Design of H-/H∞ based fault detection filter for linear uncertain systems using linear matrix inequalities

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ABSTRACT

One of the significant challenges in model-based fault detection is achieving robustness against disturbances and model uncertainties while ensuring sensitivity to faults. This study proposes an optimized approach for designing fault detection filters for discrete-time linear systems with norm-bounded model uncertainties. The design leverages the H-/H∞ optimization framework and is expressed through linear matrix inequality constraints. The filter is designed to produce a residual signal that balances two opposing objectives: minimizing the impact of disturbances and model uncertainties while maximizing fault sensitivity. The effectiveness of the proposed method is demonstrated through simulations involving sensor and actuator fault detection in the well-known three-tank system. Simulation results illustrate the method's ability to maintain robustness against disturbances and uncertainties while effectively detecting faults in the system.

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1. INTRODUCTION

Fault detection is essential in robotics and automation systems to ensure reliability, safety, environmental sustainability, and achieve desired performance levels [1]. It identifies issues early, minimizes downtime, prevents accidents, and maintains product quality. In the industry, it increases productivity, ensures operational continuity, and contributes to sustainability by preventing waste and inefficiencies. Robotics and automation systems frequently interact with humans or hazardous materials. Detecting faults early prevents accidents, protects workers, and minimizes risks. As industries adopt smart manufacturing, fault detection becomes integral to real-time monitoring, self-diagnosis, and autonomous decision-making, key principles of Industry 4.0. Meeting these requirements often increases both system complexity and cost. Faults or abnormal behaviors in such complex systems can degrade performance and potentially lead to hazardous situations, posing risks to human safety and financial loss. Thus, early detection and identification of abnormal system behaviors are essential to prevent these adverse outcomes [2]–[5]. Over the past two decades, numerous advancements in resilient fault detection (FD) system design have been made, broadly categorized into model-based and model-free approaches [6]–[9]. Among model-based methods, observer-based techniques have gained popularity due to their simpler structure and relatively lower design complexity [10], [11]. These approaches utilize a fault detection filter (FDF) to generate a residual, defined as the difference between the system's measured outputs and the estimated outputs derived from its model. By comparing the residual against a predefined threshold, the occurrence of faults can be identified [12], [13].

The presence of external disturbances and model uncertainties complicates the residual generation process, often producing non-zero residuals even in fault-free scenarios. Ideally, a fault-free system should yield a zero-value residual, while faulty conditions should result in a non-zero residual. However, disturbances and uncertainties may lead to false alarms, undermining the FD process. Therefore, robust residual generation is critical for effective FD [14]–[17]. Addressing disturbances and model uncertainty in model-based FD systems presents a significant challenge. To this end, the H∞ norm optimization technique has been employed to enhance residual robustness against disturbances [18]. Conversely, the H- index, which reflects the minimum fault sensitivity of the residual, has been used to design fault-sensitive FDFs, enhancing their sensitivity to faults [19]–[21]. While H∞ optimization ensures robustness against disturbances, it also reduces fault sensitivity, and similarly, H- index-based FDFs, although fault-sensitive, may amplify the influence of disturbances [22]. Balancing these trade-offs is key to achieving optimal FD performance.

An optimal FD system aims to minimize the impact of unknown disturbances (minimizing the H∞ norm) while maximizing fault sensitivity (maximizing the H- index), framing the design as a multi-objective optimization problem. A literature review indicates that most FD methods address robustness and sensitivity issues for continuous-time or discrete-time linear systems with external disturbances only [23]–[26]. However, model uncertainties in system matrices can introduce biases in the residual, necessitating careful handling to ensure robust residual generation. For uncertain continuous-time linear systems, an observer-based FD system was proposed in [27], utilizing iterative linear matrix inequalities (LMIs) to generate robust residuals. This approach provided an optimal balance between robustness to disturbances and fault sensitivity for the multi-objective optimization problem. However, the method's complexity increased due to the need to first derive a theoretically optimal solution and subsequently design the observer. In contrast, an H∞ based FD residual generator for linear systems was developed in [28], demonstrating robustness against disturbances and model uncertainty. Nevertheless, this approach did not address fault sensitivity issues. For successful FD, it is crucial to simultaneously consider fault sensitivity and robustness.

Motivated by the scarcity of solutions addressing the multi-objective optimization problem for discrete-time linear systems with norm-bounded model uncertainties, this study seeks to develop an optimal observer-based residual generator. The proposed method ensures observer stability while achieving robustness to disturbances, resilience against model uncertainties, and enhanced fault sensitivity. The existence of the proposed observer is established through sufficient conditions expressed as LMIs. The results obtained for the observer-based fault detection filter were illustrated through a simulation analysis of a three-tank system. The proposed approach can be applied to any discrete-time linear system with norm-bounded model uncertainties and disturbances. The key contributions of this research are outlined as follows:

a. Development of an H∞ observer-based filter to minimize the H∞ norm of G_{rd}, the transfer function matrix representing the disturbance-to-residual relationship, within the linear matrix inequality (LMI) framework.
b. Development of an H- observer-based filter aimed at maximizing the H- norm of G_{rf}, the transfer function matrix from fault to residual, also using the LMI framework.
c. Design of an observer-based filter utilizing the H-/H∞ optimization method, which concurrently minimizes the H∞ norm while maximizing the H- norm. This approach seeks to create an optimal observer-based residual generator that satisfies both H∞ and H- performance criteria.
d. After constructing the proposed observer, the l_2 norm is applied to assess and compare the generated residual against a defined threshold to detect faults.

The structure of the paper is as follows. Section 2 introduces the problem formulation. Section 3 details the core contribution of the research, including the derivation of the filter gain matrix. Section 4 provides simulation results showcasing the filter's effectiveness, particularly in detecting sensor and actuator faults within a three-tank system. Finally, section 5 presents concluding remarks.

