banknotes [36]. The 2013 floods in Germany caused considerable damage to a large number of banknotes, prompting the Deutsche Bundesbank to assemble a team of scientists to reconstruct these banknotes [37]. Incidents of banknotes being damaged by humans, animals, or disasters occur globally [38]–[41]. The Bureau of Engraving and Printing in the United States receives numerous requests each year to inspect damaged currency, and in 2018 alone it handled cases valued over US$40 million [42]. In the weeks following the fall of the Berlin Wall in 1989, the Ministry for State Security of East Germany (the Stasi) attempted to destroy intelligence documents, creating over 16,000 bags of shredded paper. The German government continues its efforts to recover these documents to this day [43]. Several World Heritage Sites, including Notre-Dame Cathedral in Paris, the National Museum of Brazilian which housed over 20 million artifacts and 530,000 books, the bell tower of the Novodevichy Monastery in Moscow, La Fenice Opera House in Venice, Windsor Castle in the United Kingdom, the Great Mosque of al-Nuri in Iraq, and the Umayyad Mosque in Syria, have been destroyed by fire or war [44]. The aforementioned events demonstrate the critical need for fragment recovery in various sectors, such as banking, forensic science, document restoration, and historical artifact reconstruction.

This study employed AI technology to assist in the reconstruction of banknote fragments. Common causes of banknote damage include exposure to fire, water, mechanical forces, chemicals, and explosives; damage from insects and animals; and deterioration or petrification due to burial. Figure 1 presents damaged banknotes in several different situations: Figure 1(a) shows euro banknotes damaged during floods [37]. Figure 1(b) shows $1,000 US bill shredded by a machine [38]. Figure 1(c) shows US dollars damaged by dog bites [40]. Figure 1(d) shows Australian dollars warped from heat [41]. Figure 1(e) shows New Taiwan dollars damaged by insects due to burial. Manual matching is the most direct method for reconstructing banknotes. Manually piecing together banknote fragments requires skill, time, and attention to detail. It primarily involves classifying banknote fragments on the basis of their edge shapes, sizes, and colors. An image of a complete banknote is used as a reference, and fragments are assembled on the basis of features such as shape, pattern, texture, anti-counterfeiting features, and even the traces of the damage. This process requires multiple trials and adjustments to ensure correct placement. Fragile and broken pieces might undergo secondary damage because of the need for repeated handling during manual matching. When the number of fragments is large, manual matching can necessitate considerable human resources.



(a) (b)

(c) (d) (e)

Figure 1. Damaged banknotes in several different situations: (a) Euros damaged by flooding [37], (b) US dollars shredded by machinery [38], (c) US dollars damaged by dog bites [40], (d) Australian dollars damaged by heat [41], and (e) New Taiwan dollars damaged by insects and burial

Currently, artificial intelligence research in the field of banknotes mainly focuses on the development of banknote materials and anti-counterfeiting technologies [45]–[47]. The use of AI to reconstruct damaged banknotes is a relatively underexplored area, with limited references being available. Reassembling banknote fragments is a two-dimensional, double-sided puzzle problem with only one solution [48]. Due to the need to evaluate both sides of the fragments, along with potential issues like folds, wrinkles, dirt, and missing pieces, as well as the highly complex shapes of the fragments, this task is more challenging compared to other types of puzzle problems.

In general, AI-based methods for the assembly of puzzle pieces can be categorized into two main types: methods based on shape, color, or texture and methods based on key points. Methods based on shape, color, and texture involve the extraction of the shape, color, and texture, respectively, of fragments and then the matching and assembly of fragments on the basis of the extracted information [49]–[52]. The key point-based method involves using a reference image related to the fragments, extracting features from the reference image and fragments, and matching the fragments to the reference image. This method is currently the mainstream approach in computer-assisted banknote fragment reconstruction systems [53]–[57].

Wu *et al.* [53] introduced a method involving the use of the scale-invariant feature transform (SIFT) [58] to reconstruct damaged banknotes. In this method, banknote fragments are matched with a reference image, and the random sample consensus (RANSAC) method [59] is then used to eliminate incorrect matching key points. It assumes all fragments originate from the same banknote and uses Quadratic Programming for assembly. Li *et al.* [54] extracted SIFT feature points from images of banknote fragments and complete banknotes and used the MSAC method [60] to eliminate incorrect matching key points and align the fragments. Gwo *et al.* [55] proposed a SIFT-based method that involves using a fuzzy logic approach to eliminate incorrect matching key points and developed a rotation angle estimation method to align fragments and then assemble them within an empty framework. Nabiyev *et al.* [56] used the AKAZE method to match fragments with a reference image and employed the RANSAC method to eliminate incorrect matching key points. They then used a novel minimum-convex-hull-based method to align the fragments before assembling them into an empty framework. Yilmaz and Nabiyev [57] implemented the SIFT, speeded-up robust features, binary robust invariant scalable key points, and AKAZE methods on a newly established dataset and then compared the results of these methods by using a Borda-count-based selection method.

Studies have indicated that dense or direct featureless simultaneous localization and mapping (SLAM) algorithms, such as large-scale direct monocular SLAM (LSD-SLAM), have promising prospects for application in geometric computer vision tasks involving the use of complete images. These algorithms differ from conventional feature-based methods that rely on extracting and matching unique features within images. Instead, SLAM algorithms act directly on pixel intensities, thereby enabling robust and accurate estimations of camera motion and scene geometry. The present study used a deep convolutional neural network called HomographyNet [61], which can directly produce the homographic related to two images, for matching images of banknote fragments.

2. METHOD

In the present study, banknotes damaged by fire, water, insects, oil, dye, and other causes were placed in an extraction cabinet to eliminate their moisture and sterilize them. The banknote fragments were then manually separated. Subsequently, each fragment was subjected to serial number and anti-counterfeiting feature checks to confirm its authenticity. Finally, after the authenticity of the fragments was verified, digital image processing was conducted.

To minimize the risk of further damage from the manual handling of fragile fragments, this study used an automated robotic arm for picking up and placing the banknote fragments. After the operation procedures were input to the robotic arm by an operator, the motor-driven robotic arm started to pick up and place the banknote fragments. The robotic arm shown in Figure 2, which was employed in this study, weighed 5.5 kg and comprised three joints with six degrees of freedom. It consisted of a mechanical body, a controller, a driver, a direct current motor, sensors, and a cycloidal pinwheel reducer. The robot could handle loads of up to 3 kg. It had a maximum working radius of 645 mm and a repeat positioning accuracy of ±0.05 mm. The tip of the robotic arm was equipped with a suction cup. When a natural background is used for banknote reconstruction, the clutters in the images and the need to convert from 3D to 2D may compromise image data, making reconstruction unnecessarily challenging. To address this problem, this study designed a fixed work area using the robotic arm and a digital camera to create an image database of the banknote fragments.

