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Kinematic Modeling and Simulation of a 2-R Robot by Using Solid Works and Verification by MATLAB/Simulink

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ABSTRACT

Simulation of robot systems which is getting very popular, especially with the lowering cost of computers, can be used for layout evaluation, feasibility studies, presentations with animation and off-line programming. Object staging modelisation using robots holds, wether for the object or the robot, the following models: The geometric one, the kinematics one and the dynamic one. To do so, the modelisation of a 2-R robot type is being implemented. Comparing between two robot postures with the same trajectory (path) and for the same length of time and establishing a computing code to obtain the kinematic and dynamic parameters are the main tasks. SolidWorks and Matlab/Simulink softwares are used to check the theory and the robot motion simulation. The verification of the obtained results by both softwares allows us to, qualitatively evaluate ,underline the rightness of the chosen model and to get the right conclusions. The results of simulations were discussed. An agreement between the two softwares is certainly obtained.

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1. INTRODUCTION

Robotics is a special engineering science which deals with designing, modeling, controlling and robots' utilization. Nowadays robots accompany people in everyday life and take over their daily routine procedures. The range of robots' utilization is very wide, from toys through office and industrial robots finally to very sophisticated ones needed for space exploration.

A large family of manufacturing equipment among the variety, which exists, is the one which supplies the motion required by a manufacturing process, such as: arc-welding, spray painting, assembly, pick and place, cutting, polishing, milling, drilling etc. Of this class of equipment, an increasingly popular type is the industrial robot. Different manipulator configurations are available as rectangular,cylindrical, spherical, revolute and horizontal jointed. A horizontal revolute configuration robot, 2-R Robot has two degrees of freedom in which two horizontal, is generally suited for small parts insertion tasks for assembly lines like electronic component insertion [2]. Although the final aim is real robotics, it is often very useful to perform simulations prior to investigations with real robots. This is because simulations are easier to setup, less expensive, faster and more convenient to use. Building up new robot models and setting up experiments only takes a few hours. A simulated robotics setup is less expensive than real robots and real world setups, thus allowing a better design exploration. Simulation often runs faster than real robots while all the parameters are easily displayed on screen [3].

The possibility to perform real-time simulations becomes particularly important in the later stages of the design process. The final design can be verified before one embarks on the costly and time consuming process of building a prototype [4].

The need for accurate and computationally efficient manipulator dynamics has been extensively emphasized in recent years. Modeling and simulation of robot systems by using various program softwares will facilitate the process of designing, constructing and inspecting the robots in the real world. Simulation is important for robot programmers to evaluate, predict the behavior of robot, in addition to verify and optimize the path planning of the process [5]. Moreover, this will save time and money and play important role in the evaluation of manufacturing automation [6]. Being able to simulate opens a wide range of options for solving many problems creatively. You can investigate, design, visualize, and test an object or even if it does not exists [7].

In this work, two axis 2-R robot systems for operation pick and place will be designed and developed using Solidworks program as shown in figure 1. The structure will be built depending on the principles of solid bodies modeling with SD technology [8, 9].To emphasize the obtained results in Solidworks program, simulation by using MATLAB/Simulink software will be carried out. The Results of both sofwares will be presented and discussed. In the paper, the equations of kinematics for 2-R robot with the robot dynamics for each joint were developed with D-H formulation.

The paper is organized as follows: First, an introduction to robotics, robot kinematics is presented in section 2. In section 3, the inverse kinematics of the robot is presented. Fourthly, the dynamics is presented in section 4. Sections 5 and 6, the simulation and results are presented followed by the conclusions and the references.