2. PROBLEM FORMULATION

The discrete-time linear system in (1) is adopted to formulate the problem being solved in this paper.

$$\begin{aligned} x(k+1) &= Ax(k) + E_f f(k) + E_d d(k) + (B + \Delta B)u(k) + \Delta A x(k) \\ y(k) &= Cx(k) + F_f f(k) + F_d d(k) + (D + \Delta D)u(k) + \Delta C x(k) \end{aligned} \tag{1}$$

Let $x(k) \in R^n$ represent the state vector, $u(k) \in R^p$ the control input vector, and $y(k) \in R^m$ the measurement vector. The disturbance vector $d(k)$ is l_2 norm bounded, such that $\|d(k)\|_2 \leq \delta_d$, while $f(k)$ is the l_2 norm bounded fault vector to be detected. The matrices E_d, F_d, E_f, and F_f define the locations where the disturbance and fault vectors influence the system dynamics, respectively. The matrices A, B, C, and D are the nominal system matrices with compatible dimensions, and ΔA, ΔB, ΔC, and ΔD represent norm-bounded model uncertainties, given as (2).

$$\begin{bmatrix} \Delta A & \Delta B \\ \Delta C & \Delta D \end{bmatrix} = \begin{bmatrix} H_1 \sum G_1 & H_1 \sum G_2 \\ H_2 \sum G_1 & H_2 \sum G_2 \end{bmatrix} \tag{2}$$

where $\sum$ is an unknown scalar constant holds the condition, i.e., $\sum^T \sum \leq I$. The assumptions listed below are used consistently throughout this work [18]:

A1: System (1) is observable; **A2:** $\begin{bmatrix} A - e^{j\theta} I & E_d \\ C & F_d \end{bmatrix}$ has full row rank, while $\theta \in [0,2\pi]$; **A3:** $(A + \Delta A)$ is stable

As initially introduced, the model-based FD system comprises two subsystems: a residual generator and a residual evaluator with thresholding and decision logic. An observer-based FD filter is used for generating the residual, which is expressed as (3):

$$\begin{aligned} \hat{x}(k+1) &= A\hat{x}(k) + L\big(y(k) - \hat{y}(k)\big) + Bu(k) \\ r(k) &= y(k) - \hat{y}(k) \end{aligned} \tag{3}$$

$(\hat{y}(k) = C\hat{x}(k) + Du(k)) \in R^m$ and $\hat{x}(k) \in R^n$ represent the estimated output and the state estimation vectors, respectively. The residual signal is denoted by $r(k)$, and the filter gain L serves as the design parameter for the proposed FD filter. The dynamics of the filter are described by the state estimation error vector, $e(k) = x(k) - \hat{x}(k)$. The following equations represent the error dynamics and the residual:

$$e(k+1) = (A - LC)e(k) + \big(E_f - LF_f\big) f(k) + (E_d - LF_d)d(k) + (\Delta A - L\Delta C)x(k) + (\Delta B - L\Delta D)u(k) \tag{4}$$

$$r(k) = Ce(k) + F_f f(k) + F_d d(k) + \Delta C x(k) + \Delta D u(k) \tag{5}$$

Undesired behavior in FD theory is caused by model uncertainty and disturbance, which influences the estimation process and makes the residual sensitive to faults, control input, and the system's state [29]. For the sake of simplicity, the dynamics of (4) are governed by two new vectors:

$$\bar{x}(k) = \begin{bmatrix} e(k) \\ x(k) \end{bmatrix} \text{ and } \bar{u}(k) = \begin{bmatrix} u(k) \\ d(k) \end{bmatrix}$$

Then, an augmented system is represented as (6), (7):

$$\bar{x}(k+1) = \bar{A}\bar{x}(k) + \bar{B}\bar{u}(k) + \bar{E}_f f(k) \tag{6}$$

$$r(k) = \bar{C}\bar{x}(k) + \bar{D}\bar{u}(k) + F_f f(k) \tag{7}$$

where

$$\bar{A} = \begin{bmatrix} A - LC & \Delta A - L\Delta C \\ 0 & A + \Delta A \end{bmatrix}; \bar{B} = \begin{bmatrix} \Delta B - L\Delta D & E_d - LF_d \\ B + \Delta B & E_d \end{bmatrix}; \bar{C} = [C \quad \Delta C]; \bar{D} = [\Delta D \quad F_d]; \bar{E}_f = \begin{bmatrix} E_f - LF_f \\ E_f \end{bmatrix}$$

The residual signal in (7) can be represented in the frequency domain.

$$r(z) = G_{r\bar{u}}(z)\bar{u}(z) + G_{rf}(z)f(z) \tag{8}$$

where $G_{r\bar{u}}(z) = \bar{C}(zI - \bar{A} + L\bar{C})^{-1}(\bar{B} - L\bar{D}) + \bar{D}$ and $G_{rf}(z) = \bar{C}(zI - \bar{A} + L\bar{C})^{-1}(\bar{E}_f - LF_f) + F_f$
$G_{r\bar{u}}(z)$ and $G_{rf}(z)$ are the transfer function matrices from $\bar{u}(k)$ and $f(k)$ to $r(k)$, respectively. The influence of disturbance and model uncertainty on the residual is measured by H_∞ norm and is represented as (9):

$$H_\infty = \|G_{r\bar{u}}(z)\|_\infty = \sup_{\theta \in [0,2\pi]} \bar{\sigma}(G_{r\bar{u}}(z)) = \sup_{\bar{u}(k) \in l_2, \|\bar{u}\|_2 \neq 0} \left\{ \frac{\sum_{k=0}^{\infty} r^T(k) r(k)}{\sum_{k=0}^{\infty} \bar{u}^T(k) \bar{u}(k)} \right\} \tag{9}$$

Robustness against disturbance and model uncertainty is expressed by (10) [30].

$$\|G_{r\bar{u}}(z)\|_\infty < \gamma; \ \gamma > 0 \tag{10}$$

γ represents the maximum effect of model uncertainty and disturbance on the residual, and the value of γ should be smaller. Likewise, the effect of fault on the residual is characterized by H_-index [31].

$$H_- = \|G_{rf}(e^{j\theta})\|_- = \inf_{\theta \in [0,2\pi]} \underline{\sigma}[G_{rf}(e^{j\theta})] \tag{11}$$

The residual's sensitivity to the fault is illustrated by (12).

$$\|G_{rf}(e^{j\theta})\|_- > \beta \quad ; \beta > 0 \tag{12}$$

β denotes the worst-case fault sensitivity measurement of the residual signal. A larger value of β shows that residual is more sensitive to a fault.

