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Zeroing neurodynamics proportional-integral-derivative controller for order-2 system: design, analysis, and verification

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ABSTRACT

This paper presents a zeroing neurodynamics proportional-integral-derivative (PID) controller for an order-2 single-input single-output (SISO) affine-in-control nonlinear system. The controller is derived from zeroing neurodynamics (ZN) and reformulates PID gains through ZN parameters, establishing an explicit correspondence to the closed-loop poles of the error dynamics. This enables systematic pole placement for stability and performance tuning, eliminating empirical trial-and-error. Theoretical analysis proves exponential convergence of the tracking error. The inverted pendulum, a benchmark system with strong geometric nonlinearity, is used to verify the controller. Simulations with large initial offsets and constant references show that the proposed controller maintains stable tracking in operating conditions where the conventional PID controller exhibits substantial performance degradation. The resulting framework provides a model-based and theoretically grounded alternative for nonlinear control.

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1. INTRODUCTION

Order-2 nonlinear systems constitute a fundamental and ubiquitous class of systems in control engineering, which are used to model diverse physical phenomena ranging from electromechanical devices to vehicle dynamics [1]. Precise trajectory tracking control for such systems remains challenging because of their inherent nonlinearities. While advanced control techniques such as optimal control [2], robust control [3], and sliding mode control [4] have emerged, proportional-integral-derivative (PID) control remains widely used in industry owing to its structural simplicity and intuitive tuning [5]–[7]. However, its performance can degrade significantly for systems with strong nonlinearities or complex reference signals, primarily because it does not explicitly use model knowledge and its parameters are usually selected empirically [8], [9].

To adapt PID controllers for nonlinear systems, various tuning and stability frameworks have been developed. For instance, internal-model-control tuning methods often simplify complex nonlinear dynamics into low-order linear systems with time delays, potentially degrading transient performance [10]. To further guarantee stability without over-simplification, parameter-manifold approaches were proposed to achieve global or semi-global stabilization for order-2 and n-dimensional nonlinear systems [11]–[12]. Although these works established a solid theoretical foundation for PID-based nonlinear control, their control laws remain tied to classical PID structures and empirical gain selection.

This paper addresses the control of an order-2 SISO affine-in-control nonlinear system by proposing and analyzing a zeroing neurodynamics PID (ZN-PID) controller derived from the zeroing neurodynamics formula [13]–[17]. The effectiveness of the ZN formula has been validated in numerous applications [18]–[21]. The core contribution is the integration of exact model-based linearization [22]–[24] with a reformulated PID structure. The conventional PID gains are redefined through ZN parameters and directly related to the closed-loop pole locations. The benchmark inverted pendulum, which exhibits strong geometric nonlinearity, is selected as the representative system.

The zeroing neurodynamics PID controller design methodology leverages recursively defined zeroing functions to construct a higher-order error dynamic. By strategically setting a specific zeroing function to zero and incorporating the exact knowledge of the system dynamics, we derive the general control law. This law naturally decomposes into a model compensation term responsible for linearizing the plant dynamics and a PID control term ensuring asymptotic error convergence. Crucially, the parameters of the zeroing neurodynamics PID controller (i.e., λ_1, λ_2, and λ_3) are transformed into equivalent PID gains (i.e., K_{p}, K_{d}, and K_{i}), and a direct correspondence between these parameters and the closed-loop poles of the resulting order-3 error dynamics is established. This intrinsic link provides a theoretically grounded alternative to the often cumbersome trial-and-error tuning associated with conventional PID controllers.

Simulation studies compare the proposed controller with a conventionally tuned PID controller [25] on the inverted pendulum. The results show superior tracking by ZN-PID for large initial offsets and constant references. In particular, the ZN-PID controller maintains the pendulum equilibrium under high-amplitude references, whereas the conventional PID controller loses balance because of the severe geometric nonlinearity.

The remainder of this paper is structured as follows. Section 2 introduces the general model of order-2 systems and the inverted pendulum model. Section 3 presents the ZN-PID derivation and its inverted-pendulum formulation. Section 4 proves the exponential stability of the controller. Section 5 reports comparative simulation results. Section 6 concludes the paper.

Finally, the main contributions of the paper are listed as follows.