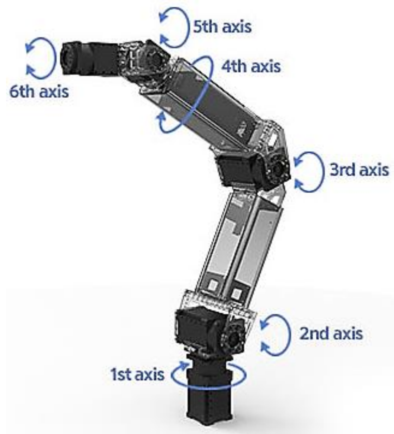


Figure 2. Robotic arm

The robotic arm picked up each banknote fragment individually and moved it to a photography area, where a digital camera photographed the fragment and transmitted the image to an AI matching system. The AI matching system conducted image analysis and sent feedback to the robotic arm, and the robotic arm moved to the next appropriate position on the basis of this feedback to continue the process of handling banknote fragments. The workflow of the robotic arm is shown in Figure 3. First, banknote fragments are divided into *n* piles, and *n*+*n* coordinates are set for the robotic arm to pick up and place the fragments. After the robotic arm detects that the photography of the first pile of banknote fragments is completed, it moves to the next pile of fragments. This process is repeated until the photography of the *n*-th pile of banknotes is completed. The AI matching system performs the following preprocessing steps on the images of banknote fragments: flipping the images of the reverse sides to match those of the front sides, removing backgrounds, eliminating white edges caused by tearing, reducing image noise, enhancing images, analyzing features, detecting edges, and sampling colors. Finally, the preprocessed images of the banknote fragments are reconstructed, and a report is generated. Figure 4 illustrates the reconstruction process for damaged banknotes.

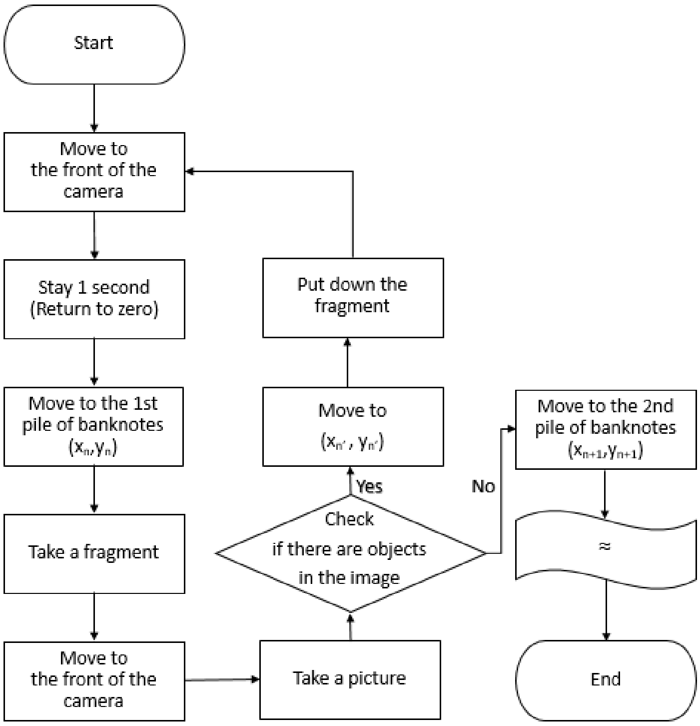


Figure 3. Workflow of the robotic arm

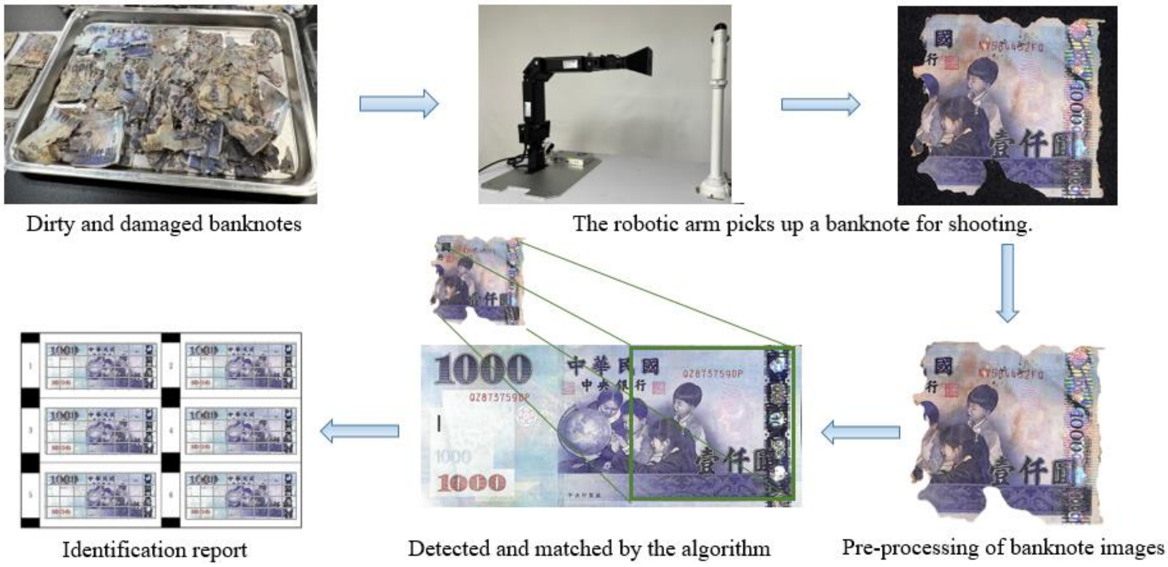


Figure 4. Schematic of the reconstruction of damaged banknotes

Deep learning models have achieved state-of-the-art performance in computer vision tasks, such as image classification, object detection, and image segmentation. The successive layers of convolutional neural networks can capture complex image features and learn task-specific characteristics. The transformation involving the rotation of two images around the camera center is homographic. A homography is crucial for constructing a panorama [62]. The present study employed HomographyNet, a VGG-type network proposed by DeTone *et al.* [61], to extract features to determine the original orientation and position of fragments on a complete banknote. The HomographyNet algorithm directly generates homographic related to two images. It uses the four-point planar projection transformation parameter, namely H_{4pt} [63], and is trained in an end-to-end manner, which eliminates the need for conventional two-stage homography estimation involving corner estimation and robust homography assessment. The parameter H_{4pt} integrates the scaling and shearing components of a homography and thus is more suitable for deep learning frameworks than is the conventional 3×3 parameter H. The architecture of HomographyNet is similar to that of VGGNet [64] in that HomographyNet consists of 3×3 convolutional blocks with the batch norm [65] and rectified linear unit functions. The structure of HomographyNet, shown in Figure 5, has eight convolutional layers, with max pooling layers following every two convolutional layers. Additionally, it contains two fully connected layers.