The solution of an optimal FD filter design based on $\frac{H_-}{H_\infty}$ optimization for the nominal system (system uncertainty, $\Delta s = 0$) can be easily obtained by solving a single Riccati equation [18]. Unfortunately, the Riccati equation cannot solve $\frac{H_-}{H_\infty}$ optimization problems for dynamic systems with model uncertainties ($\Delta s \neq 0$). The multi-objective optimization problem for linear systems subject to disturbance and model uncertainty is addressed in this study using the LMI technique. Using $\frac{H_-}{H_\infty}$ optimization, the objective is to design an optimal FD filter by determining the filter gain matrix L in a way that (a) makes the augmented system (6) asymptotically stable, (b) makes the residual (7) robust to disturbance and model uncertainties in the H_∞ sense, and (c) makes the residual (7) fault-sensitive.

3. SYNTHESIS OF OPTIMAL FD FILTER

In this section, an optimal FD filter is designed for system (1) in the LMI framework. First, separate solutions of H_∞ and H_-index conditions in (10) and (12) are obtained, followed by an algorithm for solving mixed $\frac{H_-}{H_\infty}$ optimization problem. For onward discussion, the following lemmas help to derive the main results.

Lemma 1 [18]: The observer error dynamics (4) is asymptotically stable and meets the condition (10) for the linear system (1) with zero model uncertainty in the system matrices if the following LMI is true for ($f(k) = 0$) then there exists a scalar $\gamma \geq \gamma_{\min}$, matrix L and $P = P^T > 0$.

$$\begin{bmatrix} -P & P(A-LC) & P(E_d - LF_d) & 0 \\ (A-LC)^T P & -P & 0 & C^T \\ (E_d - LF_d)^T P & 0 & -\gamma I & {F_d}^T \\ 0 & C & F_d & -\gamma I \end{bmatrix} < 0$$

The above lemma provides the necessary and sufficient condition for (10) and ensures that $\|G_{r\bar{u}}(z)\|_\infty < \gamma$.

Lemma 2 [18]: The observer error dynamics (4) is asymptotically stable and meets the condition (12) for the linear system (1) with zero model uncertainty in the system matrices if the following LMI is true for ($\bar{u}(k) = 0$), then there exists a scalar $\beta \leq \beta_{\max}$, matrix L and $P = P^T > 0$.

$$\begin{bmatrix} P - (A-LC)^T P(A-LC) - C^T C & C^T F_f + (A-LC)^T P(E_f - LF_f) \\ (E_f - LF_f)^T P(A-LC) + {F_f}^T C & \beta^2 I - {F_f}^T F_f - (E_f - LF_f)^T P(E_f - LF_f) \end{bmatrix} > 0$$

The above lemma guarantees that the minimum fault sensitivity of the residual is greater than a constant, i.e., $\|G_{rf}(e^{j\theta})\|_- > \beta$.

Lemma 3 [30]: If there exists a symmetric positive definite matrix P, and an arbitrary positive scalar $\varepsilon > 0$ that satisfy $(\varepsilon I - H^T P H)^{-1} > 0$ then

$$(W + H\Sigma G)^T P(W + H\Sigma G) \leq W^T P W + W^T P H(\varepsilon I - H^T P H)^{-1} H^T P W + \varepsilon G^T G$$

Lemma 4 [32]: The following conditions are equivalent when the Schur complement principle is applied to several symmetric matrices A_{11}, A_{12} and A_{22}.

If $A_{11} < 0$ then $\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} < 0$ if and only if $A_{22} - A_{21}(A_{11})^{-1}A_{12} < 0$

If $A_{22} < 0$ then $\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} < 0$ if and only if $A_{11} - A_{12}(A_{22})^{-1}A_{21} < 0$

Theorem 1: Consider system (1) with model uncertainties ($\Delta A, \Delta B, \Delta C, \Delta D \neq 0$), under the assumptions A1 and A2, if there exist scalars $\beta > 0, \gamma > 0$, a filter gain matrix L, a symmetric matrix $P > 0$ and a scalar $\varepsilon > 0$ such that the augmented system (6) is asymptotically stable and the following matrix inequalities hold, then conditions (10) and (12) are satisfied.

$$\begin{bmatrix} {H_2}^T H_2 + {H_3}^T P H_3 - \varepsilon I & {H_2}^T C_0 + {H_3}^T P A_0 & {H_2}^T D_0 + {H_3}^T P B_0 \\ {C_0}^T H_2 + {A_0}^T P H_3 & {C_0}^T C_0 + {A_0}^T P A_0 + \varepsilon {\underline{G_1}}^T \underline{G_1} - P & {C_0}^T D_0 + {A_0}^T P B_0 + \varepsilon {\underline{G_1}}^T \underline{G_2} \\ {D_0}^T H_2 + {B_0}^T P H_3 & {D_0}^T C_0 + {B_0}^T P A_0 + \varepsilon {\underline{G_2}}^T \underline{G_1} & {D_0}^T D_0 + {B_0}^T P B_0 + \varepsilon {\underline{G_2}}^T \underline{G_2} - \gamma^2 I \end{bmatrix} < 0 \tag{13}$$

$$\begin{bmatrix} -{H_2}^T H_2 - {H_3}^T P H_3 + \varepsilon I & {H_2}^T \bar{C}_o + {H_3}^T P \bar{A}_o & {H_2}^T F_{fo} + {H_3}^T P \bar{E}_{fo} \\ {\bar{C}_o}^T H_2 + {\bar{A}_o}^T P H_3 & -{\bar{C}_o}^T \bar{C}_o - {\bar{A}_o}^T P \bar{A}_o - \varepsilon {\underline{G_1}}^T \underline{G_1} + P & -{\bar{C}_o}^T F_{fo} - {\bar{A}_o}^T P \bar{E}_{fo} \\ {F_{fo}}^T H_2 + {\bar{E}_{fo}}^T P H_3 & -{F_{fo}}^T \bar{C}_o - {\bar{E}_{fo}}^T P \bar{A}_o & -{F_{fo}}^T F_{fo} - {\bar{E}_{fo}}^T P \bar{E}_{fo} + \beta^2 I \end{bmatrix} > 0 \tag{14}$$

In addition to solving (13) and (14), optimal filter gain L, can be determined by solving the following optimization problem:

$$\max J = \frac{\beta}{\gamma} \tag{15}$$

Proof of the Theorem

For the augmented system (6) and (7), (10) can be expressed as (16):

$$\|G_{r\bar{u}}\|_\infty < \gamma \leftrightarrow \sum_{k=0}^{\infty} [r^T(k) r(k) - \gamma^2 \bar{u}^T(k) \bar{u}(k)] < \gamma; \quad f(k) = 0 \tag{16}$$