- Controller derivation: A ZN design formula is used to derive a ZN-PID controller for order-2 systems.
- PID gains reformulation: The analysis establishes a correspondence between PID gains and the closed-loop poles of the order-3 error dynamics, providing a basis for pole-placement tuning.
- Stability proof: A rigorous proof verifies the exponential stability of the ZN-PID controller.
- Comparative simulations: Comparative simulation experiments on an inverted pendulum validate the effectiveness of the proposed controller.

2. ORDER-2 SYSTEM AND INVERTED PENDULUM

In this section, the general form of an order-2 system and the inverted pendulum model are given. First, the general form of a SISO affine-in-control order-2 system is described as,

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = f(x_1(t), x_2(t)) + g(x_1(t), x_2(t))u(t), \\ \;y(t) = x_1(t), \end{cases} \tag{1}$$

where $x_1(t) \in \mathbb{R}$, $x_2(t) \in \mathbb{R}$, and $\boldsymbol{x}(t) = [x_1(t), x_2(t)]^{\mathrm{T}}$ are state variables of the system; $u(t) \in \mathbb{R}$ denotes the control input of the system; $y(t) \in \mathbb{R}$ denotes the output of the system; $f : \mathbb{R}\times\mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R}\times\mathbb{R} \to \mathbb{R}\backslash\{0\}$ are transformation functions of state variables, and $g(x_1(t), x_2(t))$ is a positive or negative coefficient term. Given an order-2 system described by (1) and a desired output (reference signal) $y_{\mathrm{d}}(t) \in \mathbb{R}$, our goal is to find a $u(t)$ as input, which makes the output tracking error $e(t) = y_{\mathrm{d}}(t) - y(t)$ approach zero. For simplicity, in the remainder of this section, $x_1(t)$ and $x_2(t)$ are abbreviated as x_1 and x_2, respectively. All system parameters are real numbers.

The inverted pendulum system [26] is formulated as an order-2 nonlinear system within the structure of (1). The state variable x_1 is the angular displacement of the pendulum from the vertical equilibrium, and x_2 is the angular velocity. The force applied to the cart is the control input $u(t)$. Then, the functions $f(x_1, x_2)$ and

$g(x_1, x_2)$ in (1) are identified as

$$\begin{cases} f(x_1, x_2) = \dfrac{(M+m)g_{\mathrm{a}} \sin x_1 - ml \sin x_1 \cos x_1 x_2^2}{l\left(M + m\sin^2 x_1\right)}, \\ g(x_1, x_2) = -\dfrac{\cos x_1}{l\left(M + m\sin^2 x_1\right)}, \end{cases} \tag{2}$$

where $M > 0$ and $m > 0$ are respectively the mass of the cart and the pendulum; $l > 0$ is the pendulum length; g_{a} is the gravitational acceleration. To ensure $g(x_1, x_2) \neq 0$, the angular displacement x_1 is constrained to $x_1 \in (-\pi/2, \pi/2)$, which guarantees $\cos x_1 > 0$, ensuring the non-singularity of $g(x_1, x_2)$.

3. ZEROING NEURODYNAMICS PID CONTROLLER

In this section, the general ZN-PID design procedure and its inverted-pendulum formulation are given.

3.1. General form

First, according to [27], we define the following zeroing functions recursively:

$$\begin{cases} z_0(t) = \displaystyle\int_0^t e(\tau)\mathrm{d}\tau, \\ z_1(t) = \dot{z}_0(t) + \lambda_1 z_0(t) = e(t) + \lambda_1 \displaystyle\int_0^t e(\tau)\mathrm{d}\tau, \\ z_2(t) = \dot{z}_1(t) + \lambda_2 z_1(t) = \dot{e}(t) + (\lambda_1 + \lambda_2)e(t) + \lambda_1\lambda_2 \displaystyle\int_0^t e(\tau)\mathrm{d}\tau, \\ z_3(t) = \dot{z}_2(t) + \lambda_3 z_2(t) \\ \qquad = \ddot{e}(t) + (\lambda_1 + \lambda_2 + \lambda_3)\dot{e}(t) + (\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3)e(t) + \lambda_1\lambda_2\lambda_3 \displaystyle\int_0^t e(\tau)\mathrm{d}\tau, \end{cases} \tag{3}$$