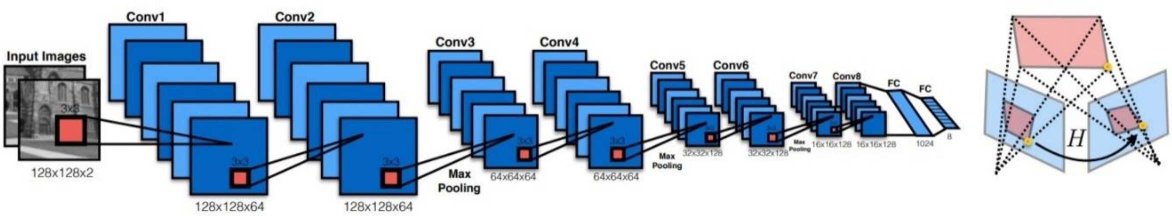


Figure 5. Deep image homography estimation - HomographyNet (modified from [61])

A large amount of data is required to train a deep convolutional network from scratch to compute features related to fragments. To obtain sufficient training data, this study randomly cut images of complete banknotes into 100,000 images of front and back fragments and then applied random projective transformations to generate numerous labeled training examples. Initially, a square patch A was cropped out from the original image I, with the four corners of the image being perturbed within a random value in the range of $[-\rho, \rho]$. The four corresponding transformation relationships were defined as the homography H^{AB}. $(H^{AB})^{-1}$ was applied to the original image I to generate a distorted image I'. A patch B corresponding to image patch A was cropped out from this distorted image I'. The four-point parameters H^{AB} served as training labels linked to the ground truth and were saved in a four-point parameterized format as H^{*}_{4pt}. Two images with a homography relationship and a size of 128×128×2 were stacked into channels for input into HomographyNet to obtain a four-point parameterized output, $\widetilde{H}_{4pt}$.

The reconstruction of damaged banknote fragments has the following characteristics. First, unlike unknown historical artifact fragments, the images of complete banknotes are available and can be used as prior information. Second, if the content of two banknote fragments overlaps, they originate from different banknotes. Third, because damaged banknotes typically have missing fragments, fragments from the same batch that cannot be directly joined but do not overlap can be considered as part of the same banknote. This approach maximizes the surface area of a single banknote to increase the redeemable amount. According to Standards for Redemption of Soiled or Mutilated Currency Notes and Coins Unfit for Circulation published by the Central Bank of Taiwan, if the remaining parts of a damaged banknote constitute more than three-quarters of the original banknote, the full amount can be redeemed. If the remaining parts constitute more than half but less than three-quarters of the original banknote, half the full amount is redeemed. For all aligned fragments, this study extracted local features related to shape and content to determine their paired relationships. The constraints between fragments were considered to evaluate alignment and assign weights to edges for global matching. Finally, a greedy search method was employed to find a simple graph with the maximum compatible edge sets. The clustered fragments were embedded into a framework, with overlapping clusters filling the next empty frame. According to the regulations of the Bank of Taiwan, the remaining proportion was calculated based on the table in Figure 6. Priority is given to assembling damaged and soiled banknotes that can reach an area of 3/4 or more, and only then attempting to piece together those that cover more than 1/2 of the area. Subsequently, the redeemable amount was automatically calculated.

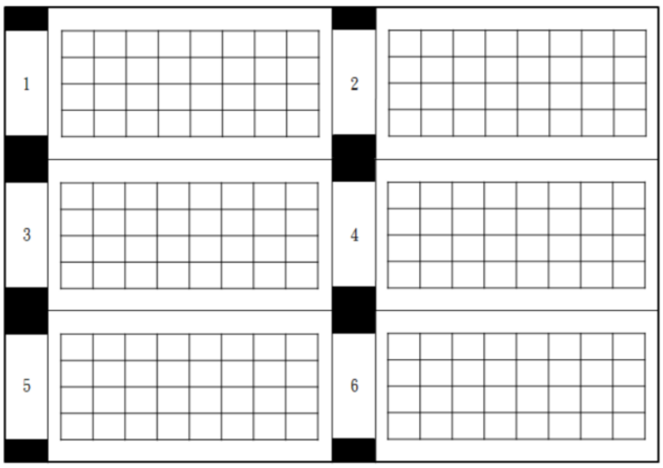


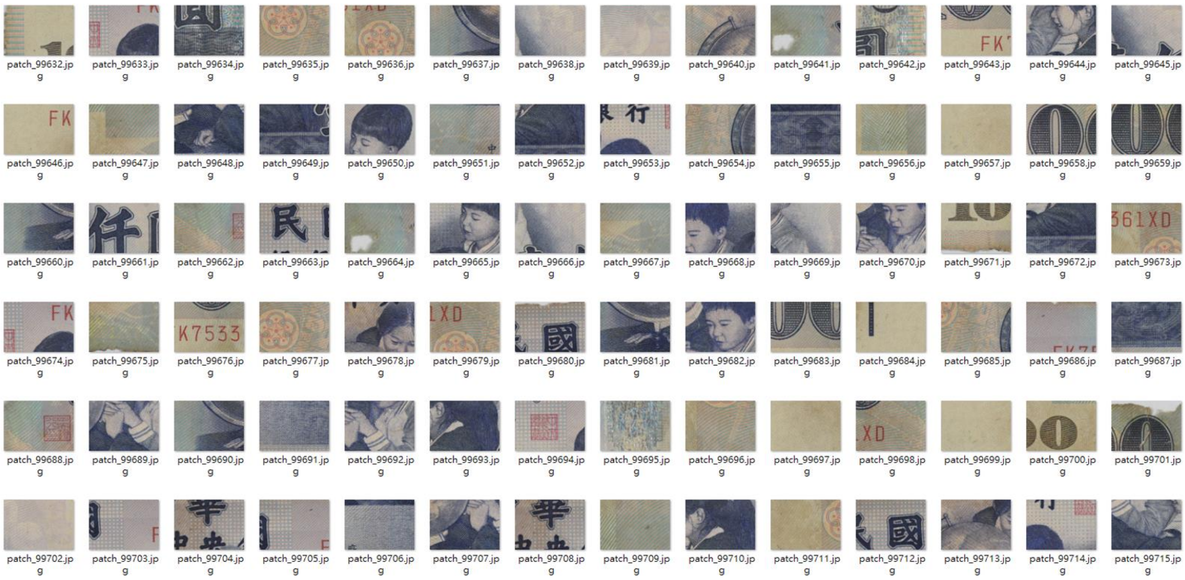
Figure 6. Identification analysis table

3. RESULTS AND DISCUSSION

To evaluate the performance of HomographyNet on a dataset of damaged banknotes, this study designed two experiments to conduct homographic comparison tests between the deep learning approach based on HomographyNet and a conventional feature-based method, SIFT. The system was trained using intact NT$1,000 banknotes from the AN-I edition and damaged NT$1,000 banknotes from the AN-II edition separately to obtain diversified test results. The objective was to determine whether HomographyNet, trained with a synthetic dataset, has performance comparable to or better than SIFT in the application of reconstruction of damaged banknotes.