Defining a Lyapunov function, $V(\bar{x}(k)) = \bar{x}^T(k) P \bar{x}(k) > 0$ where $P = \text{diag}[P_1, P_2] > 0$. Suppose $P > 0$, the necessary stability condition listed below is ensured.

$$\sum_{k=0}^{\infty} \Big(V(\bar{x}(k+1)) - V(\bar{x}(k)) \Big) < 0 \tag{17}$$

The control objective (10) and H_∞ FD filter stability is ensured by combining (16) and (17), which will yield

$$\sum_{k=0}^{\infty} [r^T(k) r(k) + V(\bar{x}(k+1)) - V(\bar{x}(k)) - \gamma^2 \bar{u}^T(k) \bar{u}(k)] < 0 \tag{18}$$

From equation (6) and (7), it is easy to write

$$[\bar{x}^T(k) \quad \bar{u}^T(k)] \left(\begin{bmatrix} \bar{A}^T \\ \bar{B}^T \end{bmatrix} P [\bar{A} \quad \bar{B}] + \begin{bmatrix} \bar{C}^T \\ \bar{D}^T \end{bmatrix} [\bar{C} \quad \bar{D}] + \begin{bmatrix} -P & 0 \\ 0 & -\gamma^2 I \end{bmatrix} \right) \begin{bmatrix} \bar{x}(k) \\ \bar{u}(k) \end{bmatrix} < 0 \tag{19}$$

The constant and uncertain system matrices are divided as follows to avoid any ambiguity:

$$\begin{bmatrix} \bar{C} & \bar{D} \\ \bar{A} & \bar{B} \end{bmatrix} = \begin{bmatrix} C_0 & D_0 \\ A_0 & B_0 \end{bmatrix} + \begin{bmatrix} \Delta\bar{C} & \Delta\bar{D} \\ \Delta\bar{A} & \Delta\bar{B} \end{bmatrix} \tag{20}$$

where $\begin{bmatrix} C_0 & D_0 \\ A_0 & B_0 \end{bmatrix} = \begin{bmatrix} C & 0 & 0 & F_d \\ A - LC & 0 & 0 & E_d - LF_d \\ 0 & A & B & E_d \end{bmatrix}$;

$$\begin{bmatrix} \Delta\bar{C} & \Delta\bar{D} \\ \Delta\bar{A} & \Delta\bar{B} \end{bmatrix} = \begin{bmatrix} H_2 \\ H_1 - LH_2 \\ H_1 \end{bmatrix} \Sigma [0 \quad G_1 \quad G_2 \quad 0] = \begin{bmatrix} H_2 \\ H_3 \end{bmatrix} \Sigma [\underline{G_1} \quad \underline{G_2}] = \underline{H} \,\Sigma\, \underline{G}$$

Representing the above matrices as:

$$\underline{A_o} = \begin{bmatrix} C_0 & D_0 \\ A_0 & B_0 \end{bmatrix}; A_o = \begin{bmatrix} A - LC & 0 \\ 0 & A \end{bmatrix}; B_o = \begin{bmatrix} 0 & E_d - LF_d \\ B & E_d \end{bmatrix}; C_o = [C \quad 0]; D_o = [0 \quad F_d]; \underline{H} = \begin{bmatrix} H_2 \\ H_3 \end{bmatrix};$$
$$H_3 = \begin{bmatrix} H_1 - LH_2 \\ H_1 \end{bmatrix}; \underline{G} = [\underline{G_1} \quad \underline{G_2}]; \underline{G_1} = [0 \quad G_1]; \underline{G_2} = [G_2 \quad 0]; \underline{\overline{P}} = \begin{bmatrix} -P & 0 \\ 0 & -\gamma^2 I \end{bmatrix}; \underline{P} = \begin{bmatrix} I & 0 \\ 0 & P \end{bmatrix}$$

Applying Lemma 3 on (19) using (20) turns to (21).

$$[\bar{x}^T(k) \quad \bar{u}^T(k)]\left(\left(\underline{A}_o + \underline{H}\,\Sigma\,\underline{G}\right)^T \underline{P}\left(\underline{A}_o + \underline{H}\,\Sigma\,\underline{G}\right) + \overline{P}\right)\begin{bmatrix}\bar{x}(k)\\ \bar{u}(k)\end{bmatrix} \tag{21}$$

$$\left(\underline{A}_o + \underline{H}\Sigma\underline{G}\right)^T \underline{P}\left(\underline{A}_o + \underline{H}\Sigma\underline{G}\right) + \overline{P} \le \underline{A}_o{}^T \underline{P}\underline{A}_o + \underline{A}_o{}^T \underline{P}\underline{H}\left(\varepsilon I - \underline{H}^T\underline{P}\underline{H}\right)^{-1}\underline{H}^T\underline{P}\underline{A}_o + \varepsilon\underline{G}^T\underline{G} + \overline{P} \tag{22}$$

Applying Lemma 4 on (22) will yield (23).

$$\begin{bmatrix}\underline{H}^T\underline{P}\underline{H} - \varepsilon I & \underline{H}^T\underline{P}\underline{A}_o\\ \underline{A}_o{}^T\underline{P}\underline{H} & \underline{A}_o{}^T\underline{P}\underline{A}_o + \varepsilon\underline{G}^T\underline{G} + \overline{P}\end{bmatrix} < 0 \tag{23}$$

Expanding (23), one can write as (24).

$$\begin{bmatrix}H_2{}^TH_2 + H_3{}^TPH_3 - \varepsilon I & H_2{}^TC_0 + H_3{}^TPA_0 & H_2{}^TD_0 + H_3{}^TPB_0\\ C_0{}^TH_2 + A_0{}^TPH_3 & C_0{}^TC_0 + A_0{}^TPA_0 + \varepsilon\underline{G_1}^T\underline{G_1} - P & C_0{}^TD_0 + A_0{}^TPB_0 + \varepsilon\underline{G_1}^T\underline{G_2}\\ D_0{}^TH_2 + B_0{}^TPH_3 & D_0{}^TC_0 + B_0{}^TPA_0 + \varepsilon\underline{G_2}^T\underline{G_1} & D_0{}^TD_0 + B_0{}^TPB_0 + \varepsilon\underline{G_2}^T\underline{G_2} - \gamma^2 I\end{bmatrix} < 0 \tag{24}$$