where λ_1, λ_2, and λ_3 are the controller parameters that satisfy $\lambda_i \in \mathbb{C}$ and $\mathrm{Re}(\lambda_i) > 0$ ($i \in \{1, 2, 3\}$); $z_0(t)$, $z_1(t)$, $z_2(t)$, and $z_3(t)$ are order-0 through order-3 zeroing functions. Besides, $e(t)$, $\dot{e}(t)$, and $\ddot{e}(t)$ represent the output tracking error and its order-1 and order-2 time derivatives, respectively. Based on the state equations of the general form of the order-2 system in (1), their specific expressions are derived as follows:

$$\begin{cases} e(t) = y_{\mathrm{d}}(t) - y(t) = y_{\mathrm{d}}(t) - x_1(t), \\ \dot{e}(t) = \dot{y}_{\mathrm{d}}(t) - \dot{y}(t) = \dot{y}_{\mathrm{d}}(t) - x_2(t), \\ \ddot{e}(t) = \ddot{y}_{\mathrm{d}}(t) - \ddot{y}(t) = \ddot{y}_{\mathrm{d}}(t) - \dot{x}_2(t) = \ddot{y}_{\mathrm{d}}(t) - f(x_1, x_2) - g(x_1, x_2)u(t). \end{cases} \tag{4}$$

Then, to simplify the form of the derived control law and to facilitate the comparison between the zeroing neurodynamics PID controller and the conventional PID controller, we define the following coefficients:

$$\begin{cases} K_{\mathrm{d}} = \lambda_1 + \lambda_2 + \lambda_3, \\ K_{\mathrm{p}} = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3, \\ K_{\mathrm{i}} = \lambda_1\lambda_2\lambda_3. \end{cases} \tag{5}$$

If the parameters λ_i are arbitrary complex numbers, the corresponding coefficients K_{d}, K_{p}, K_{i} might also become complex, rendering the controller physically unrealizable. To address this, we impose an additional requirement: if any λ_i is complex, it must appear in a complex-conjugate pair. From (5), it is straightforward to verify that this ensures all coefficients are real-valued, thus guaranteeing the physical realizability of the controller.

In addition, to avoid the explicit integral item in the control input, a new state variable $x_3(t)$ which satisfies $\dot{x}_3(t) = e(t)$ and $x_3(0) = 0$ is introduced into the original order-2 system (1), and $\boldsymbol{x}_{\mathrm{e}}$ is defined as $[x_1, x_2, x_3]^{\mathrm{T}}$. Then the extended order-2 system is developed as below:

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = f(x_1, x_2) + g(x_1, x_2)u(t), \\ \dot{x}_3(t) = y_{\mathrm{d}}(t) - x_1(t), \\ y(t) = x_1(t). \end{cases} \tag{6}$$

Finally, by setting $z_3(t) = 0$ in (3) and combining it with (4) and (5), we obtain the general form of the zeroing neurodynamics PID controller as follows:

$$\begin{aligned} u(t) &= \frac{1}{g(x_1, x_2)}(\ddot{y}_\mathrm{d}(t) - f(x_1, x_2) + K_\mathrm{d}(\dot{y}_\mathrm{d}(t) - x_2(t)) + K_\mathrm{p}(y_\mathrm{d}(t) - x_1(t)) + K_\mathrm{i}x_3(t)) \\ &= \frac{1}{g(x_1, x_2)}\left(\ddot{y}_\mathrm{d}(t) - f(x_1, x_2) + K_\mathrm{d}\dot{e}(t) + K_\mathrm{p}e(t) + K_\mathrm{i}\int_0^t e(\tau)\mathrm{d}\tau\right) \\ &= \frac{1}{g(x_1, x_2)}(u_\mathrm{mp}(t) + u_\mathrm{pid}(t)), \end{aligned} \tag{7}$$

where $u_\mathrm{mp}(t) = \ddot{y}_\mathrm{d}(t) - f(x_1, x_2)$ is the model compensation term; $u_\mathrm{pid}(t) = K_\mathrm{d}(\dot{y}_\mathrm{d}(t) - x_2(t)) + K_\mathrm{p}(y_\mathrm{d}(t) - x_1(t)) + K_\mathrm{i}x_3(t)$ is the PID control term.

