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Optimizing robot anomaly detection through stochastic differential approximation and Brownian motion

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ABSTRACT

This paper presents an adaptive approximation method for detecting anomalous patterns in extensive data streams gathered by mobile robots operating in rough terrain. Detecting anomalies in such dynamic environments poses a significant challenge, as it requires continuous monitoring and adjustment of robot movement, which can be resource intensive. To address this, a cost-effective solution is proposed that incorporates a threshold mechanism to track transitions between different regions of the data stream. The approach utilizes stochastic differential approximation (SDA) and optimistic optimization of Brownian motion to determine optimal parameter values and thresholds, ensuring efficient anomaly detection. This method focuses on minimizing the movement cost of the robots while maintaining accuracy in anomaly identification. By applying this technique, robots can dynamically adjust their movements in response to changes in the data stream, reducing operational expenses. Moreover, the temporal performance of the data stream is prioritized, a key factor often overlooked by conventional search engines. This paper demonstrates how the approach enhances the precision of anomaly detection in resource-constrained environments, making it particularly beneficial for real-time applications in rugged terrains.

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1. INTRODUCTION

The primary signal and sensor processing issue is anomaly identification during the data stream [1]. This research's primary focus is identifying anomalies within data streams, particularly emphasizing the element of time. The objective is to enhance the efficiency of representing a specific subset of temporal data streams through a sequential design of experiments, facilitating accurate and rapid anomaly detection [2], [3]. A significant challenge in time-based anomaly identification, especially in the context of mobile robots used in rough terrain rescue missions, is the associated cost of transitioning the data stream from one geographical region to another [4]. This cost primarily arises from the movement of robots. Various approximation algorithms have been developed to address this issue, with the Brownian motion algorithm being a prominent choice due to its experience in managing time and cost-effective model construction [5]. However, the standard Brownian motion algorithm faces challenges, including handling vast datasets, memory limitations, and the inability to adapt to a behavior-based system with infinite variance [6], [7]. To address the challenges of real-time anomaly detection in dynamic, resource-constrained environments like rugged terrains, the authors propose a novel approach leveraging stochastic differential approximation (SDA) and optimistic optimization of Brownian motion [8]. This method approximates the shortest path characterized by Brownian motion within a defined time interval, achieving greater accuracy by minimizing path length. The study aims to provide a cost-effective, adaptive solution that balances energy efficiency, operational cost, and real-time response while optimizing robot movement for precise anomaly detection.

Brownian motion is defined as the variation in the path length measurement, characterized by its continuous and natural nature and adaptability to showcase differentiation in erratic motions [9], [10]. In presenting the minimum Brownian motion using stochastic differential approximation and optimistic optimization, the path is depicted in time intervals, each further divided into sub-intervals [11]. The critical objective is to determine how each sub-interval should be represented within the entire time interval. This involves the selection of a mathematical model, in this case, stochastic differential equations (SDE), and an optimization technique, specifically an optimistic optimization algorithm [12]. In summary, this paper introduces an innovative approach to tackle the challenges associated with time-based anomaly identification, particularly in mobile robots, by presenting the stochastic differential approximation and optimistic optimization of Brownian motion as a more effective alternative to the traditional Brownian motion algorithm [13]. The proposed approach provides several key contributions to enhance cost efficiency and improve anomaly detection accuracy in robotic systems. One of its primary advantages is the significant reduction in expenses related to the movement of mobile robots, particularly during the process of detecting anomalies in data streams [14]. By employing a mathematical model that approximates the minimal Brownian motion path, the method effectively minimizes unnecessary robot movements, thereby conserving both energy and resources.

Brownian motion, often associated with random movement patterns, is streamlined here to limit robot movement, focusing on pathways with minimal deviation. This targeted movement strategy not only saves operational costs but also extends the operational lifespan of robotic components by reducing wear and tear on the machinery [15]. In addition to cost savings, the approach employs an advanced mathematical framework based on stochastic differential equations (SDEs) to refine the precision of the minimal path approximation. By leveraging SDEs, the system can dynamically adjust to unpredictable factors that impact robot movement and data collection [16]. This added layer of mathematical rigor enhances the accuracy of the Brownian motion model, ensuring that the robots follow a path close to the minimal distance required to detect anomalies effectively. The use of SDEs ensures that the approach adapts well to fluctuating conditions, which are common in real-world applications where robotic systems encounter varied terrains and obstacles [17]–[20]. This results in a robust system where the accuracy of anomaly detection remains high, even under challenging conditions.

To improve the approach, an optimistic optimization technique is incorporated to effectively tackle continuous optimization challenges associated with Brownian motion [16]. This framework plays a vital role in identifying the optimal paths for robots, striking a balance between minimizing path length and maximizing the probability of anomaly detection. By focusing on optimizing the robot's path, the method significantly increases the chances of accurately identifying anomalies without necessitating extensive movement. This optimization process does not solely focus on identifying anomalies but also emphasizes efficient resource allocation, minimizing computational power, and ultimately reducing the time and cost involved in anomaly detection. An additional aspect of this methodology is its emphasis on accurately pinpointing the most likely minimum path of Brownian motion within a specific time frame, a feature that significantly enhances the effectiveness of anomaly detection [21]. The proposed approach effectively narrows down the set of probable paths, focusing on accurately identifying the minimum path that exhibits Brownian motion characteristics. This ensures that anomalies in extensive data streams are detected promptly and with minimal resource consumption, reducing unnecessary robotic movements. The time-bound nature of this identification process further enhances the system's responsiveness, enabling real-time detection of irregularities. By maintaining high precision in anomaly detection while operating under resource constraints, the approach becomes highly suitable for applications where both accuracy and operational efficiency are critical, such as autonomous navigation, disaster response, and environmental monitoring in challenging terrains.