The SIFT directly uses full images to estimate the homography of planar projection transformation, whereas the deep learning method uses small data patches and requires the segmentation of full images. In this study, images of complete banknotes were randomly cut into 100,000 images of front and back fragments for deep learning, as shown in Figure 7. Figure 7(a) shows an intact NT$1,000 banknote from the AN-I series, while Figure 7(b) shows a damaged NT$1,000 banknote from the AN-II series. The brightness (0.5–2.0), color (0.8–1.2), and gamma shift (0.8–1.2) of the images were randomly varied during training, and the L1 photometric loss function was employed. Testing was then performed using a randomly created test set of banknote fragments.

(a)



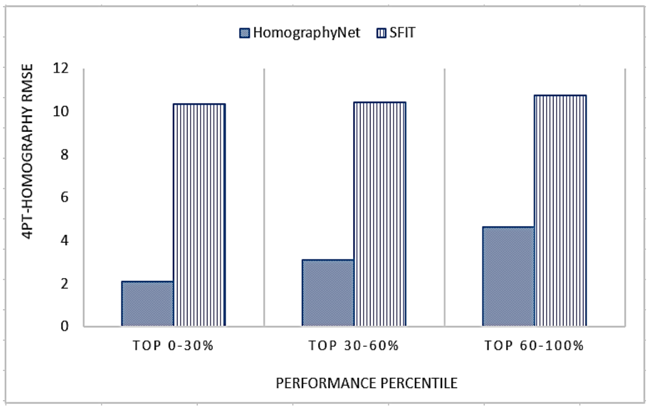
(b)

Figure 7. Randomly cut banknote fragment images (a) intact NT$1,000 banknote from AN-I edition and (b) damaged NT$1,000 banknote from AN-II edition

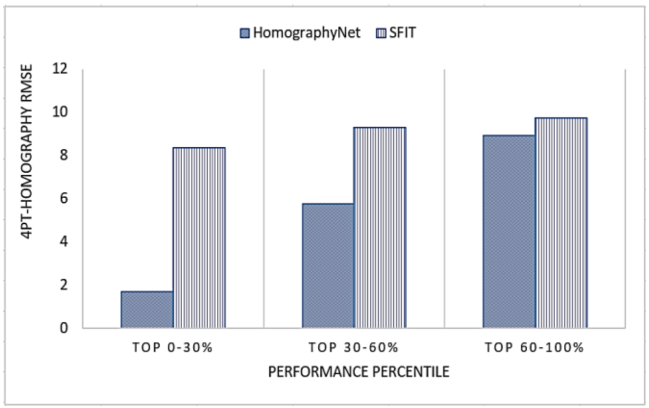
The evaluation metric RMSE originates from the mean corner error in both adopted algorithms. The estimated homography $\widetilde{H}_{4pt}$ and ground-truth homography H^*_{4pt} were compared using (1), in which L_H represents the Euclidean $L2$ norm between $\widetilde{H}_{4pt}$ and H^*_{4pt}. A lower RMSE indicates superior performance.

$$L_H = \frac{1}{2}\left\|\widetilde{H}_{4pt} - H^*_{4pt}\right\|_2^2 \quad (1)$$

Figure 8 illustrates the performance percentile analysis results for the two adopted algorithms. Figures 8(a) and 8(b) display the results obtained for the intact NT$1,000 banknote from the AN-I edition and the damaged NT$1,000 banknote from the AN-II edition, respectively. HomographyNet considerably outperformed SIFT on the AN-I edition. The RMSE values of the SIFT were consistently greater than 10, whereas the RMSE values of HomographyNet were only 2.11 and 4.65 in the best 30% and the worst 40% of the time, respectively. For the AN-II edition, the RMSE values of the SIFT were as high as 8.37 and 9.75 in the best 30% and worst 40% of the time, respectively. In contrast, the RMSE value for HomographyNet is only 1.69 in the best 30% of the time. Although the RMSE value reached 8.92 in the worst 40% of the time, its performance still surpassed that of the feature-based method (SIFT). The homography estimation is visualized in Figure 9. The original image is deformed by randomly perturbing its four corners. The deformed image is then restored using different homography estimation methods, with lower RMSE values indicating better performance.



(a)



(b)

Figure 8. 4pt-homography RMSE values from the two adopted algorithms: (a) intact NT$1,000 banknote from AN-I edition and (b) damaged NT$1,000 banknote from AN-II edition

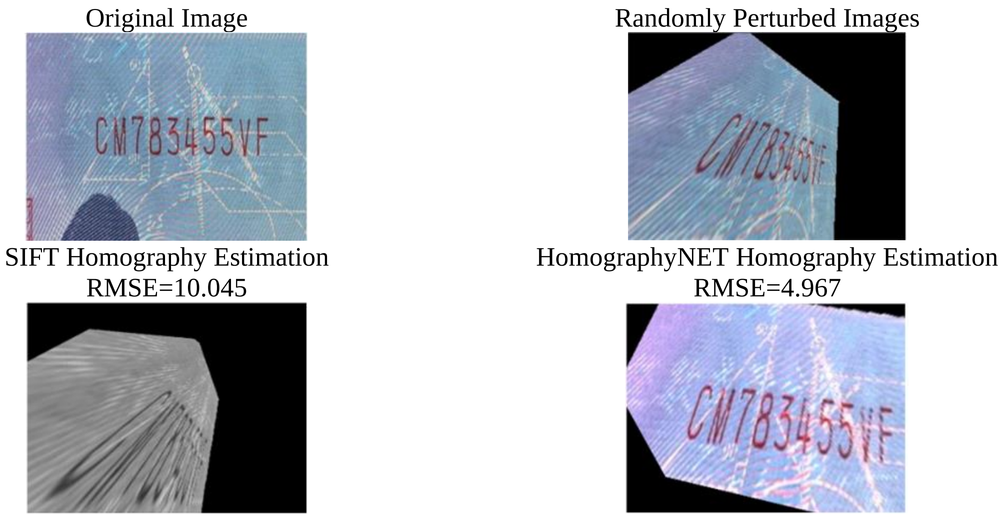


Figure 9. Homography estimation: SIFT vs HomographyNet

The traditional feature-based method, SIFT, can extract a large number of invariant feature points, performing stably under different lighting conditions and exhibiting favorable scale, rotation, and viewpoint invariance. However, during the testing process of this study, the SIFT exhibited a low matching speed and performed relatively poorly. An example of recognition failure is shown in Figure 10. The main reasons for its poor performance were as follows: i) high computational complexity, which was caused by the SIFT using Gaussian kernels and DOG operators to extract and detect feature points, resulting in lengthy computation times; ii) inaccurate positioning of corners during the extraction of feature points, which resulted in the feature points being inaccurately reflected in the image structure, leading to incorrect matches; and iii) sensitivity of the SIFT to noise, which caused poor performance when processing images of dirty or damaged banknotes. By contrast, HomographyNet exhibited suitable testing performance. This algorithm employs supervised learning and is trained using a large quantity of prelabeled data. During the learning processes, the algorithm examines its errors, corrects its understanding, and enhances the accuracy of its predictions, thereby enhancing its reconstruction performance.