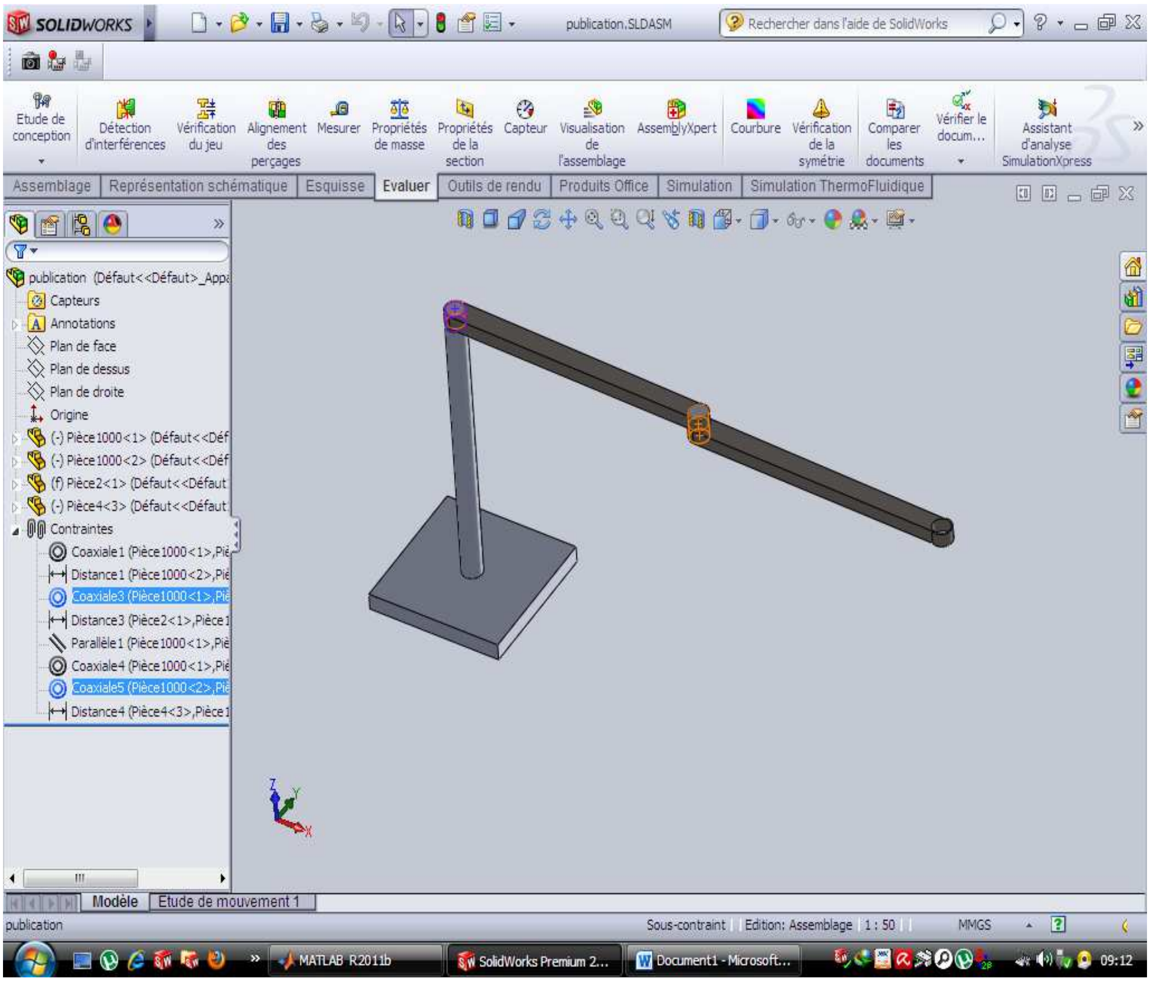


Figure 1. 2-R robot using SolidWorks

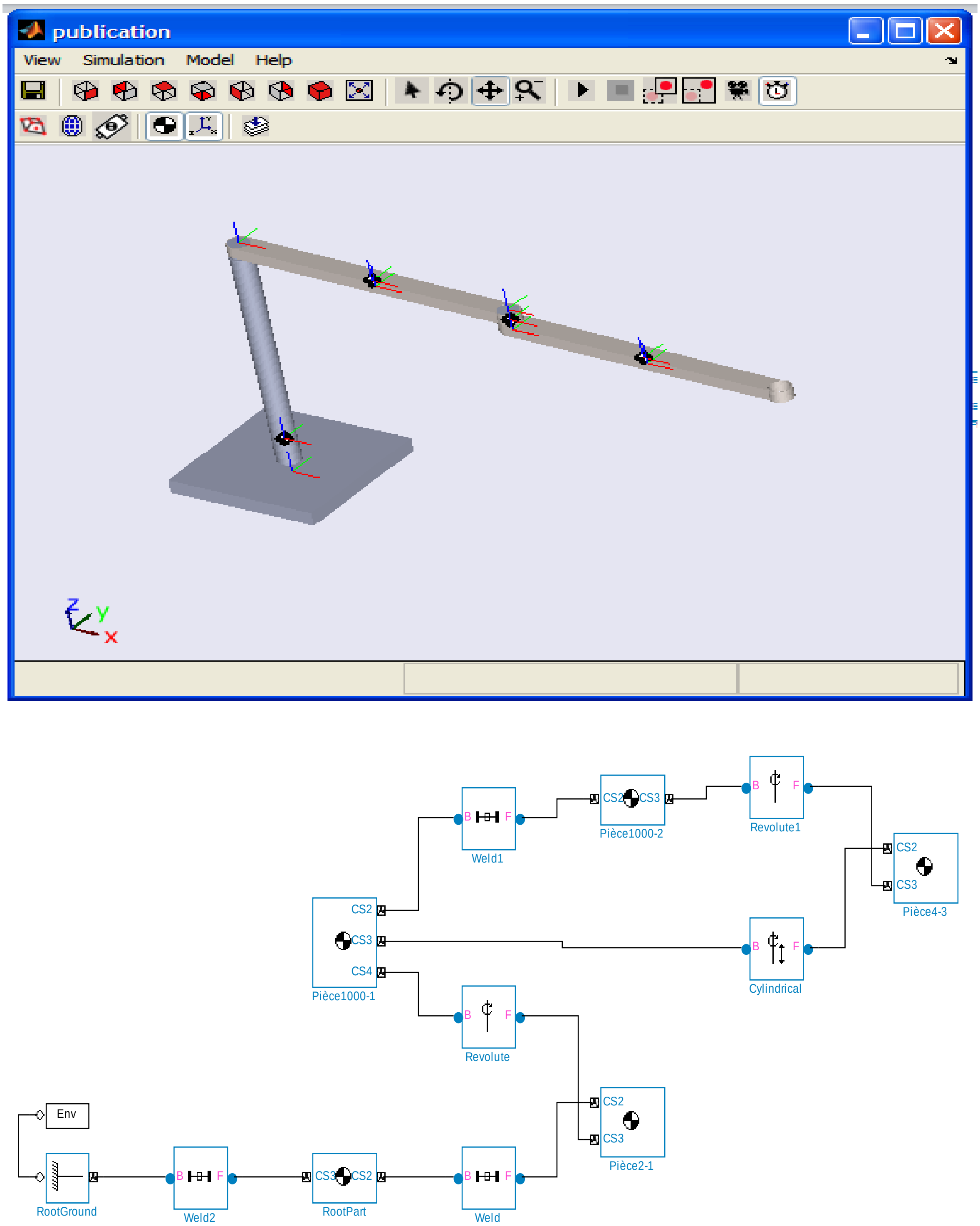


Figure 2. 2-R Robot modeled in MATLAB/Simulink

2. ROBOTKINEMATICS:

The Denavit-Hartenberg (D-H) parameters for 2-R robot specified in **figure 3** are defined in table:

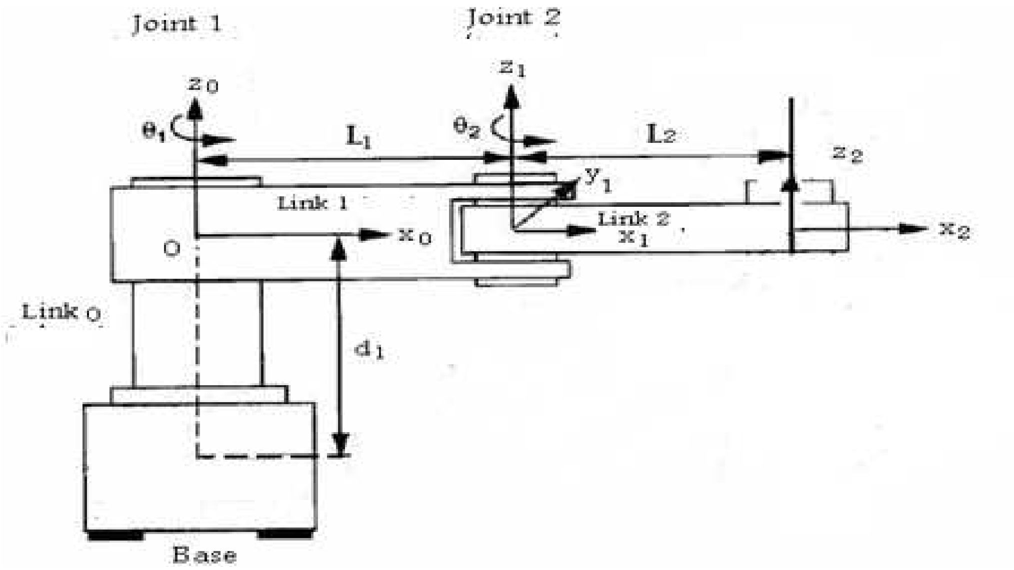


Figure 3. D-H Parameters for two- joint 2-R Robot

Table 1. D-H parameters of 2-R Robot

Link	a_i	α_i	d_i	θ_i
1	l_1	0	0	θ_1^*
2	l_2	0	0	θ_2^*

$$T_1^0 = \begin{bmatrix} c_1 & -s_1 & 0 & l_1c_1 \\ s_1 & c_1 & 0 & l_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

$$T_2^1 = \begin{bmatrix} c_2 & -s_2 & 0 & l_2c_2 \\ s_2 & c_2 & 0 & l_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

After multiplication and use of addition matrices, one gets the total transformation matrix:

$$T_2^0 = \begin{bmatrix} c_{12} & -s_{12} & 0 & l_1c_1 + l_2c_{12} \\ s_{12} & c_{12} & 0 & l_1s_1 + l_2s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

3. INVERSE KINEMATICS OF THE ROBOT:

3.1 Inverse solution for position:

Desired location of the 2-R Robot:

$$T_H^R = \begin{pmatrix} n_X & o_X & a_X & p_X \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{4}$$

<u>The joint variable θ_2 :</u>

The final equation representing the robot is:**[10]:**

$$T_2^0 = T_H^R \tag{5}$$

<u>We get:</u>

$$p_X = l_1 c_1 + l_2 c_{12} \tag{6}$$

$$p_y = l_1 s_1 + l_2 s_{12} \tag{7}$$

From equation 8 and equation 9:

$$c_2 = \frac{1}{2l_1 l_2}(p_X{}^2 + p_y{}^2 - l_1^2 - l_2^2) \tag{8}$$

$$s_2 = \pm\sqrt{1 - c_2{}^2} \tag{9}$$

$$\theta_2 = \pm a\tan\frac{s_2}{c_2} \tag{10}$$

<u>The joint variable θ_1:</u>

Rearranging equation (6) and equation (7) yields:

$$p_X = (l_1 + l_2 c_2)c_1 - l_2 s_2 s_1 \tag{11}$$

$$p_y = l_2 s_2 c_1 + (l_1 + l_2 c_2)s_1 \tag{12}$$

Solving equations (11) and (12) by Kramer's rule

$$\Delta = \begin{bmatrix} l_1 + l_2 c_2 & -l_2 s_2 \\ l_2 s_2 & l_1 + l_2 c_2 \end{bmatrix} = (l_1 + l_2 c_2)^2 + (l_2 s_2)^2 \tag{13}$$

<u>And we have:</u>

$$p_X^2 + p_y^2 = (l_1 + l_2 c_2)^2 + (l_2 s_2)^2 \tag{14}$$

$$\Delta s_1 = \begin{bmatrix} l_1 + l_2 c_2 & p_x \\ l_2 s_2 & p_y \end{bmatrix} \tag{15}$$

$$\Delta c_1 = \begin{bmatrix} p_x & -l_2 s_2 \\ p_y & l_1 + l_2 c_2 \end{bmatrix} \tag{16}$$

$$s_1 = \frac{\Delta s_1}{\Delta} = \frac{(l_1 + l_2 c_2) p_y - l_2 s_2 p_x}{p_X^2 + p_y^2} \tag{17}$$

$$c_1 = \frac{\Delta c_1}{\Delta} = \frac{(l_1 + l_2 c_2) p_x + l_2 s_2 p_y}{p_X^2 + p_y^2} \tag{18}$$

$$s_2 = \pm\sqrt{1 - {c_2}^2}$$

$$\theta_1 = a\tan\frac{s_1}{c_1} = a\tan\frac{(l_1 + l_2 c_2) p_y \mp l_2 s_2 p_x}{(l_1 + l_2 c_2) p_x \pm l_2 s_2 p_y} \tag{19}$$

The sign ($\pm$) indicates the two postures elbow up and elbow down.