2. METHOD

Consider the data stream $f = \{ 1,2,3 \ldots \ldots n\}$ where f creates the series of random variables are $A_1^f, A_2^f, A_3^f \ldots \ldots A_n^f$ occurs with the data sample space A. Each data stream is dependent on either one of the hypotheses such as $H_0 \; or \; H_1$. The observations made by two different probability distribution functions on data sample space A of data stream are D_1 *and* D_2. Taken H_0 for $D_1$where $H_0 \rightarrow A_n^f$ is true for H_0. Similarly, H_1 for D_2 where $H_1 \rightarrow A_n^f$ is true for H_1 where n denotes the set of integers. H_0 is positive when the specific data stream defines the movement of data stream is normal and H_1 is positive when the movement of data stream is target. Suppose for the taken data stream, H_1 is true and its probability is ρ and H_0 is true with its probability $(1-\rho)$ where $\rho \in (0,1)$. When these criteria are not satisfied for ρ, it shows that target data stream happens to be too rare case. During the expectation of hypothesis P_i. The general structure of Brownian motion is shown in Figure 1.

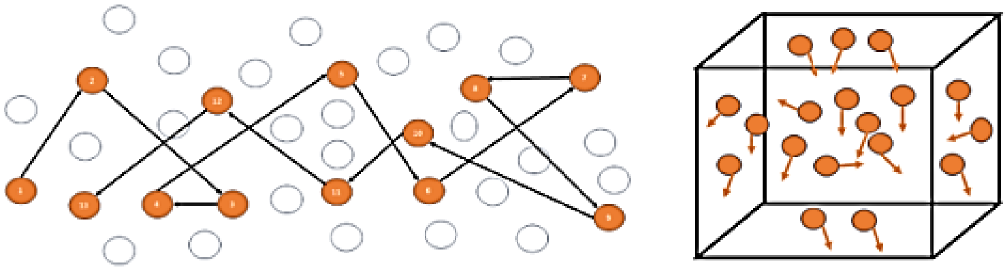

Figure 1. Structure of Brownian motion [21]

To reduce the movement cost of the mobile robot during the data stream from one point to another point, the problem observed and formulated during the data movements from stream $W_1$to the next stream $W_n + 1$, so it exists with cost λ_W within distribution $D\ \forall\ W(i.e.)\lambda_W \geq 0$ and $€[\lambda_W] = \tilde{\lambda}$ is finite [22]. Assume that movement cost λ_W and the remarks $O_T^{W'}$ are commonly independent ∀ W,W'&T. The movement cost is expensive due to the movement of the robot while the data stream is monitored during the new data stream [23]. If the observations are not taken, then it is said to be zero time and if the observations are monitored and the time are recorded then it shows the observation for movement cost. This can also be related to energy.

Problem 1: Consider the Brownian motion $(Br(t))_{t\geq 0}$ with a Hurst value Hu > 1/2. Stochastic linear equations of this type are investigated in the format in (1).

$$dA(t) = S(A(t),t)dt + T(A(t),t)dBr(t), \tag{1}$$

$A(t_0) = A_0$, Whereas $t_0 \in (0,T), A_0$ is an Unpredictable vector in $\mathbb{Q}^n$ and the subsequent criteria hold true with likelihood 1 for the randomly generated functions S and T in (2),

$$S \epsilon E(\mathbb{Q}^n * (0,T), \mathbb{Q}^n), T \epsilon E^1(\mathbb{Q}^n * (0,T), \mathbb{Q}^n) \tag{2}$$

for every $t \in (0,T)$ the functions $S(\cdot,t), \frac{\partial T(\cdot,t)}{\partial x^i}, \frac{\partial T(\cdot,t)}{\partial t}$ for all i in [1, 2..., n] are localized Lipschitz.

Consider the supplementary partial differential (3) along the route on $\mathbb{Q}^n * \mathbb{Q} * (0,T)$,

$$\frac{\partial L}{\partial y}(x,y,t) = T(L(x,y,t),t) \tag{3}$$

$L(X_0, Y_0, t_0) = A_0$ Wherein $X_0$is an array of random elements in some set $\mathbb{Q}^n$ and Y_0 is an independent variable in some set $\mathbb{Q}$. It derives from differentiated equation theory that in the neighbourhood N of (X_0, Y_0, t_0) a local solution $L \in E^1(\mathbb{Q}^n * \mathbb{Q} * (0,T), \mathbb{Q}^n)$with Lipschitz limited variations in the parameter x occurs with likelihood 1 in (4).

$$\det\left(\frac{\partial L^i}{\partial x^j}(x,y,t)\right) \neq 0 \tag{4}$$

$For\,(y,z,t) \in N$

$$\frac{\partial^2 L}{\partial y^2}(y,z,t) = \sum_{j=1}^{n} \frac{\partial T}{\partial y^j}(L(x,y,t),t)T^j(L(x,y,t),t) \tag{5}$$

From (5), on [0, T], additionally consider the path wise differential equation (in matrix form) as in (6).