The following is a detailed discussion for components in the proposed controller. First, by leveraging exact knowledge of the system dynamics $f(x_1, x_2)$, the model compensation term $u_\mathrm{mp}(t)$ linearizes the nonlinear order-2 system and simplifies the tracking problem. The inclusion of $\ddot{y}_\mathrm{d}(t)$ directly embeds the desired trajectory's acceleration profile into the control input. Then, the PID control term $u_\mathrm{pid}(t)$ ensures the asymptotic convergence of the tracking error. Besides, the conventional PID gains are refactored into the tuning parameters λ_i ($i \in \{1, 2, 3\}$), establishing an explicit correspondence between these parameters and the closed-loop poles of an order-3 error system. This point is to be discussed in detail in Section 4.

It is worth noting that, unlike optimization-based control strategies or online learning algorithms that require iterative computation at every sampling time, the ZN-PID controller only involves basic arithmetic operations. Its computational complexity is $O(1)$ per time step and it can be implemented on standard embedded microcontrollers.

3.2. Zeroing neurodynamics PID controller for the inverted pendulum

Combining (2) and (7), the ZN-PID controller for the inverted pendulum is obtained as

$$\begin{aligned} u(t) = &-\frac{l\left(M + m\sin^2 x_1\right)}{\cos x_1}\left(\frac{m\sin x_1 \cos x_1 x_2^2}{\left(M + m\sin^2 x_1\right)} + \ddot{y}_\mathrm{d}(t) - \frac{(M + m)g_\mathrm{a}\sin x_1}{l\left(M + m\sin^2 x_1\right)}\right. \\ &\left. + K_\mathrm{d}(\dot{y}_\mathrm{d}(t) - x_2(t)) + K_\mathrm{p}(y_\mathrm{d}(t) - x_1(t)) + K_\mathrm{i}x_3(t)\right). \end{aligned} \tag{8}$$

4. STABILITY ANALYSIS

In this section, we prove the stability of the ZN-PID controller. Specifically, the stability of the ZN-PID controller is described by the following theorem.

Theorem 1. When the states of system (6) and parameters in functions $f(x_1, x_2)$ and $g(x_1, x_2)$ are accurately measured, the output tracking error $e(t)$ of the zeroing neurodynamics PID controller (7) exponentially converges to zero.

Proof. By substituting (7) for the $u(t)$ in (6), we get

$$\ddot{e}(t) + K_\mathrm{d}\dot{e}(t) + K_\mathrm{p}e(t) + K_\mathrm{i}\int_0^t e(\tau)\mathrm{d}\tau = 0, \tag{9}$$

which is equivalent to

$$\dddot{e}(t) + K_\mathrm{d}\ddot{e}(t) + K_\mathrm{p}\dot{e}(t) + K_\mathrm{i}e(t) = 0. \tag{10}$$

Then, by performing Laplace transform [28] on (10), we obtain

$$(s^3 + K_\mathrm{d}s^2 + K_\mathrm{p}s + K_\mathrm{i})E(s) = (s^2 + K_\mathrm{d}s + K_\mathrm{p})e(0) + (s + K_\mathrm{d})\dot{e}(0) + \ddot{e}(0). \tag{11}$$

Therefore, the expression of $E(s)$ is

$$E(s) = \frac{(s^2 + K_\mathrm{d}s + K_\mathrm{p})e(0) + (s + K_\mathrm{d})\dot{e}(0) + \ddot{e}(0)}{s^3 + K_\mathrm{d}s^2 + K_\mathrm{p}s + K_\mathrm{i}}. \tag{12}$$

From Routh-Hurwitz stability criterion [29], the necessary and sufficient conditions for all roots of the characteristic equation $\Phi(s) = s^3 + K_\mathrm{d}s^2 + K_\mathrm{p}s + K_\mathrm{i} = 0$ to have negative real parts are: $K_\mathrm{d} > 0$, $K_\mathrm{p} > 0$, $K_\mathrm{i} > 0$, and $K_\mathrm{d}K_\mathrm{p} - K_\mathrm{i} > 0$. Since $\mathrm{Re}(\lambda_i) > 0$ ($i \in \{1, 2, 3\}$), from (5), we know that all coefficients K_d, K_p, and K_i are strictly positive, and $K_\mathrm{d}K_\mathrm{p} - K_\mathrm{i} = (\lambda_1 + \lambda_2 + \lambda_3)(\lambda_1\lambda_2 + \lambda_1\lambda_3) + \lambda_2\lambda_3(\lambda_2 + \lambda_3) > 0$. Therefore, all roots of $\Phi(s) = 0$ have negative real parts, which means the solution $e(t)$ is a linear combination of exponentially decaying terms. Thus, we have $\lim_{t\to\infty} e(t) = 0$, proving the exponential stability of the zeroing neurodynamics PID controller at the origin of the augmented error dynamics. □