Equation of elbow up:

$$\theta_2 = -a\tan\frac{s_2}{c_2} \tag{20}$$

$$\theta_1 = a\tan\frac{+p_x l_2 s_2 + p_y (l_1 + l_2 c_2)}{p_x (l_1 + l_2 c_2) - (p_y l_2 s_2)} \tag{21}$$

Equation of elbow down:

$$\theta_2 = +a\tan\frac{s_2}{c_2} \tag{22}$$

$$\theta_1 = a\tan\frac{-p_x l_2 s_2 + p_y (l_1 + l_2 c_2)}{p_x (l_1 + l_2 c_2) + (p_y l_2 s_2)} \tag{23}$$

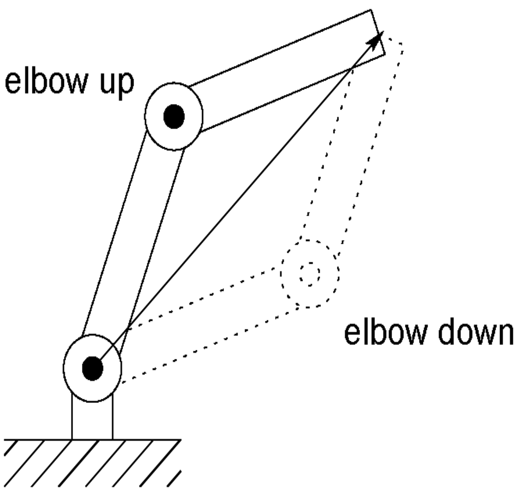


Figure 4. The two postures of 2-R Robot

4. ROBOT DYNAMICS:

4.1 Kinetic energy:

$$K_1 = \frac{1}{2} m_1 l_1^2 \dot{\theta}_1^2 \tag{24}$$

$$K_2 = \frac{1}{2} m_2 l_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 l_2^2 (\dot{\theta}_1 + \dot{\theta}_2)^2 + m_2 l_1 l_2 (\dot{\theta}_1^2 + \dot{\theta}_1 \dot{\theta}_2) \cos\theta_2 \tag{25}$$

4.2 Potentialenergy:

$$V_1 = m_1 g l_1 \sin\theta_1 \tag{26}$$

$$V_2 = m_2 g y_2 = m_2 g [l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2)] \tag{27}$$

4.3 Joints Torque:

$$\begin{aligned}\tau_1 &= m_2 l_2^2 (\ddot{\theta}_1 + \ddot{\theta}_2) + m_2 l_1 l_2 c_2 (2\ddot{\theta}_1 + \ddot{\theta}_2) \\ &+ (m_1 + m_2) l_1^2 \ddot{\theta}_1 - m_2 l_1 l_2 s_2 \dot{\theta}_2^2 - 2 m_2 l_1 l_2 s_2 \dot{\theta}_1 \dot{\theta}_2 + m_2 l_2 g c_{12} + (m_1 + m_2) l_1 g c_1\end{aligned} \tag{28}$$

$$\tau_2 = m_2 l_1 l_2 c_2 \ddot{\theta}_1 + m_2 l_1 l_2 s_2 \dot{\theta}_1^2 + m_2 l_2 g c_{12} + m_2 l_2^2 (\ddot{\theta}_1 + \ddot{\theta}_2) \tag{29}$$

5. SIMULATION AND RESULTS:

5.1 Application:

This application will bear the energetic comparison during the movement of the 2-R Robot with $m_1 = m_2 = 16.92 Kg$ and $l_1 = l_2 = 1m$, for the two postures. From the study of movement of the two postures « elbow down» and « elbow up» for the same nature of the trajectory (path) , reaching the same desired position during the same interval of time of 1s.as shown in the following diagram:

<u>Mass parameters:</u>

The mass is shown Figure 5.

<u>The initial position (home position):</u>

The initial position was obtained from the matrix of the homogeneous transformation (3) when $\theta_1 = 0°$ and $\theta_2 = 0°$ as it's shown in the following Figure 6 and 7.