$$dC(t) = \left(\frac{\partial L}{\partial x} C(t), Br(t), t\right)^{-1} \left[S(L(C(t), Br(t), t), t) - \frac{\partial L}{\partial t} C(t), Br(t), t\right] dt, \quad (6)$$

$$C(t_0) = C_0 \quad (7)$$

From (7), a maximum range, has a distinct local solution as $(t_0^1, t_0^2) \subseteq (0, t)$ with $t_0 \epsilon (t_0^1, t_0^2)$. Here, a use of stochastic application of the formula to the randomized function $Q(z, t) = L(C(t), y, t)$ and the Brownian motion Br in (8),

$$L(C(t), Br(t), t) - L(C(t_0), Br(t_0), t_0) \quad (8)$$

$$= \sum_{j=0}^{n} \int_{t_0}^{t} \left(\frac{\partial L}{\partial x^j}(C(s), Br(s), s)\right) dY^j(s) + \int_{t_0}^{t} \left(\frac{\partial L}{\partial x}(C(s), Br(s), s)\right) dBr(s) + \int_{t_0}^{t} \left(\frac{\partial L}{\partial t}(C(s), Br(s), s)\right) ds$$

$$= \int_{t_0}^{t} S(L(C(s), Br(s), s), s) ds + \int_{t_0}^{t} S(L(C(s), Br(s), s), s) dBr(s). \quad (9)$$

Hence, $A(t) \coloneqq L(C(t), Br(t), t)$ proves.

$$A(t) = A_0 + \int_{t_0}^{t} S(A(s), s) ds + \int_{t_0}^{t} T(A(s), s) dBr(s). \quad (10)$$

For each $Z \in \mathbb{Z}$,a treat the path wise differential equation of (6) (represented as a matrix) as an approximation Br_Z(t) of the original process $Br(t)$ and the (11),

$$dC_z(t) = \left(\frac{\partial L}{\partial x} C_z(t), Br_Z(t), t\right)^{-1} \begin{bmatrix} S(L(C_z(t), Br_Z(t), t), t) \\ -\frac{\partial L}{\partial t} C_z(t), Br_Z(t), t \end{bmatrix} dt$$

$$C_z(t_0) = C_0 \quad (11)$$

A distinct localized equilibrium $C_N$on the maximum period of presence $(t1, t2) \subseteq (t_0^1, t_o^2)$. Using the stochastic formula, $Q(z, t) = L(C(t), y, t)$ for the random function $Q(z, t)$ and the procedure BN as in (12),

$$L(C_Z(t), Br_Z(t), t) - L(C_Z(t_0), Br_Z(t_0), t_0) \quad (12)$$

$$= \sum_{j=0}^{n} \int_{t_0}^{t} \left(\frac{\partial L}{\partial x^j}(C_Z(s), Br_Z(s), s)\right) dC_Z{}^j(s) + \int_{t_0}^{t} \left(\frac{\partial L}{\partial x}(C_Z(s), Br_Z(s), s)\right) dBr_Z(s)$$

$$+ \int_{t_0}^{t} \left(\frac{\partial L}{\partial t}(C_Z(s), Br_Z(s), s)\right) ds$$

$$= \int_{t_0}^{t} S(L(C_Z(s), Br_Z(s), s), s) ds + \int_{t_0}^{t} S(L(C_Z(s), Br_Z(s), s), s) dBr_Z(s).$$

Hence, $A_Z(t) \coloneqq L(C_Z(t), Br_Z(t), t)$ proves in (13),

$$A_Z(t) = A_0 + \int_{t_0}^{t} S(A_Z(s), s) ds + \int_{t_0}^{t} T(A_Z(s), s) dBr_Z(s) \quad (13)$$

The subsequent path wise condition arises from the proposed theorem

$$\lim_{Z \to \infty} \sup \left\| C_Z(t) - C(t) \right\| = 0$$

since L follows that

$$\lim_{Z \to \infty} sup \left\| A_Z(t) - A(t) \right\| = 0$$

This study effectively demonstrates the connection between the outcomes and approximations obtained from the provided stochastic differential equation and their association with the path wise differential equation, as referenced in sources [4]–[7]. More specifically, it showcases how the solution C for the path wise differential (7) can be inferred from the response A corresponding to the stochastic equation (1). It's crucial to emphasize that this statement is made under the assumption that equation 6 possesses a unique global solution, thereby guaranteeing the uniqueness of the local solution for equation 1.

This paper establishes a clear link between the solutions and estimations of stochastic and path wise differential equations [24], [25]. Additionally, it underscores the significance of unique conditions, particularly in the context of (6) and its implications for equation 1. Let L represent the solution to (2). It is well established that the solution is invertible in the neighborhood N of (X_0, Y_0, t_0).

$(x, y, t) \in N, \longmapsto L\big((x, y, t), y, t\big)$has the opposite.