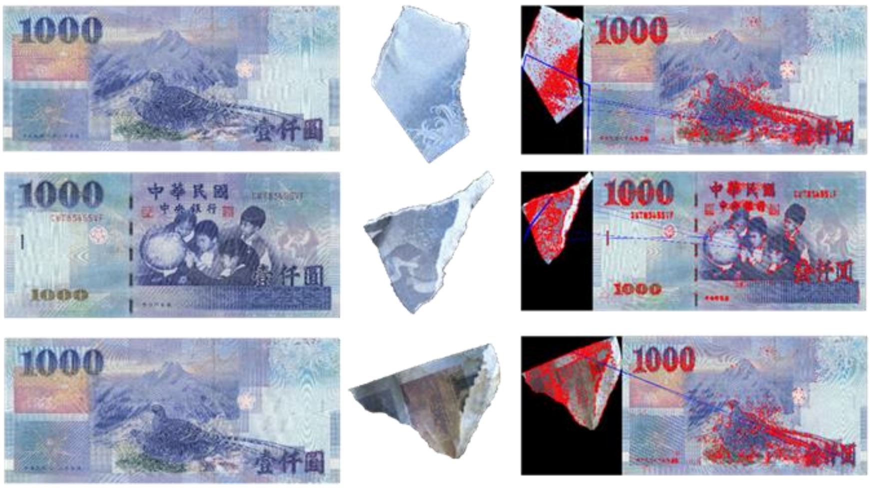
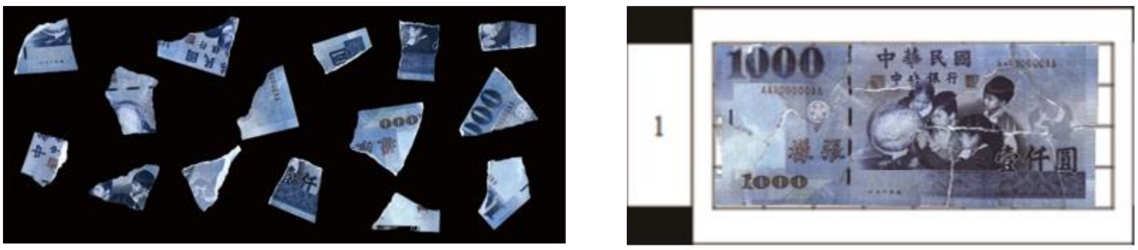


Figure 10. Case of recognition failure with the SIFT

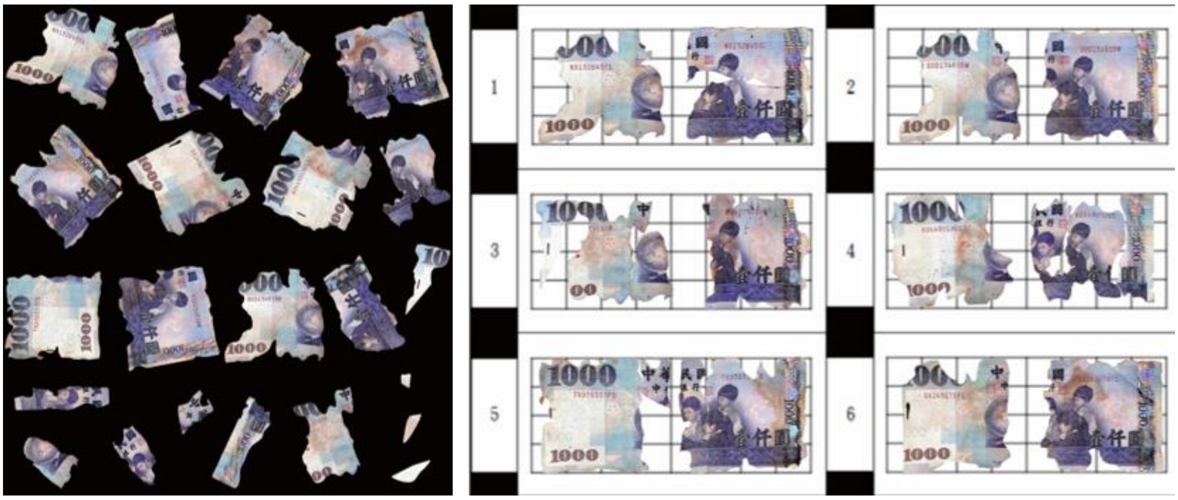
Subsequently, two experiments were conducted using a sample banknote and a real banknote. First, the sample banknote was manually torn into pieces, and all fragments were photographed for reconstruction. As shown in Figure 11(a), there are a total of 15 fragments. A blank template with the same size as the original sample banknote was used to reconstruct the sample banknote, the reconstruction results are shown in Figure 11(b). All 15 fragments were correctly aligned. Next, the images of the damaged banknote fragments were fed into the reconstruction system to understand the assembly status of the real fragments.



(a) (b)

Figure 11. Sample banknote reconstruction (a) sample banknote fragments and (b) reconstruction results

A total of 22 real banknote fragments damaged by insect infestation were photographed into the reconstruction system, as shown in Figure 12(a). These fragments could be manually assembled into six banknotes, and the reconstruction results are shown in Figure 12(b). A total of 19 fragments were correctly aligned, whereas the remaining three pieces were too small for detecting reliable alignments and thus were not embedded into the template. Figure 12 reveals that slight deformation occurred for some fragments because of the original curvature of the banknotes and the interpolation effects during image processing. The experimental results indicate that because of the influence of dirt, missing parts, and folds, the success rate of assembling real banknote fragments was lower than that of assembling manually torn sample fragments. Nevertheless, the automated system developed in this study for assembling damaged banknotes provided satisfactory reconstruction outcomes.



(a) (b)

Figure 12. Reconstruction of damaged banknotes: (a) damaged banknote fragments and (b) reconstruction results

4. CONCLUSION

Assembling fragments of damaged banknotes is a complex computer vision problem. This study developed an AI system for assembling damaged banknotes by using an image registration technique that involves using deep neural networks to estimate a planar projection transformation homography. The transformation results obtained with the developed system were verified by comparing them with those obtained using the SIFT, a conventional feature-based method. Experiments proved that the supervised learning approach of this study outperformed the SIFT in terms of efficiency and can be applied in practice. In the future, the research team of this study will continue to explore the use of other regression models in banknote reconstruction. Different regression models have different advantages in terms of size and computational complexity; thus, the adoption of different models might lead to the discovery of more suitable system architectures, thereby enhancing operational efficiency and reconstruction performance.