Rewriting the above matrix inequality as (25).

$$\begin{bmatrix}H_2{}^T & H_3{}^T & 0\\ C_0{}^T & A_0{}^T & \underline{G_1}^T\\ D_0{}^T & B_0{}^T & \underline{G_2}^T\end{bmatrix}\begin{bmatrix}I & 0 & 0\\ 0 & P & 0\\ 0 & 0 & \varepsilon I\end{bmatrix}\begin{bmatrix}H_2 & C_0 & D_0\\ H_3 & A_0 & B_0\\ 0 & \underline{G_1} & \underline{G_2}\end{bmatrix} - \begin{bmatrix}\varepsilon I & 0 & 0\\ 0 & P & 0\\ 0 & 0 & \gamma^2 I\end{bmatrix} < 0 \tag{25}$$

Applying the Schur complement lemma given below, (25) can be represented as (26).

$$\begin{bmatrix}-\varepsilon I & 0 & 0 & H_2{}^T & H_3{}^T & 0\\ 0 & -P & 0 & C_0{}^T & A_0{}^T & \underline{G_1}^T\\ 0 & 0 & -\gamma^2 I & D_0{}^T & B_0{}^T & \underline{G_2}^T\\ H_2 & C_0 & D_0 & -I & 0 & 0\\ H_3 & A_0 & B_0 & 0 & -P^{-1} & 0\\ 0 & \underline{G_1} & \underline{G_2} & 0 & 0 & -\varepsilon^{-1}I\end{bmatrix} < 0 \tag{26}$$

The nonlinear inequality is transformed into linear inequality form by performing matrix equivalent transformation as (27).

$$\begin{bmatrix}-\varepsilon I & 0 & 0 & H_2{}^T & H_3{}^TP & 0\\ 0 & -P & 0 & C_0{}^T & A_0{}^TP & \underline{G_1}^T\varepsilon\\ 0 & 0 & -\gamma^2 I & D_0{}^T & B_0{}^TP & \underline{G_2}^T\varepsilon\\ H_2 & C_0 & D_0 & -I & 0 & 0\\ PH_3 & PA_0 & PB_0 & 0 & -P & 0\\ 0 & \varepsilon\underline{G_1} & \varepsilon\underline{G_2} & 0 & 0 & -\varepsilon I\end{bmatrix} < 0 \tag{27}$$

By inserting the $H_3, A_0, B_0, C_0, D_0, \underline{G_1}, \underline{G_2}$ and P matrices in the above matrix, which completes the proof of the first part of Theorem 1. Similarly, H_- index-based fault sensitivity condition (12) can be derived as (28).

$$\left\|G_{rf}\right\|_- > \beta \leftrightarrow \textstyle\sum_{k=0}^{\infty}[r^T(k)r(k) > \beta^2 f^T(k)f(k)] \quad ; \bar{u}(k) = 0 \tag{28}$$

Considering the Lyapunov function defined earlier, $V(k) = \bar{x}^T(k)P\bar{x}(k) > 0,\ P > 0$. The control objective (12) and H_- index filter stability is ensured by (29).

$$\textstyle\sum_{k=0}^{\infty}\left[r^T(k)r(k) - V\left(\bar{x}(k+1)\right) + V\left(\bar{x}(k)\right) - \beta^2 f^T(k)f(k)\right] > 0 \tag{29}$$

After mathematical simplification, one can write (29) as (30).

$$\textstyle\sum_{k=0}^{\infty}\left[r^T(k)r(k) - \beta^2 f^T(k)f(k) - V\left(\bar{x}(k+1)\right) + V\left(\bar{x}(k)\right)\right] < 0 \tag{30}$$

Substituting matrices from (6) and (7) into (30) by taking $\bar{u}(k) = 0$, can be written in matrix form as (31) and (32).

$$[\bar{x}^T \quad f^T]\left(\begin{bmatrix}\bar{A}^T\\ \bar{E}_f{}^T\end{bmatrix}P[\bar{A} \quad \bar{E}_f]+\begin{bmatrix}\bar{C}^T\\ F_f{}^T\end{bmatrix}[\bar{C} \quad F_f]+\begin{bmatrix}-P & 0\\ 0 & -\beta^2 I\end{bmatrix}\right)\begin{bmatrix}\bar{x}\\ f\end{bmatrix}<0 \tag{31}$$

$$[\bar{x}^T \quad f^T]\left(\begin{bmatrix}\bar{C}^T & \bar{A}^T\\ F_f{}^T & \bar{E}_f{}^T\end{bmatrix}\begin{bmatrix}I & 0\\ 0 & P\end{bmatrix}\begin{bmatrix}\bar{C} & F_f\\ \bar{A} & \bar{E}_f\end{bmatrix}+\begin{bmatrix}-P & 0\\ 0 & -\beta^2 I\end{bmatrix}\right)\begin{bmatrix}\bar{x}\\ f\end{bmatrix}<0 \tag{32}$$

The constant and uncertain matrices are separated as:

$$\begin{bmatrix}\bar{C} & F_f\\ \bar{A} & \bar{E}_f\end{bmatrix}=\begin{bmatrix}\bar{C}_o & F_{fo}\\ \bar{A}_o & \bar{E}_{fo}\end{bmatrix}+\begin{bmatrix}\Delta\bar{C} & \Delta F_f\\ \Delta\bar{A} & \Delta\bar{E}_f\end{bmatrix} \tag{33}$$

where $\begin{bmatrix}\bar{C}_o & F_{fo}\\ \bar{A}_o & \bar{E}_{fo}\end{bmatrix}=\begin{bmatrix}C & 0 & F_f\\ A-LC & 0 & E_f-LF_f\\ 0 & A & E_f\end{bmatrix}$ and $\begin{bmatrix}\Delta\bar{C} & \Delta F_f\\ \Delta\bar{A} & \Delta\bar{E}_f\end{bmatrix}=\begin{bmatrix}H_2\\ H_1-LH_2\\ H_1\end{bmatrix}\Sigma[0 \quad G_1 \quad 0]$