K stands for the mapping that provides

$K(L(x, y, t), y, t) = x$ and $L(K(x, y, t), y, t) = y$

In area N around us, the matrix equivalence holds in (14).

$$\frac{\partial K}{\partial y}(y, z, t) = \left(\frac{\partial L}{\partial x}(K(y, z, t), y, t)\right)^{-1} \tag{14}$$

By using (2), the obtain (15) and (16).

$$\frac{\partial L}{\partial y}(z, y, t) = -\sum_{i=0}^{n} \frac{\partial K}{\partial z^i}(z, y, t) T^i(z, t) \tag{15}$$

$$\frac{\partial L}{\partial t}(z, y, t) = -\sum_{i=0}^{n} \frac{\partial K}{\partial z^i}(z, y, t) \frac{\partial L^i}{\partial t}(K(z, y, t), y, t) \tag{16}$$

By plugging the parameters of the $\mathbb{S}^{n+1}$-valued procedure $(A(t), Br(t))$ into the stochastic equation for an expression K (z, y, t), get (17),

$$\begin{aligned} &K(A(t), Br(t), t) - K(A(t_0), Br(t_0), t_0) \\ &= \sum_{j=0}^{n} \int_{t_0}^{t} \left(\frac{\partial K}{\partial z^j}(A(s), Br(s), s)\right) dZ^j(s) + \int_{t_0}^{t} \left(\frac{\partial K}{\partial z}(A(s), Br(s), s)\right) dBr(s) + \\ &\int_{t_0}^{t} \left(\frac{\partial K}{\partial t}(A(s), Br(s), s)\right) ds \\ &= \sum_{j=0}^{n} \int_{t_0}^{t} \left(\frac{\partial K}{\partial z^j}(A(s), Br(s), s)\right) F(A(s), s) ds \\ &\qquad - \sum_{j=0}^{n} \int_{t_0}^{t} \frac{\partial K}{\partial z^j}(A(s), Br(s), t) \frac{\partial L^j}{\partial t}(K(A(s), Br(s), s), Br(s), s) ds \end{aligned} \tag{17}$$

But $A(s) = L(K(A(s), Br(s), s), Br(s), s)$ and (14) holds, hence

$$C(t) \coloneqq K(A(t), Br(t), t) \tag{18}$$

satisfies the path-wise (6) on an individual scale. Similarly, it may establish that $A_Z(t) \coloneqq L(C_Z(t), Br_Z(t), t)$ satisfies the path-wise (11) on a particular scale. Let Br approximate a proportionate Brownian motion B_z. Let $S, T\colon \mathbb{S}^n * (0, t)$ be predetermined in (19) and (20),

$$A(t) = A_0 + \int_{t_0}^{t} S(A(s), s) ds + \int_{t_0}^{t} T(A(s), s) dBr(s). \tag{19}$$

$$A_Z(t) = A_0 + \int_{t_0}^{t} S(A_Z(s), s) ds + \int_{t_0}^{t} T(A_Z(s), s) dBr_Z(s). \tag{20}$$

where $Z \in \mathbb{Z}$, accepts, with high probability, a single local solution on the same interval (t_1, t_2) (where t_0 is outside of Z but still part of the interval). In addition, the following approximate result has been obtained in (21).

$$Approx.\left(\lim_{Z\to\infty}\sup\|A_Z(t)-A(t)\|=0\right)=1 \quad (21)$$

Problem 2 Find the nearer location from the source point where the data stream W begins and define optimal path for the data stream W moves from one point to another point.

Proof (a) The anticipated quantity of near-optimal locations for any b is constrained in (22),

$$\mathbb{Q}[\mathcal{N}_b(\eta\ )] \leq 6\eta^2 2^b \quad (22)$$

It fixes the value of $\eta_b \triangleq \eta_\varepsilon\left(\frac{1}{2^b}\right)$. The speed of growth of $\mathcal{N}_b(\eta_b)$, the quantity of η_b-near-optimal locations in [0, 1] of the form $k/2^b$, is measured by the near-optimality dimension in dimension one with the pseudo-distance$l(m,n)=\eta_\varepsilon(|n-m|)$.It shows that, in general, this quantity grows at a constant rate regarding b. This implies that there exists a metric where the Brownian is Lipschitz with likelihood at least $1-\varepsilon$ and has a near-optimality aspect = 0 with $E\ =\ \mathcal{O}(log(1/\varepsilon))$.

The $\mathcal{O}(log(1/\varepsilon))$ term, originating from the standard DOO error for deterministic function optimisation, and a different $\mathcal{O}(log(1/\varepsilon))$ term, originating from the need to adjust our pseudo-distance $\mathcal{L}\ to\ \varepsilon$ such that the Brownian is $\mathcal{L}$-Lipschitz with likelihood 1- ε, together represent the finalised study difficulty bound. Combining these two bounds yields an upper limit on sample complexity of $\mathcal{O}(log^2(1/\varepsilon))$.

Proof (b) A Brownian motion whose optimum O is reached for the initial time at the location described as is denoted by U and the Brownian meander T_0^+can be defined as in (23),

$$T_0^+ \triangleq \frac{O-W(t1-t.t1)}{\sqrt{t1}} \quad (23)$$

$$T_1^+ \triangleq \frac{O-W(t\ 1+t(1-t1))}{\sqrt{1-t1}} \quad (24)$$

Then the theorem 1 declares that $T^+ \leqq T_0^+ \leqq T_1^+$ and t1 changes regardless of both $T_0^+\ and\ T_1^+$.