<u>Starting from the initial position:</u>

$p_X = 2 \; p_y = 0$

<u>To the desired position:</u>

$p_X = 0.86 \; p_y = 1.5$

<u>The equation of movement for (elbow down) for the initial and desired position:</u>

The equation of movement for the borrowed path "elbow down" to reach the desired positions given by the relations**[11]:**

$$\theta_1 = (10t^4 + 10t^3 + 10t^2)° \tag{30}$$

$$\theta_2 = (50t^4 + 10t^3)° \tag{31}$$

<u>The equation of movement for (elbow up) for the initial and desired position:</u>

The equation of movement for the borrowed path "elbow up" to reach the desired position is given by the relations**[11]:**

$$\theta_1 = (60t^4 + 20t^3 + 10t^2)° \tag{32}$$

$$\theta_2 = (-50t^4 - 10t^3)° \tag{33}$$

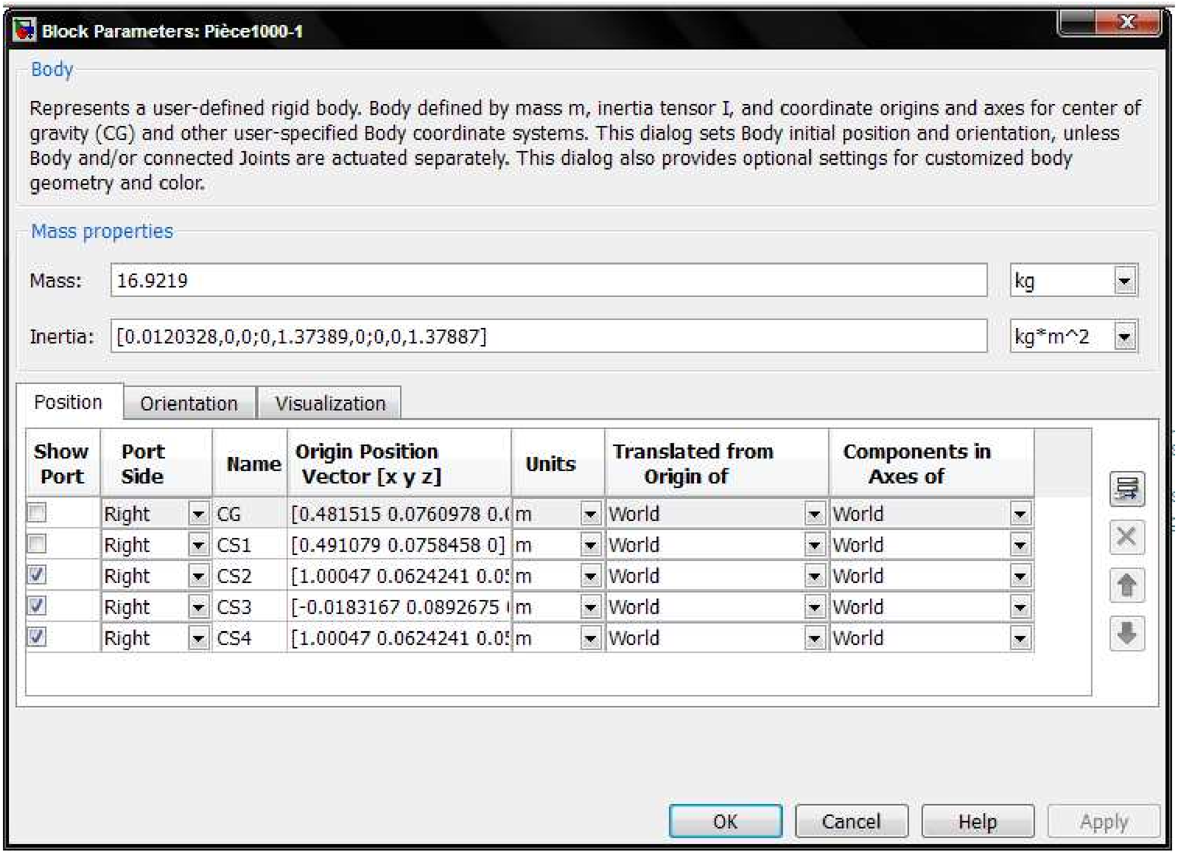


Figure 5. mass parameters of link

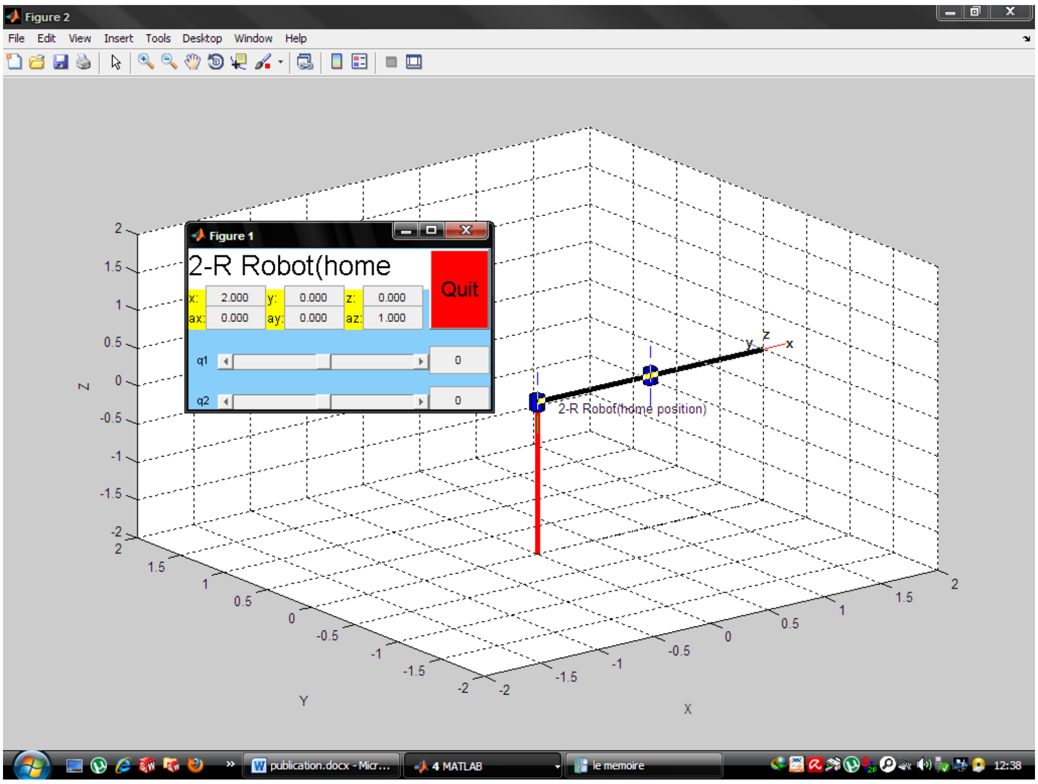


Figure 6. The home position of 2-R Robot(two postures):Matlab/Simulink.