Defining the new matrices:

$$\bar{\bar{A}}_o=\begin{bmatrix}\bar{C}_o & F_{fo}\\ \bar{A}_o & \bar{E}_{fo}\end{bmatrix};\ \bar{A}_o=\begin{bmatrix}A-LC & 0\\ 0 & A\end{bmatrix};\ \bar{E}_{fo}=\begin{bmatrix}E_f-LF_f\\ E_f\end{bmatrix};\ F_{fo}=F_f;\ \bar{C}_o=[C \quad 0];\ \underline{H}=\begin{bmatrix}H_2\\ H_3\end{bmatrix};$$
$$H_3=\begin{bmatrix}H_1-LH_2\\ H_1\end{bmatrix};\ \underline{G}=[\underline{G_1} \quad 0];\ \underline{G_1}=[0 \quad G_1];\ \underline{P}=\begin{bmatrix}I & 0\\ 0 & P\end{bmatrix};\qquad \overline{\underline{P1}}=\begin{bmatrix}-P & 0\\ 0 & -\beta^2 I\end{bmatrix}$$

It is simple to write (32) as follows using Lemma 3:

$$[\bar{x}^T \quad f^T]\left((\bar{\bar{A}}_o+\underline{H}\,\Sigma\,\underline{G})^T\underline{P}(\bar{\bar{A}}_o+\underline{H}\,\Sigma\,\underline{G})+\overline{\underline{P1}}\right)\begin{bmatrix}\bar{x}\\ f\end{bmatrix}<0 \tag{34}$$

$$\bar{\bar{A}}_o{}^T\underline{P}\bar{\bar{A}}_o+\bar{\bar{A}}_o{}^T\underline{PH}(\varepsilon I-\underline{H}^T\underline{PH})^{-1}\underline{H}^T\underline{P}\bar{\bar{A}}_o+\varepsilon\underline{G}^T\underline{G}+\overline{\underline{P1}}<0 \tag{35}$$

According to H_-index criteria ($\left\|G_{rf}(e^{j\theta})\right\|_- > 0$), the above inequality is written as:

$$-\bar{\bar{A}}_o{}^T\underline{P1}\bar{\bar{A}}_o-\bar{\bar{A}}_o{}^T\underline{P1H}(\varepsilon I-\underline{H}^T\underline{P1H})^{-1}\underline{H}^T\underline{P1}\bar{\bar{A}}_o-\varepsilon\underline{G}^T\underline{G}-\overline{\underline{P1}}>0 \tag{36}$$

By applying Lemma 4, (36) becomes (37).

$$\begin{bmatrix}\varepsilon I-\underline{H}^T\underline{PH} & \underline{H}^T\underline{P}\bar{\bar{A}}_o\\ \bar{\bar{A}}_o{}^T\underline{PH} & -\bar{\bar{A}}_o{}^T\underline{P}\bar{\bar{A}}_o-\varepsilon\underline{G}^T\underline{G}-\overline{\underline{P1}}\end{bmatrix}>0 \tag{37}$$

Expanding matrix inequality (37)

$$\begin{bmatrix}-H_2{}^TH_2-H_3{}^TPH_3+\varepsilon I & H_2{}^T\bar{C}_o+H_3{}^TP\bar{A}_o & H_2{}^TF_{fo}+H_3{}^TP\bar{E}_{fo}\\ \bar{C}_o{}^TH_2+\bar{A}_o{}^TPH_3 & -\bar{C}_o{}^T\bar{C}_o-\bar{A}_o{}^TP\bar{A}_o-\varepsilon\underline{G_1}^T\underline{G_1}+P & -\bar{C}_o{}^TF_{fo}-\bar{A}_o{}^TP\bar{E}_{fo}\\ F_{fo}{}^TH_2+\bar{E}_{fo}{}^TPH_3 & -F_{fo}{}^T\bar{C}_o-\bar{E}_{fo}{}^TP\bar{A}_o & -F_{fo}{}^TF_{fo}-\bar{E}_{fo}{}^TP\bar{E}_{fo}+\beta^2 I\end{bmatrix}>0 \tag{38}$$

This concludes the proof of Theorem 1's second part. FD filter gain can be calculated by solving the LMIs (27) and (38) for the optimization problem (15).

$$L=P_1{}^{-1}X_1 \tag{39}$$

3.1. Residual evaluation and threshold

In the second step of the FD process, the generated residual is evaluated using l_2 signal norm and further compared with the threshold, $J_{th}>0$. The residual (3) that was generated using the proposed FD filter can be shown as (40):

$$r(k)=r_d(k)+r_u(k)+r_f(k) \tag{40}$$

In fault-free case, $r_f(k) = 0$, then the residual evaluation function becomes $J = \|r_d(k) + r_u(k)\|_2^2$. Thus, the threshold can be computed as $J_{th} = \sup\|r_d(k) + r_u(k)\|_2^2$. In the end, the evaluated residual (J) is compared with the threshold (J_{th}) and the fault alarm is released when the following condition is satisfied:

$J > J_{th}$; fault alarm
$J \leq J_{th}$; fault-free

4. APPLICATION TO A THREE-TANK SYSTEM

A three-tank system application is used in this study. The system is often used to illustrate the principles of process control, system dynamics, and fault detection. In such a system, the liquid levels in the tanks and the flow rates between them are managed using sensors, actuators, and controllers. Automation plays a critical role in this setup by ensuring the precise regulation of these variables to achieve desired outcomes, such as maintaining specific liquid levels or flow rates. Using advanced automation technologies, such as programmable logic controllers (PLCs) and distributed control systems (DCS), the three-tank system can operate autonomously, adjusting valves and pumps based on real-time feedback from level and flow sensors. This level of automation improves accuracy, reduces manual intervention, and ensures consistent operation even in complex scenarios. Moreover, integrating fault detection algorithms into the system enhances reliability by identifying anomalies like sensor malfunctions, leaks, or blockages, enabling proactive maintenance. Thus, the automation of a three-tank system serves as a foundational model for understanding and implementing control strategies in larger industrial processes such as chemical manufacturing, water treatment, and oil refining.