For each positive integer, it establishes a maximum constraint on the predicted amount of η-near-optimal positions b>0 and any values of $\eta > 0$.

$$\mathbb{Q}[\mathcal{N}_b(\eta\ \ )] = \ \mathbb{Q}\left[\sum_{a=0}^{2^b} 1\left\{W\left(\frac{a}{2^b}\right) > O-\eta\right\}\right] = \ \sum_{a=0}^{2^b}\mathbb{Q}\left[1\left\{W\left(\frac{a}{2^b}\right) > O-\eta\right\}\right]$$
$$= \sum_{a=0}^{2^b}\mathbb{Q}\left[1\left\{\left\{W\left(\frac{a}{2^b}\right) > O-\eta\ \cap\frac{a}{2^b}\leq t1\right\}\cup\left\{W\left(\frac{a}{2^b}\right) > O-\eta\cap\frac{a}{2^b} > t1\right\}\right\}\right]$$
$$= \sum_{a=0}^{2^b}\mathbb{Q}\left[1\left\{T_0^+\left(1-\frac{k}{t12^b}\right) < \frac{\eta}{\sqrt{t1}}\cap\frac{a}{2^b}\leq t1\right\}\right] + \sum_{a=0}^{2^b}\mathbb{Q}\left[1\left\{T_1^+\left(\frac{\frac{a}{2^b}-t1}{1-t1}\right) < \frac{\eta}{\sqrt{1-t1}}\cap\frac{a}{2^b} > t1\right\}\right]$$

Since t1 changes regardless of $T_0^+\ and\ T_1^+$, utilizing the above equation with C= (T_0^+, T_1^+), D=t1 and function in (25),

$$fun\colon (c_0, c_1), d \to \sum_{a=0}^{2^b}\left(\left[1\left\{c_0\left(1-\frac{k}{t12^b}\right) < \frac{\eta}{\sqrt{d}}\cap\frac{a}{2^b}\leq d\right\}\right] +\ 1\left\{c_1\left(\frac{\frac{a}{2^b}-t1}{1-d}\right) < \frac{\eta}{\sqrt{1-d}}\cap\frac{a}{2^b} > d\right\}\right) \quad (25)$$

it has sufficient evidence to assert that.

$$\mathbb{Q}[\mathcal{N}_b(\eta\ )] = \ \mathbb{Q}[f(C,D)] \leq \sup\mathbb{Q}[f(C,v)]$$
$$\leq \sup\left\{\sum_{a=0}^{2^b}\mathbb{Q}\left[1\left\{T_0^+\left(1-\frac{k}{v2^b}\right) < \frac{\eta}{\sqrt{v}}\cap\frac{a}{2^b}\leq v\right\}\right]\right\} + \sup\left\{\sum_{a=0}^{2^b}\mathbb{Q}\left[1\left\{T_1^+\left(\frac{\frac{a}{2^b}-v}{1-v}\right) < \frac{\eta}{\sqrt{1-v}}\cap\frac{a}{2^b} > v\right\}\right]\right\}$$
$$= \sup\left\{\sum_{a=0}^{\lfloor v^{2^b}\rfloor}\mathbb{R}\left\{T_0^+\left(1-\frac{k}{v2^b}\right) < \frac{\eta}{\sqrt{v}}\right\}\right\} + \sup\left\{\sum_{a=\lfloor v^{2^b}\rfloor}^{2^b}\mathbb{R}\left\{T_1^+\left(\frac{\frac{a}{2^b}-v}{1-v}\right) < \frac{\eta}{\sqrt{1-v}}\right\}\right\}$$
$$= 2\sup\left\{\sum_{a=0}^{\lfloor v^{2^b}\rfloor}\mathbb{R}\left\{T_0^+\left(1-\frac{k}{v2^b}\right) < \frac{\eta}{\sqrt{v}}\right\}\right\}$$
$$= 2\sup\{\beta_1+\beta_2+\beta_3+\beta_4\} \quad (26)$$

where (26) is expanded as

$$\beta_1 = \sum_{a=0}^{\lfloor 2^b\eta^2 \rfloor} \mathbb{R}\left[T_0^+\left(1-\frac{k}{v2^b}\right) < \frac{\eta}{\sqrt{v}}\right],\ \beta_2 = \sum_{a=\lceil 2^b\eta^2\rceil}^{\left\lfloor \frac{v2^b}{2}\right\rfloor} \mathbb{R}\left[T_0^+\left(1-\frac{k}{v2^b}\right) < \frac{\eta}{\sqrt{v}}\right],$$

$$\beta_3 = \sum_{a=\left\lceil \frac{v2^b}{2}\right\rceil}^{\lfloor v2^b\rfloor - \lceil 2^b\eta^2\rceil} \mathbb{R}\left[T_0^+\left(1-\frac{k}{v2^b}\right) < \frac{\eta}{\sqrt{v}}\right],\ \ \beta_4 = \sum_{a=\lfloor v2^b\rfloor - \lceil 2^b\eta^2\rceil}^{\lfloor v2^b\rfloor} \mathbb{R}\left[T_0^+\left(1-\frac{k}{v2^b}\right) < \frac{\eta}{\sqrt{v}}\right].$$

Given that 1 is the highest possible likelihood, it may simply place a limit on $\beta_1 and\ \beta_4$ as $2^b\eta^2$, to obtain that $\beta_1 + \beta_4 \leq 2(2^b\eta^2)$. By accumulating across the Brownian meander distribution parameters, it can now place upper and lower bounds on the rest of the possibilities occurring in the aforementioned formula.