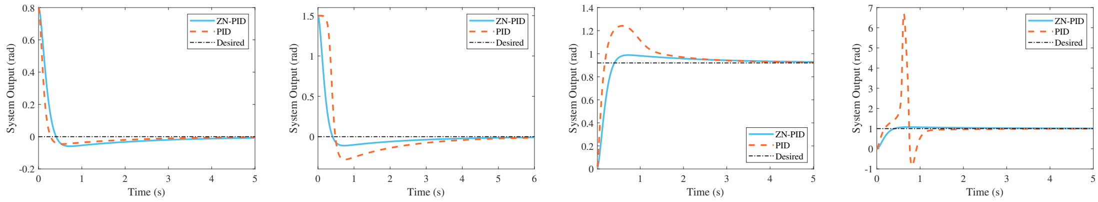
In fact, from (12) and the definitions in (5), the characteristic polynomial can be factorized as $(s + \lambda_1)(s + \lambda_2)(s + \lambda_3) = 0$, which means that the negatives of the ZN parameters λ_i are directly the poles of the dynamics equation of the error. Consequently, unlike the classical Ziegler–Nichols PID tuning method, which typically determines controller gains by driving the system to a marginally stable oscillatory state and empirically estimating the ultimate gain and oscillation period, the proposed ZN-PID framework establishes an explicit analytical correspondence between controller parameters and closed-loop pole locations. Therefore, the tuning process is transformed from heuristic oscillation-based gain adjustment into a systematic pole-placement procedure with clear interpretations of convergence rate and transient-response behavior. In this sense, the proposed method avoids the trial-and-error nature and near-instability operation commonly associated with classical Ziegler–Nichols tuning, while providing a theoretically grounded and more predictable PID tuning methodology for nonlinear order-2 systems.

5. SIMULATIVE EXPERIMENTS AND VERIFICATIONS

In this section, we use MATLAB programs [30] to execute relevant simulation experiments to verify the effectiveness of proposed zeroing neurodynamics PID controllers compared with conventional PID controllers. All simulations are performed using MATLAB R2022a and on a Windows 10 personal computer equipped with an Intel Core i5-10300H (2.50 GHz) CPU and 16 GB RAM, and the type and maximum step size of the solver are ode15s and 10^{-4} s, respectively. The general form of control input of conventional PID controllers is described as $u_\mathrm{PID}(t) = \sigma_g[\widetilde{K}_\mathrm{d}(\dot{y}_\mathrm{d}(t) - x_2(t)) + \widetilde{K}_\mathrm{p}(y_\mathrm{d}(t) - x_1(t)) + \widetilde{K}_\mathrm{i}x_3(t)]$, where $\sigma_g = \mathrm{sgn}(g(x_1, x_2)) \in \{-1, +1\}$, and the Ziegler-Nichols method or other experience-based parameter tuning methods is used to obtain the PID parameters.