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REFERENCES

[1] N. A. Rasheed and J. Nordin, "A survey of computer methods in reconstruction of 3D archaeological pottery objects," *International Journal of Advanced Research*, vol. 3, no. 3, pp. 712–714, 2015.

[2] S. Joseph, I. Hipiny, H. Ujir, S. F. S. Juan, and J. L. Minoi, "Performance evaluation of SIFT against common image deformations on iban plaited mat motif images," *Indonesian Journal of Electrical Engineering and Computer Science (IJEECS)*, vol. 23, no. 3, pp. 1470–1477, Sep. 2021, doi: 10.11591/ijeecs.v23.i3.pp1470-1477.

[3] F. Kleber and R. Sablatnig, "A survey of techniques for document and archaeology artefact reconstruction," in *Proceedings of the International Conference on Document Analysis and Recognition*, 2009, pp. 1061–1065, doi: 10.1109/ICDAR.2009.154.

[4] A. Rajendran, M. Endo, Y. Katsuda, K. Hidaka, and H. Sugiyama, "Programmed two-dimensional self-assembly of multiple DNA origami jigsaw pieces," *ACS Nano*, vol. 5, no. 1, pp. 665–671, Dec. 2011, doi: 10.1021/nn1031627.

[5] G. Yao *et al.*, "Gold-nanoparticle-mediated jigsaw-puzzle-like assembly of supersized plasmonic DNA origami," *Angewandte Chemie - International Edition*, vol. 54, no. 10, pp. 2966–2969, Jan. 2015, doi: 10.1002/anie.201410895.

[6] F. Kasmin, Z. Othman, and S. S. S. Ahmad, "Pixel-wise classification using support vector machine for binarization of degraded

historical document image," *Indonesian Journal of Electrical Engineering and Computer Science (IJEECS)*, vol. 15, no. 3, pp. 1329–1336, Sep. 2019, doi: 10.11591/ijeecs.v15.i3.pp1329-1336.

[7] N. Memon and A. Pal, "Automated reassembly of file fragmented images using greedy algorithms," *IEEE Transactions on Image Processing*, vol. 15, no. 2, pp. 385–393, Feb. 2006, doi: 10.1109/TIP.2005.863054.

[8] S. I. M. Ali, M. Nihad, H. M. Sharaf, and H. Farouk, "Machine learning for text document classification-efficient classification approach," *IAES International Journal of Artificial Intelligence (IJ-AI)*, vol. 13, no. 1, pp. 703–710, Mar. 2024, doi: 10.11591/ijai.v13.i1.pp703-710.

[9] A. K. Abasi, A. T. Khader, M. A. Al-Betar, S. Naim, M. A. Awadallah, and O. A. Alomari, "Text documents clustering using modified multi-verse optimizer," *International Journal of Electrical and Computer Engineering (IJECE)*, vol. 10, no. 6, pp. 6361–6369, Dec. 2020, doi: 10.11591/IJECE.V10I6.PP6361-6369.

[10] Z. Labd, S. Bahassine, K. Housni, F. Z. A. H. Aadi, and K. Benabbes, "Text classification supervised algorithms with term frequency inverse document frequency and global vectors for word representation: A comparative study," *International Journal of Electrical and Computer Engineering (IJECE)*, vol. 14, no. 1, pp. 589–599, Feb. 2024, doi: 10.11591/ijece.v14i1.pp589-599.

[11] S. G. Desta and G. S. Lehal, "Automatic spelling error detection and correction for Tigrigna information retrieval: a hybrid approach," *Bulletin of Electrical Engineering and Informatics*, vol. 12, no. 1, pp. 387–394, Feb. 2023, doi: 10.11591/eei.v12i1.4209.

[12] S. B. Ng, K. W. Ng, R. W. O. K. Rahmat, and Y. J. Yoong, "Blending of three-dimensional geometric model shapes," *Indonesian Journal of Electrical Engineering and Computer Science (IJEECS)*, vol. 27, no. 1, pp. 102–109, Jul. 2022, doi: 10.11591/ijeecs.v27.i1.pp102-109.

[13] M. S. Hamid, N. A. Manap, R. A. Hamzah, A. F. Kadmin, S. F. A. Gani, and A. I. Herman, "A new function of stereo matching algorithm based on hybrid convolutional neural network," *Indonesian Journal of Electrical Engineering and Computer Science (IJEECS)*, vol. 25, no. 1, pp. 223–231, Jan. 2022, doi: 10.11591/ijeecs.v25.i1.pp223-231.

[14] E. M. Dastin-van Rijn *et al.*, "Uncovering biomarkers during therapeutic neuromodulation with PARRM: Period-based Artifact Reconstruction and Removal Method," *Cell Reports Methods*, vol. 1, no. 2, Jun. 2021, doi: 10.1016/j.crmeth.2021.100010.

[15] R. T. H. Kumar and C. K. Narayanappa, "Improved feature exctraction process to detect seizure using CHBMIT-dataset," *International Journal of Electrical and Computer Engineering (IJECE)*, vol. 11, no. 1, pp. 827–843, Feb. 2021, doi: 10.11591/ijece.v11i1.pp827-843.

[16] T. Hachad, A. Sadiq, F. Ghanimi, L. Hachad, and A. Laguidi, "Towards a new method of estimating the student attention based on the eye gaze," *Indonesian Journal of Electrical Engineering and Computer Science (IJEECS)*, vol. 28, no. 2, pp. 867–877, Nov. 2022, doi: 10.11591/ijeecs.v28.i2.pp867-877.

[17] N. Parvin and P. Kavitha, "Content based image retrieval using feature extraction in JPEG domain and genetic algorithm," *Indonesian Journal of Electrical Engineering and Computer Science (IJEECS)*, vol. 7, no. 1, pp. 226–233, Jul. 2017, doi: 10.11591/ijeecs.v7.i1.pp226-233.

[18] R. A. Mustafa, H. S. Chyad, and R. A. Haleot, "Palm print recognition based on harmony search algorithm," *International Journal of Electrical and Computer Engineering (IJECE)*, vol. 11, no. 5, pp. 4113–4124, Oct. 2021, doi: 10.11591/ijece.v11i5.pp4113-4124.

[19] S. Kuila, N. Dhanda, and S. Joardar, "Feature extraction of electrocardiogram signal using machine learning classification," *International Journal of Electrical and Computer Engineering (IJECE)*, vol. 10, no. 6, pp. 6598–6605, Dec. 2020, doi: 10.11591/IJECE.V10I6.PP6598-6605.