$$\mathbb{R}[T_0^+(t) < c] = 2\sqrt{2\pi}\int_0^c \frac{d\exp\left(-\frac{d^2}{2t}\right)}{t\sqrt{t*2\pi}}\int_0^d \frac{\exp\left(-\frac{d^2}{2(1-t)}\right)}{t\sqrt{(1-t)*2\pi}}\,dd\,dc \leq \frac{2}{t\sqrt{(1-t)(t.2\pi)}}\int_0^c d^2\exp\left(-\frac{d^2}{2t}\right)dd$$
$$\leq \frac{2c^3}{3t\sqrt{(1-t)(t.2\pi)}} \leq \frac{2c^3}{3t(1-t)\sqrt{(1-t)(t.2\pi)}} = \frac{2}{3\sqrt{2\pi}}\left(\frac{c}{\sqrt{t(1-t)}}\right)^3$$

The limit is then applied to the sum of $\beta_2 and\ \beta_3$.

$$\beta_2 + \beta_3 = \sum_{a=\lceil 2^b\eta^2\rceil}^{\left\lfloor \frac{v2^b}{2}\right\rfloor} \mathbb{R}\left[T_0^+\left(1-\frac{k}{v2^b}\right) < \frac{\eta}{\sqrt{v}}\right] + \sum_{a=\left\lceil \frac{v2^b}{2}\right\rceil}^{\lfloor v2^b\rfloor - \lceil 2^b\eta^2\rceil} \mathbb{R}\left[T_0^+\left(1-\frac{k}{v2^b}\right) < \frac{\eta}{\sqrt{v}}\right]$$

$$\leq \sum_{a=\lceil 2^b\eta^2\rceil}^{\left\lfloor \frac{v2^b}{2}\right\rfloor} \frac{2}{3\sqrt{2\pi}}\left(\frac{\frac{\eta}{\sqrt{v}}}{\sqrt{(1-\frac{a}{v2^b}\sqrt{\frac{a}{v2^b}})}}\right)^3 + \sum_{a=\left\lceil \frac{v2^b}{2}\right\rceil}^{\lfloor v2^b\rfloor - \lceil 2^b\eta^2\rceil} \frac{2}{3\sqrt{2\pi}}\left(\frac{\frac{\eta}{\sqrt{v}}}{\sqrt{(1-\frac{a}{v2^b}\sqrt{\frac{a}{v2^b}})}}\right)^3$$

$$\leq \sum_{a=\lceil 2^b\eta^2\rceil}^{\left\lfloor \frac{v2^b}{2}\right\rfloor} \frac{1}{6\sqrt{\pi}}\left(\frac{\frac{\eta}{\sqrt{v}}}{\sqrt{\sqrt{\frac{a}{v2^b}}}}\right)^3 + \sum_{a=\left\lceil \frac{v2^b}{2}\right\rceil}^{\lfloor v2^b\rfloor - \lceil 2^b\eta^2\rceil} \frac{1}{6\sqrt{\pi}}\left(\frac{\frac{\eta}{\sqrt{v}}}{\sqrt{1-\frac{a}{v2^b}}}\right)^3 \leq$$

$$\sum_{a=\lceil 2^b\eta^2\rceil}^{\left\lfloor \frac{v2^b}{2}\right\rfloor} \frac{1}{6\sqrt{\pi}}\left(\frac{\frac{\eta}{\sqrt{v}}}{\sqrt{\sqrt{\frac{a}{v2^b}}}}\right)^3 + \sum_{a=\lceil 2^b\eta^2\rceil}^{\lfloor v2^b\rfloor - \left\lceil \frac{2^b v}{2}\right\rceil} \frac{1}{6\sqrt{\pi}}\left(\frac{\frac{\eta}{\sqrt{v}}}{\sqrt{\frac{\lfloor 2^b v\rfloor}{v2^b} + \frac{a}{v2^b}}}\right)^3$$

Swapping out the indexing as $g = -g' + \lfloor v2^b \rfloor$, discover the following,

$$\beta_2 + \beta_3 \leq \frac{(2^b\eta^2)^{3/2}}{6\sqrt{\pi}}\left(\sum_{a=\lceil 2^b\eta^2\rceil}^{\left\lfloor \frac{v2^b}{2}\right\rfloor} \frac{1}{g^{3/2}} + \sum_{a=\lceil 2^b\eta^2\rceil}^{\lfloor v2^b\rfloor - \left\lceil \frac{2^b v}{2}\right\rceil} \frac{1}{g^{3/2}}\right)$$

$$\leq \frac{(2^b\eta^2)^{\frac{3}{2}}}{3\sqrt{\pi}}\sum_{a=\lceil 2^b\eta^2\rceil}^{\propto} \frac{1}{g^{\frac{3}{2}}}$$
$$\leq \frac{(2^b\eta^2)^{\frac{3}{2}}}{3\sqrt{\pi}}\frac{3}{\sqrt{\lceil 2^b\eta^2\rceil}} \leq \frac{1}{\sqrt{\pi}}2^b\eta^2 \leq 2^b\eta^2,$$

where it was utilized for anything in the previous row $g_0 > 0$,

$$\sum_{g=g_0}^{\propto} \frac{1}{g^{\frac{3}{2}}} \leq \frac{1}{g_0^{\frac{3}{2}}} + \sum_{g=g_0+1}^{\propto} \int_{g-1}^{g} \frac{1}{v^{\frac{3}{2}}}\,dv = \frac{1}{g_0^{\frac{3}{2}}} + \int_{g_0}^{\propto} \frac{1}{v^{\frac{3}{2}}}\,dv = \frac{1}{g_0^{\frac{3}{2}}} + \frac{2}{\sqrt{g_0}} \leq \frac{3}{\sqrt{g_0}}$$