This section presents simulation results that demonstrate the effectiveness of the proposed FD method. For simulation purposes, abrupt and intermittent faults are introduced in the sensors and actuators of the advanced three-tank system illustrated in Figure 1. Such faults significantly degrade system performance and are included in the study to evaluate the capability of the proposed method in identifying these critical issues. Modeling errors from the system linearization process are incorporated as norm-bounded model uncertainties. The behavior of the three-tank system is described by the following set of nonlinear equations, which capture its dynamics.

$$\begin{aligned} A\dot{h}_1 &= Q_1 - Q_{13} \\ A\dot{h}_2 &= Q_2 + Q_{32} - Q_{20} \\ A\dot{h}_1 &= Q_{13} - Q_{32} \end{aligned} \tag{41}$$

with

$$\begin{aligned} Q_{13} &= a_1 s_{13} sgn(h_1 - h_3)\sqrt{2g|h_1 - h_3|} \\ Q_{32} &= a_3 s_{23} sgn(h_3 - h_2)\sqrt{2g|h_3 - h_2|} \\ Q_{20} &= a_2 s_0 \sqrt{2gh_2} \end{aligned}$$

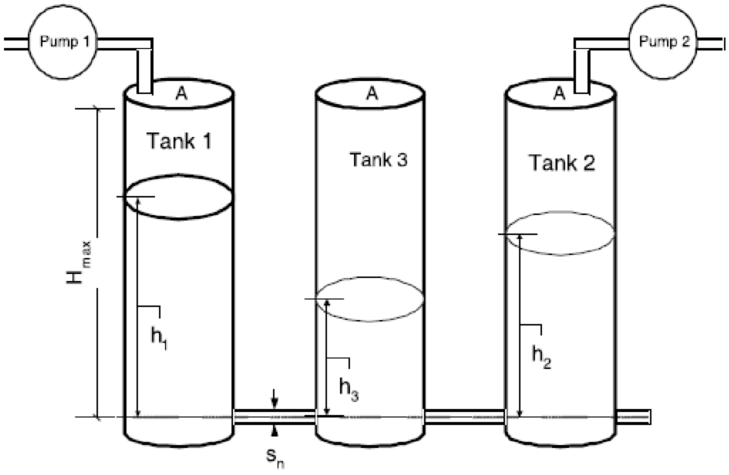

Figure 1. A three-tank system [33]

The process outputs $y(k)$, represented by h_3, h_2, h_1, corresponding to the water levels in the respective tanks. The process inputs $u(k)$ are denoted by Q_1 and Q_2, while Q_{ij} represents the flow rate of water from tank i-th to tank j-th. Additionally, s_{13} and s_{23} refer to the cross-sectional areas of the pipes connecting the respective tanks. The cross-sectional area of the pipe connected to Tank 2 is s_0. sgn denotes the signum function, which is defined as

$$\text{sgn}(x) = \begin{cases} -1 & if x < 0 \\ 0 & if x = 0 \\ 1 & if x > 0 \end{cases}$$

$$s_{13} = s_{23} = s_0 = s_n$$

The system's primary parameters and coefficients are shown in Table 1. In the three-tank system, an unknown disturbance arises from water falling into the tanks from the pumps. Additionally, the sensors used to measure water levels introduce noise measurement. For fault detection (FD), a linear model of the system is derived by applying Taylor series expansion and linearizing the dynamics around the equilibrium or operating point. This process results in a linear nominal model of the discrete-time system in state-space form, as shown in (1). The linearization is performed at the operating points $h_1 = 45\text{cm}$, $h_2 = 15\text{cm}$ and $h_3 = 30\text{cm}$. Nominal matrices are obtained after linearizing the nonlinear model of the system.

$$A = \begin{bmatrix} 0.9915 & 0 & 0.0084 \\ 0 & 0.9807 & 0.0082 \\ 0.0084 & 0.0082 & 0.9833 \end{bmatrix}; B = \begin{bmatrix} 0.0065 & 0.0008 \\ 0.0008 & 0.0065 \\ 0 & 0 \end{bmatrix}; \ E_d = \begin{bmatrix} 0.25 & 0 & 0 \\ 0 & 0.25 & 0 \\ 0 & 0 & 0.25 \end{bmatrix};$$

$$C = \text{diag}[1,1,1]; \ D = 0; \ E_f = B; \ F_d = F_f = \text{diag}[1,1,1]$$

The linearization process incorporates modelling errors known as norm-bounded model uncertainty into the system matrices, which are denoted as:

$$H_1 = H_2 = \begin{bmatrix} -0.01 & 0 & 0 \\ 0 & -0.01 & 0 \\ 0 & 0 & -0.01 \end{bmatrix}; G_1 = \begin{bmatrix} 0.01 & 0 & 0.015 \\ 0 & 0.01 & 0.015 \\ 0.01 & 0.01 & 0.05 \end{bmatrix}; G_2 = \begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 0.02 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Table 1. Three-tank system's parameters [33]

Parameters	Value	Unit
A	154	cm^2
s_n	0.5	cm^2
H_{max}	62	cm
$Q1_{max}$	100	cm^3/sec
$Q2_{max}$	100	cm^3/sec
a_1	0.46	
a_2	0.60	
a_3	0.45	

The uncertain parameter ($\sum = \text{diag}[0.9597,0.9597,0.9597]$) is chosen at random and unknown disturbance, $d(k) \in [-0.01,0.01]$ is used for simulations. The pump inflows are assumed to be constant with specified values of $Q_1 = 25.6$ cm^3/sec and $Q_2 = 39.5$ cm^3/sec. After solving the linear matrix inequalities in (13) and (14), a disturbance attenuation level of $\gamma = 1.026$ and a fault sensitivity level of $\beta = 1.9891$ are achieved. The corresponding filter gain matrix is computed using (39) and is given as

$$L = \begin{bmatrix} 0.3265 & -0.0001 & -0.0017 \\ * & 6.7344 & -0.0017 \\ * & * & 6.7265 \end{bmatrix}$$

Furthermore, the residual evaluation function is computed using l_2 norm of the residual (40) and the threshold is computed as $J_{th|f=0} = \sup_{\bar{u}(k)\in l_2} \|r(k)\|_2 = 0.03$. The residual in the sensor/actuator fault-free case is shown in Figure 2.