[20] W. Boukhari, M. Benyettou, and B. Abderrahim, "A human vision based system for biometric images recognition," *Indonesian Journal of Electrical Engineering and Computer Science (IJEECS)*, vol. 25, no. 3, pp. 1508–1517, Mar. 2022, doi: 10.11591/ijeecs.v25.i3.pp1508-1517.

[21] M. T. Ghazal and K. Abdullah, "Face recognition based on curvelets, invariant moments features and SVM," *Telkomnika (Telecommunication Computing Electronics and Control)*, vol. 18, no. 2, pp. 733–739, Apr. 2020, doi: 10.12928/TELKOMNIKA.v18i2.14106.

[22] A. Gupta, "A new copy move forgery detection technique using adaptive over-segementation and feature point matching," *Bulletin of Electrical Engineering and Informatics*, vol. 7, no. 3, pp. 345–349, Sep. 2018, doi: 10.11591/eei.v7i3.754.

[23] S. Manjunatha and M. M. Patil, "Efficient resampling features and convolution neural network model for image forgery detection," *Indonesian Journal of Electrical Engineering and Computer Science (IJEECS)*, vol. 25, no. 1, pp. 183–190, Jan. 2022, doi: 10.11591/ijeecs.v25.i1.pp183-190.

[24] M. Shivanandappa and M. M. Patil, "Extraction of image resampling using correlation aware convolution neural networks for image tampering detection," *International Journal of Electrical and Computer Engineering (IJECE)*, vol. 12, no. 3, pp. 3033–3043, Jun. 2022, doi: 10.11591/ijece.v12i3.pp3033-3043.

[25] N. A. Karim, S. A. Ali, and M. J. Jawad, "A coverless image steganography based on robust image wavelet hashing," *Telkomnika (Telecommunication Computing Electronics and Control)*, vol. 20, no. 6, pp. 1317–1325, Dec. 2022, doi: 10.12928/TELKOMNIKA.v20i6.23596.

[26] T. S. Cho, S. Avidan, and W. T. Freeman, "The patch transform," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 32, no. 8, pp. 1489–1501, Aug. 2010, doi: 10.1109/TPAMI.2009.133.

[27] S. K. Seetharamaswamy and S. K. Veeranna, "Image reconstruction through compressive sampling matching pursuit and curvelet transform," *International Journal of Electrical and Computer Engineering (IJECE)*, vol. 13, no. 6, pp. 6277–6284, Dec. 2023, doi: 10.11591/ijece.v13i6.pp6277-6284.

[28] M. I. Shujaa and A. A. Abdulmajeed, "Implementing bezier surface interpolation and N.N in shape reconstruction and depth estimation of a 2D image," *Indonesian Journal of Electrical Engineering and Computer Science (IJEECS)*, vol. 16, no. 3, pp. 1609–1616, Dec. 2019, doi: 10.11591/ijeecs.v16.i3.pp1609-1616.

[29] A. E. Fahrudin, Endarko, K. Ain, and A. Rubiyanto, "Enhanced image reconstruction of electrical impedance tomography using simultaneous algebraic reconstruction technique and K-means clustering," *International Journal of Electrical and Computer Engineering (IJECE)*, vol. 13, no. 4, pp. 3987–3997, Aug. 2023, doi: 10.11591/ijece.v13i4.pp3987-3997.

[30] K. Daurenbekov *et al.*, "Noisy image enhancements using deep learning techniques," *International Journal of Electrical and Computer Engineering (IJECE)*, vol. 14, no. 1, pp. 811–818, Feb. 2024, doi: 10.11591/ijece.v14i1.pp811-818.

[31] Z. El khattabi, Y. Tabii, and A. Benkaddour, "Video shot boundary detection using the scale invariant feature transform and RGB color channels," *International Journal of Electrical and Computer Engineering (IJECE)*, vol. 7, no. 5, pp. 2565–2673, Oct. 2017, doi: 10.11591/ijece.v7i5.pp2565-2673.

[32] Y. Zhao and M. Su, "A puzzle solver and its application in speech descrambling," in *CEA'07 Proceedings of the 2007 annual Conference on International Conference on Computer Engineering and Applications*, 2007, pp. 171–176.

[33] M. G. Chung, M. M. Fleck, and D. A. Forsyth, “Jigsaw puzzle solver using shape and color,” in *International Conference on Signal Processing Proceedings*, 1998, vol. 2, pp. 877–880, doi: 10.1109/icosp.1998.770751.

[34] M. S. AL-Huseiny and A. S. Sajit, “Transfer learning with GoogLeNet for detection of lung cancer,” *Indonesian Journal of Electrical Engineering and Computer Science (IJEECS)*, vol. 22, no. 2, pp. 1078–1086, May 2021, doi: 10.11591/ijeecs.v22.i2.pp1078-1086.

[35] P. Saengpetch, L. Pipanmemekaporn, and S. Kamolsantiroj, “Functional magnetic resonance imaging-based brain decoding with visual semantic model,” *International Journal of Electrical and Computer Engineering (IJECE)*, vol. 10, no. 6, pp. 6682–6690, Dec. 2020, doi: 10.11591/IJECE.V10I6.PP6682-6690.

[36] “Jigsaw expert saves man’s shredded cash,” *Daily Motion*, 2019. https://www.dailymotion.com/video/xg2snl (accessed Jan. 26, 2023).

[37] K. Pfaffenbach, “After the flood: reconstructing damaged banknotes,” *The Globe and Mail*, 2013. https://www.theglobeandmail.com/report-on-business/after-the-flood-reconstructing-damaged-banknotes/article13563407/ (accessed Feb. 10, 2023).

[38] M. Holohan, “This will make a great wedding story’: Parents resigned after kid shreds $1,060,” *Today*, 2018. https://www.today.com/parents/2-year-old-shreds-1-060-much-his-parents-chagrin-t138982 (accessed Feb. 10, 2023).

[39] “How do you piece together £10,000 in cut-up notes?,” *BBC News*, 2008. http://news.bbc.co.uk/2/hi/uk_news/magazine/7765701.stm (accessed Feb. 10, 2023).

[40] M. Rossiter, “Family dog eats $4,000 in cash off the kitchen counter,” *Scripps News*, 2024. https://www.scrippsnews.com/science-and-tech/animals-and-insects/family-dog-eats-4-000-in-cash-off-the-kitchen-counter.

[41] P. Johnston, “Torn, burnt or damaged cash? Here’s how to get it replaced,” *Yahoo Finance*, 2022. https://au.finance.yahoo.com/news/what-to-do-if-you-have-damaged-cash-013240512.html (accessed Feb. 26, 2023).