At long last, the obtained (27),

$$\forall v, \beta_1 + \beta_2 + \beta_3 + \beta_4 \leq 32^b \eta^2 \tag{27}$$

Hence, $\mathbb{Q}[\mathcal{N}_b(\eta)] \leq 6\eta^2 2^b$

3. RESULTS AND DISCUSSION

In the MATLAB implementation, a series of numerical tests were conducted to assess the performance of a technique for evaluating the movement cost of data streams. This process involved assigning specific cost values to the movement of data streams. One key parameter, denoted as $\hat{\lambda}$, was set to 0, simplifying the calculation by negating the contribution of λ to the cost function. The system utilized a probability distribution where $\pi = 0$ and followed a standard normal distribution of n (0,1), which was essential for detecting data stream movements within a defined range. The experiment aimed to evaluate the effectiveness of the proposed technique by simulating a series of data streams and measuring the associated movement costs. However, a notable limitation arises when λ is set to 0, as it becomes impossible to calculate the optimal movement cost under such conditions. Despite this, the experiment proceeded by generating target data stream values with a probability of 0.1 and following a normal distribution of mean 0 and variance 1, n (0,1). This probabilistic generation allowed for a controlled yet basic environment in which the technique could be evaluated. For the simulation, the standard data stream was modeled using a distribution with a mean of 0 and a higher variance of 1.7, allowing the system to compare the performance between the generated target data stream and the standard one. This difference in distribution enabled the method to assess how well it could detect and evaluate movements across varied data patterns. According to the results, the proposed method produced a 15% probability for detecting the values W_0 and W_1, marking these as key points of interest within the data stream's movement.

Despite the insights gained, there were observable shortcomings. Specifically, the movement cost for W_0 reached its maximum, along with its associated error rate. This indicates that while the method may offer some benefits, such as detecting movements in data streams, its efforts to provide optimal results under certain conditions, particularly when λ is set to 0. To enhance the experiment, it would be beneficial to introduce a more complex scenario with varied parameter values and conditions to fully assess the technique's effectiveness and limitations. Figure 2 presents a comparative visualization of three distinct data samples, labeled as sample 1, sample 2, and sample 3, each delineated by a unique color—black, red, and blue, respectively. Sample 1 exhibits a small KL divergence $C(W_0\,||W_1)$ and $C(W_1||W_0)$, indicating a predicted overshoot approaching zero. Sample 2 has a small error proportion where $P(s^n = s_0)$, and for a larger proportion, it shows a large value, denoted by (1-β)/α, which leads to data stream termination. In the case of sample 3, which has a large value, the primary goal of the analysis is to reduce the number of algorithm changes before correctly identifying the target data stream.

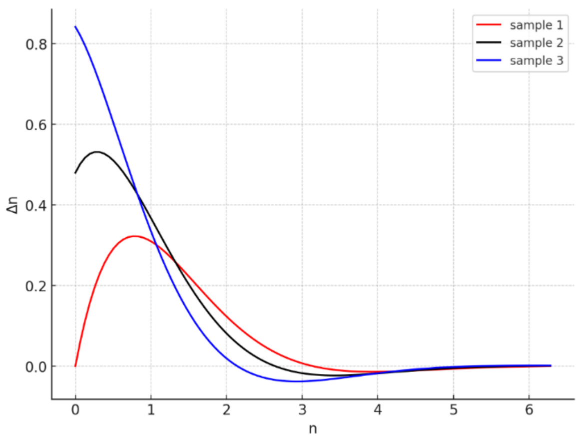

Figure 2. Samples with KL divergences

All three samples follow a similar trend with their own peculiar characteristics. Sample 1 (black) exhibits a steady, almost linear decline until it flattens out towards the end of the domain. Sample 2 (red) is characterized by a significant oscillation before sharply dropping, indicating a variable response or measurement before reaching a similar level as sample 1 towards the end. Sample 3 (blue) mirrors the oscillation seen in sample 2, but with a less pronounced initial drop and a deeper final descent. This comparison allows for an easy assessment of the similarities and differences in behavior, or responses captured by the three samples across the range of 'n' values. Figure 3 presents a time-series linear approximation of Brownian motion, reflecting data stream flow dynamics within a unit interval. The visualization begins at zero and rapidly reaches a maximum at T=0.2, after which it moderately recedes to a plateau of 0.8, sustained until T=0.6. Subsequently, the series undergoes a pronounced drop to 0.6, stabilizes momentarily, then descends precipitously to 0.2, culminating in a final downturn back to zero as T approaches 1. This portrayal suggests a piecewise linear process with distinct, sustained levels before transitioning, indicative of a system exhibiting stepwise stability before entering new phases. Figures 4 to 6 offer insights into the behavior of data stream movements and their relationship with the parameter λ.

Figure 4 demonstrates a minimum Brownian path with characteristic rise-and-fall patterns over time, displaying the inherent randomness of the motion through peaks and troughs between T=0 and T=1. In contrast, Figure 5 shows a decreasing trend in the number of movements starting from 40 at λ=0 and diminishing as λ increases, suggesting an inverse relationship. Meanwhile, Figure 6 contrasts this by depicting a direct, linear correlation between λ and the total number of observations, which increases proportionally from 110 to 145 as λ grows from 0 to 5. This comparison highlights the significance in which λ values change data stream behavior, with the rise in observations possibly pointing to increased data collection or detection capabilities as λ grows, despite the decrease in movement frequency. Together, these figures aid in identifying anomalies between different data stream paths by evaluating the movement cost over time.