Figure 3 displays the impact of an abrupt sensor fault in Tank 1 on the residual. A fault with a 10 cm offset is introduced to the sensor input of Tank 1 at $t = 80$ sec. The simulation results confirm that the fault is successfully detected. Comparable responses and successful detections are also observed for faults in the other sensors. As shown in Figure 3, the evaluation function remains below the threshold prior to the fault occurrence but exceeds the threshold when the sensor fault occurs at t = 80 sec. Similarly, Figure 4 demonstrates the response when an intermittent fault is applied to the actuator of Pump 1. The results highlight that the fault detection filter effectively identified faults in the discrete-time system, even in the presence of unknown disturbances and model uncertainty.

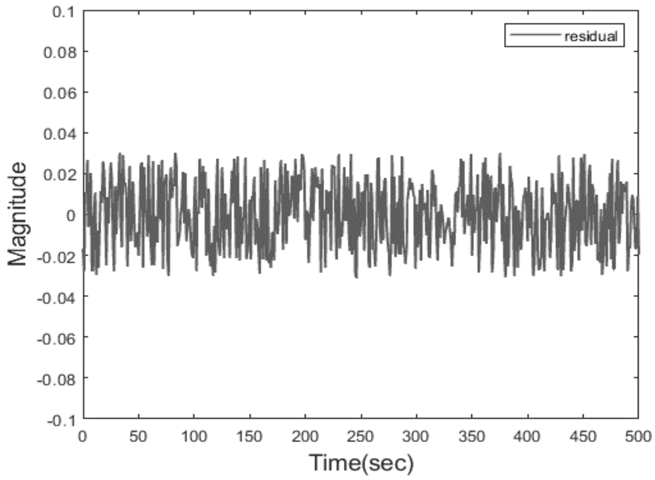

Figure 2. Residual in a fault-free case

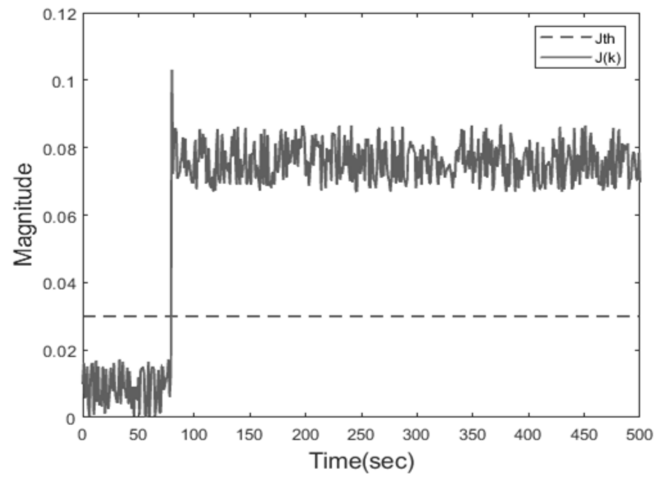

Figure 3. An abrupt sensor FD in Tank 1

Remark: In section 3, two linear matrix inequalities (LMIs) are derived for fault detection (FD) in linear uncertain systems. An H- index-based fault-sensitive filter is designed to improve the fault sensitivity of the residual. However, this filter also exhibits sensitivity to disturbances and model uncertainties. In contrast, the H∞ FD filter ensures disturbance attenuation but also provides robustness against faults. To address these challenges, a multi-objective H-/H∞ based FD filter is proposed, which simultaneously offers robustness to disturbances and model uncertainties, as well as sensitivity to faults. Rather than maximizing β and minimizing γ separately, the performance index, β/γ, is maximized in this design. It is important to note that the residual generated by the H-/H∞ based filter may be less sensitive than that produced by the H- index-based fault-sensitive filter. Similarly, the residual from the H-/H∞ based filter might be less robust to disturbances and model uncertainties compared to the residual from the H∞ based filter. Nevertheless, the proposed H-/H∞ based FD filter is advantageous as it achieves both disturbance attenuation and fault sensitivity simultaneously.

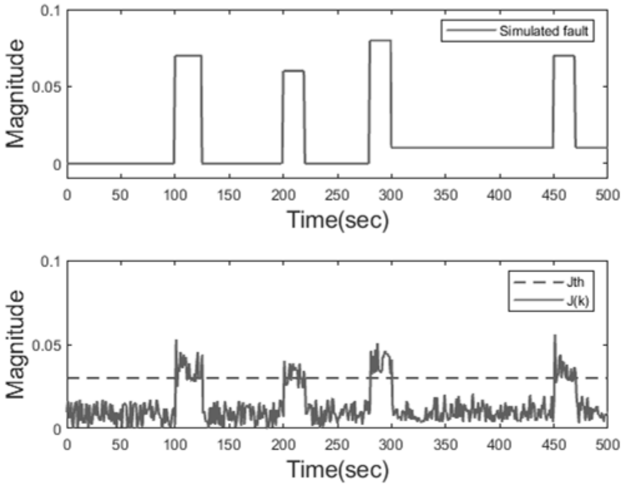

Figure 4. Simulated fault (top); Evaluated residual and threshold in actuator fault (bottom)

5. CONCLUSION

Robotics and automation systems rely on advanced fault detection mechanisms to ensure seamless operation, prevent downtime, and maintain safety standards. This paper addresses the fault detection problem for discrete-time linear systems subjected to deterministic disturbances and norm-bounded model uncertainty. The proposed FD filter generates a residual that simultaneously attenuates the effect of model uncertainty and disturbance and enhances the sensitivity to faults. The filter optimizes the H-/H∞ performance index, ensuring the best trade-off between robustness and sensitivity in all directions of the residual space. A solution to the problem is formulated using linear matrix inequality constraints. Successful actuator and sensor fault detection results of the three-tank system obtained from simulation validate the effectiveness of the proposed observer-based FD system. By identifying and addressing the faults, these systems can enhance reliability and efficiency across various industrial applications.

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AUTHOR CONTRIBUTIONS STATEMENT

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C : **C**onceptualization
M : **M**ethodology
So : **So**ftware
Va : **Va**lidation
Fo : **Fo**rmal analysis
I : **I**nvestigation
R : **R**esources
D : **D**ata Curation
O : Writing - **O**riginal Draft
E : Writing - Review & **E**diting
Vi : **Vi**sualization
Su : **Su**pervision
P : **P**roject administration
Fu : **Fu**nding acquisition

CONFLICT OF INTEREST STATEMENT

Authors state no conflict of interest

DATA AVAILABILITY
Data availability is not applicable to this paper as no new data were created in this study.

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