Then, we compare the proposed ZN-PID controller with a conventional PID controller on the inverted pendulum. The inverted-pendulum parameters are selected as $M = 1$, $m = 0.1$, $g_\mathrm{a} = 9.8$, and $l = 0.25$. For the sake of simplicity, the units of these parameters are omitted. For all cases below, the ZN-PID parameters are $\lambda_1 = 9 + 3\mathrm{i}$, $\lambda_2 = 9 - 3\mathrm{i}$, and $\lambda_3 = 0.5$, while the conventional PID gains are $\widetilde{K}_\mathrm{p} = 100$, $\widetilde{K}_\mathrm{d} = 10$, and $\widetilde{K}_\mathrm{i} = 50$. The tracking effects of two controllers are shown as Figure 1. First, the control performance in different initial states is shown as Figure 1(a) and (b). It is seen that, when the initial condition $x_1(0)$ is far from $\pi/2$, the nonlinearity of the system is subtle, thus both controllers achieve good performance. However, when $x_1(0)$ is near $\pi/2$, the nonlinearity of the system is obvious, resulting in a decrease in control performance of the conventional PID controller. In contrast, the proposed zeroing neurodynamics PID controller maintains stable tracking, highlighting its superiority in the control of highly nonlinear systems. Besides, Figure 1(c) and (d) show the response for constant signals with different amplitudes. It is seen that as the amplitude of the constant reference signal increases, the performance of the conventional PID controller deteriorates. When the amplitude exceeds a certain value, the inverted pendulum system under conventional PID control exhibits a phenomenon where the swing angle sharply increases and then sharply decreases, often causing the pendulum to lose balance and topple in actual environment. In contrast, the zeroing neurodynamics PID controller proposed in this paper illustrates its superior performance and superiority under the same conditions. Even with strong nonlinearity brought by large constant reference amplitudes, the zeroing neurodynamics PID controller maintains stability, effectively suppresses oscillations, achieves rapid and smooth convergence to the desired angular setpoint with minimal overshoot.

Table 1 summarizes the simulation results. The ZN-PID controller achieves lower root mean squared error (RMSE) for the larger constant references and the larger initial offset, while both controllers reach zero absolute steady-state error (ASSE) in all four cases.



(a) (b) (c) (d)

Figure 1. Control effects of PID and ZN-PID controllers on the inverted pendulum (a) Zero reference with $\boldsymbol{x}_{\mathrm{e}}(0) = [0.8, 0, 0]^{\mathrm{T}}$, (b) Zero reference with $\boldsymbol{x}_{\mathrm{e}}(0) = [1.5, 0, 0]^{\mathrm{T}}$, (c) constant reference with amplitude 0.92, and (d) constant reference with amplitude 1

Table 1. Quantitative Comparisons between ZN-PID Controller and Conventional PID Controller

Reference signal	Initial state	RMSE		ASSE	
		PID	ZN-PID	PID	ZN-PID
$y_{\mathrm{d}}(t) = 0.92$	$\boldsymbol{x}_{\mathrm{e}}(0) = \mathbf{0}$	0.156	**0.145**	0	0
$y_{\mathrm{d}}(t) = 1$	$\boldsymbol{x}_{\mathrm{e}}(0) = \mathbf{0}$	0.843	**0.161**	0	0
$y_{\mathrm{d}}(t) = 0$	$\boldsymbol{x}_{\mathrm{e}}(0) = [0.8, 0, 0]^{\mathrm{T}}$	**0.104**	0.129	0	0
$y_{\mathrm{d}}(t) = 0$	$\boldsymbol{x}_{\mathrm{e}}(0) = [1.5, 0, 0]^{\mathrm{T}}$	0.373	**0.221**	0	0

For simplicity, units of references, states, and errors are omitted in the above table.

6. CONCLUSION

In this paper, a zeroing neurodynamics PID controller for an order-2 affine-in-control nonlinear system has been presented and applied to an inverted pendulum. The ZN construction reformulates the PID gains through parameters that correspond directly to the poles of the order-3 error dynamics, turning controller tuning into a systematic pole-placement task. Theoretical analysis proves the exponential stability of the zeroing neurodynamics PID controller. Comparative simulations with a conventional PID controller show that the proposed controller maintains stable tracking for large initial offsets and high-amplitude constant references, where the conventional controller exhibits pronounced degradation. Future work will consider hardware validations and extensions to systems with incomplete model information.

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AUTHOR CONTRIBUTIONS STATEMENT

This journal uses the Contributor Roles Taxonomy (CRediT) to recognize individual author contributions, reduce authorship disputes, and facilitate collaboration.

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C : **C**onceptualization
M : **M**ethodology
So : **So**ftware
Va : **Va**lidation
Fo : **Fo**rmal Analysis
I : **I**nvestigation
R : **R**esources
D : **D**ata Curation
O : Writing - **O**riginal Draft
E : Writing - Review & **E**diting
Vi : **Vi**sualization
Su : **Su**pervision
P : **P**roject Administration
Fu : **Fu**nding Acquisition

CONFLICT OF INTEREST STATEMENT

Authors state no conflict of interest.

DATA AVAILABILITY

The MATLAB source codes of simulative experiments and the data that support the findings of this study are available on request from the corresponding author, Zhonghua Li.

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