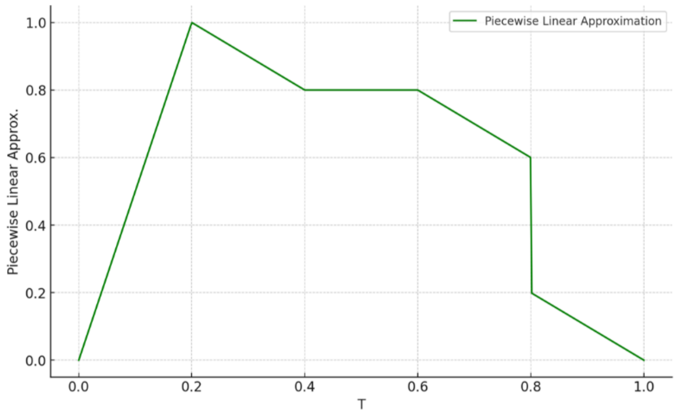

Figure 3. Stochastic differential equation with linear approximation for Brownian motion

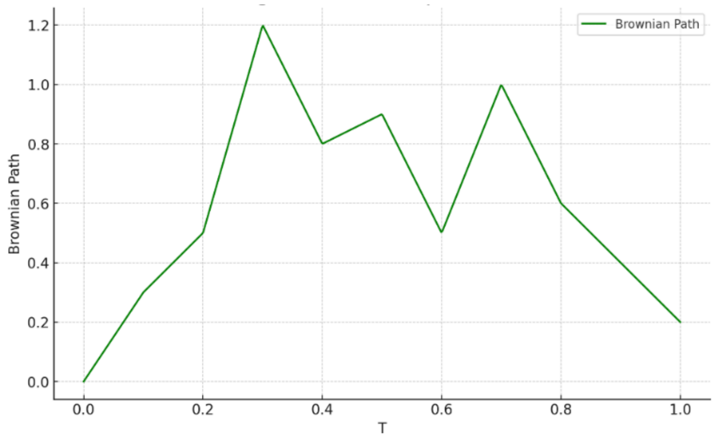

Figure 4. Brownian path with time

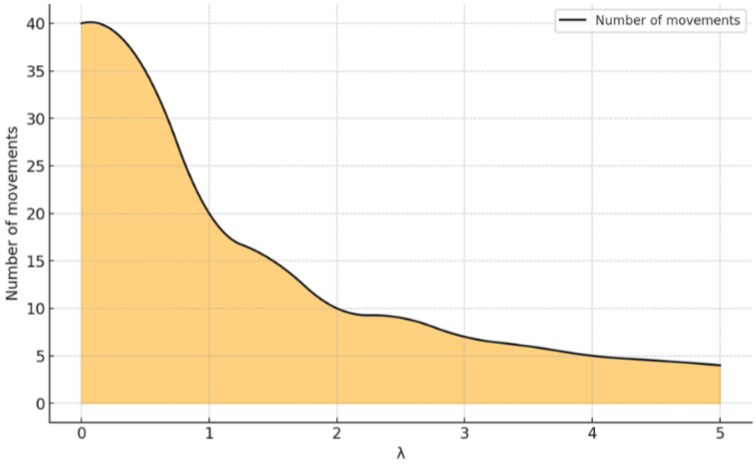

Figure 5. Data stream movements based on observations

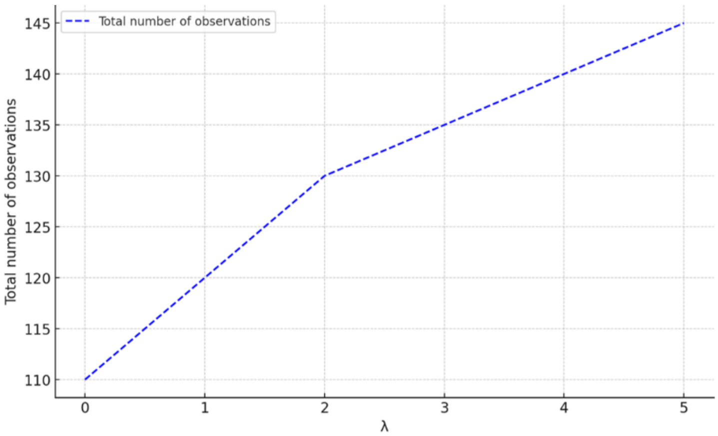

Figure 6. Total of number of observation w.r.t λ value

4. CONCLUSION

Anomaly detection is achieved by repositioning a data stream from one location to another, with movement costs evaluated through Brownian motion. The process involves applying a mathematical model based on the stochastic differential equation of Brownian motion. By optimizing the Brownian motion over time, the nearest position to the anomaly is identified. The model developed and validated in this work effectively approximates the minimum path required to detect anomalies. Its strength lies in its ability to accomplish the task within a relatively short crossing time, while accurately identifying anomalies through the observation of movement costs. This approach proves to be a valuable tool for efficiently detecting anomalies, offering both speed and accuracy in the process.

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