[42] J. Joyce, “Remaking money,” *TIME for Kids*, 2019. https://au.finance.yahoo.com/news/what-to-do-if-you-have-damaged-cash-013240512.html (accessed May 05, 2023).

[43] K. Hoyer, “Germany is still trying to crack the stasi puzzle,” *The Washington Post*, 2023. https://www.washingtonpost.com/opinions/2023/03/07/east-germany-stasi-surveillance-documents/ (accessed Aug. 12, 2023).

[44] “World heritage sites that were lost to disasters,” *TRT WORLD*, 2019. https://www.trtworld.com/europe/world-heritage-sites-that-were-lost-to-disasters-25919 (accessed Jun. 16, 2023).

[45] T. Kyrychok, A. Shevchuk, V. Nesterenko, and P. Kyrychok, “Banknote paper deterioration factors: circulation simulator method,” *BioResources*, vol. 9, no. 1, pp. 710–724, Dec. 2014, doi: 10.15376/biores.9.1.710-724.

[46] S. Aulia, L. Bagus Budhi, A. Rusdinar, and R. Yuyun Siti, “Geometric and grayscale template matching for saudi arabian riyal paper currency recognition,” *International Journal of Electrical and Computer Engineering (IJECE)*, vol. 8, no. 6, pp. 4230–4238, Dec. 2018, doi: 10.11591/ijece.v8i6.pp4230-4238.

[47] H. K. Easa, A. A. Saber, N. K. Hamid, and H. A. Saber, “Machine learning based approach for detection of fake banknotes using support vector machine,” *Indonesian Journal of Electrical Engineering and Computer Science (IJEECS)*, vol. 31, no. 2, pp. 1016–1022, Aug. 2023, doi: 10.11591/ijeecs.v31.i2.pp1016-1022.

[48] S. Yılmaz and V. V. Nabiyev, “Comprehensive survey of the solving puzzle problems,” *Computer Science Review*, vol. 50, Nov. 2023, doi: 10.1016/j.cosrev.2023.100586.

[49] J. Li, M. Barkowsky, and P. Le Callet, “Analysis and improvement of a paired comparison method in the application of 3DTV subjective experiment,” in *2012 19th IEEE International Conference on Image Processing*, Sep. 2012, pp. 629–632, doi: 10.1109/ICIP.2012.6466938.

[50] B. Kimia and H. Can Aras, “HINDSITE: a user-interactive framework for fragment assembly,” in *2010 IEEE Computer Society Conference on Computer Vision and Pattern Recognition - Workshops, CVPRW 2010*, Jun. 2010, pp. 62–69, doi: 10.1109/CVPRW.2010.5543532.

[51] D. Sholomon, O. David, and N. S. Netanyahu, “A genetic algorithm-based solver for very large jigsaw puzzles,” in *Proceedings of the IEEE Computer Society Conference on Computer Vision and Pattern Recognition*, Jun. 2013, pp. 1767–1774, doi: 10.1109/CVPR.2013.231.

[52] K. Zhang and X. Li, “A graph-based optimization algorithm for fragmented image reassembly,” *Graphical Models*, vol. 76, no. 5, pp. 484–495, Sep. 2014, doi: 10.1016/j.gmod.2014.03.001.

[53] P. H. Wu, J. J. Ding, J. M. Guo, P. J. Kang, and C. E. Pu, “Banknote reconstruction from fragments using quadratic programming and SIFT points,” in *Proceedings - IEEE International Symposium on Circuits and Systems*, Jun. 2014, pp. 570–573, doi: 10.1109/ISCAS.2014.6865199.

[54] H. Li, Y. Zheng, S. Zhang, and J. Cheng, “Solving a special type of jigsaw puzzles: Banknote reconstruction from a large number of fragments,” *IEEE Transactions on Multimedia*, vol. 16, no. 2, pp. 571–578, Feb. 2014, doi: 10.1109/TMM.2013.2291968.

[55] C. Y. Gwo, C. H. Wei, Y. Li, and N. H. Chiu, “Reconstruction of banknote fragments based on keypoint matching method,” *Journal of Forensic Sciences*, vol. 60, no. 4, pp. 906–913, Apr. 2015, doi: 10.1111/1556-4029.12777.

[56] V. V. Nabiyev, S. Yılmaz, A. Günay, G. Muzaffer, and G. Ulutaş, “Shredded banknotes reconstruction using AKAZE points,” *Forensic Science International*, vol. 278, pp. 280–295, Sep. 2017, doi: 10.1016/j.forsciint.2017.07.014.

[57] S. Yılmaz and V. V. Nabiyev, “Solving double-sided puzzles: automated assembly of torn-up banknotes evidence,” *Journal of Forensic Sciences*, vol. 64, no. 4, pp. 1078–1091, Jan. 2019, doi: 10.1111/1556-4029.13993.

[58] D. G. Lowe, “Distinctive image features from scale-invariant keypoints,” *International Journal of Computer Vision*, vol. 60, no. 2, pp. 91–110, Nov. 2004, doi: 10.1023/B:VISI.0000029664.99615.94.

[59] M. A. Fischler and R. C. Bolles, “Random sample consensus: A paradigm for model fitting with applications to image analysis and automated cartography,” *Communications of the ACM*, vol. 24, no. 6, pp. 381–395, Jun. 1981, doi: 10.1145/358669.358692.

[60] P. S. LaFollette and R. L. Stafford, “Isthmus critical points for solving jigsaw puzzles in computer vision,” *IEEE Transactions on Systems, Man and Cybernetics*, vol. 21, no. 5, pp. 1271–1278, 1991, doi: 10.1109/21.120080.

[61] D. DeTone, T. Malisiewicz, and A. Rabinovich, “Deep image homography estimation,” *arXiv:1606.03798*, Jun. 2016.

[62] M. Brown and D. G. Lowe, “Automatic panoramic image stitching using invariant features,” *International Journal of Computer Vision*, vol. 74, no. 1, pp. 59–73, Dec. 2007, doi: 10.1007/s11263-006-0002-3.

[63] S. Baker, A. Datta, and T. Kanade, “Param eterizing homographies.” Tech. Rep, Robotics Institute, Pittsburgh, 2006.

[64] Simonyan Karen and Zisserman Andrew, “Very deep convolutional networks for large-scale image recognition,” in *3rd International Conference on Learning Representations, ICLR 2015 - Conference Track Proceedings*, Sep. 2015, p. 14.

[65] S. Ioffe and C. Szegedy, “Batch normalization: accelerating deep network training by reducing internal covariate shift,” in *32nd International Conference on Machine Learning, ICML 2015*, 2015, vol. 1, pp. 448–456.

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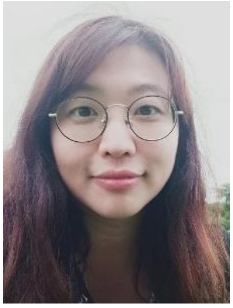

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