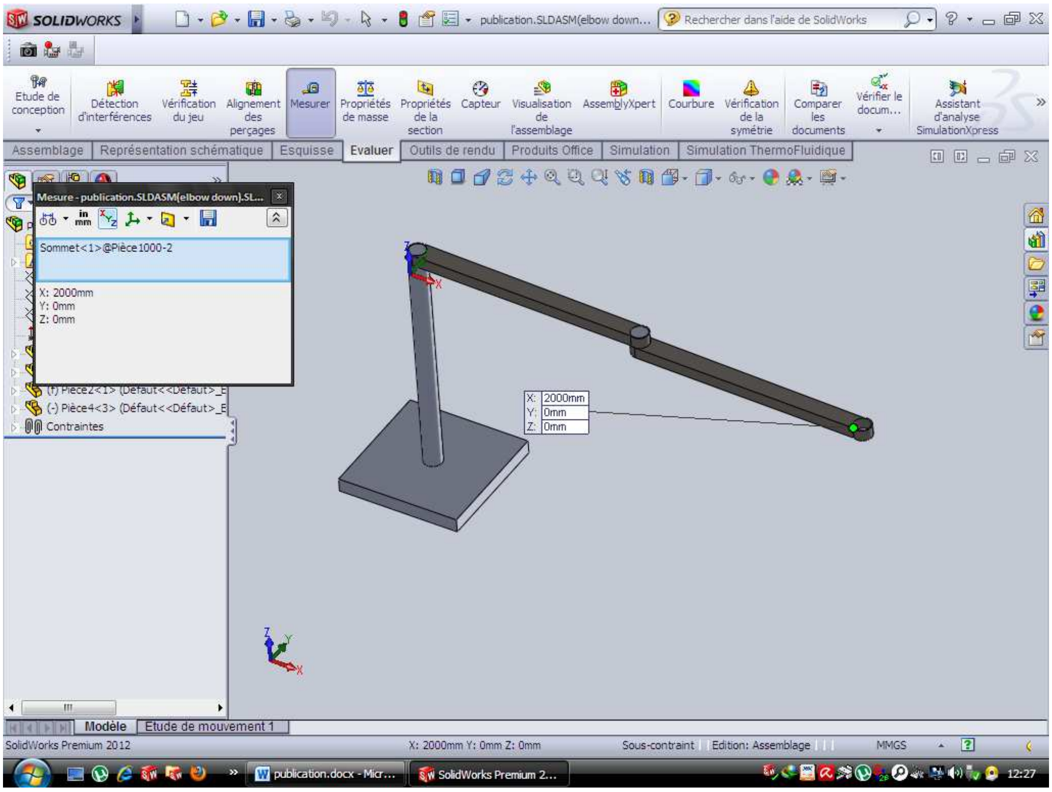


Figure 7. the home position of 2-R Robot (two postures): Solidworks.

The choice of expressions of θ_1 and θ_2 must satisfy the following conditions[11]:
-In the meantime (interval) time must verify the initial and the desired position.

- At every moment the two postures (elbow down and up) must have the same coordinates P_x **and** P_y**.**

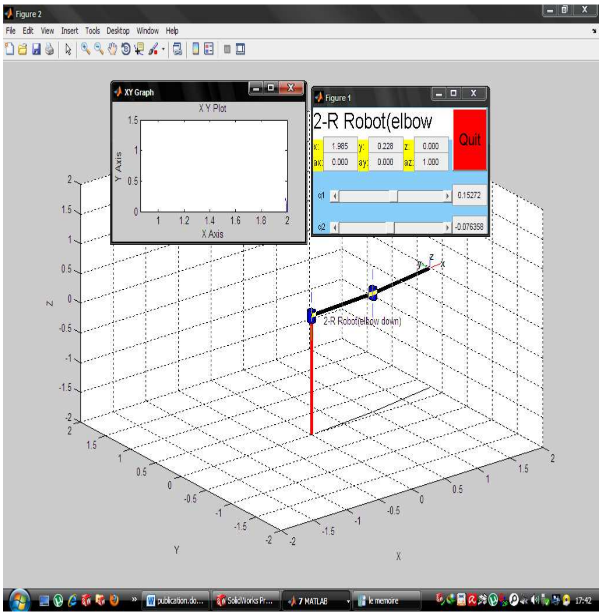
5.2 Checking of path:
Applying the relations (30), (31), (32) and (33) in (6) and (7) ,obtaining the following table

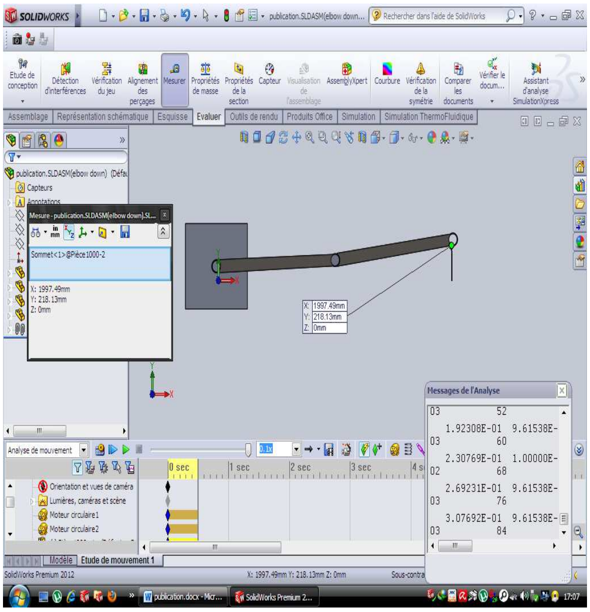
Table 2. The position of the two postures during 1s.

time (sec)	Elbow down				Elbow up			
	Tita1 (deg)	Tita2 (deg)	p_X (m)	p_y (m)	Tita1 (deg)	Tita2 (deg)	p_X (m)	p_y (m)
0	0	0	2	0	0	0	2	0
0.1	0.11	0.015	1.99	0.004	0.126	-0.015	1.99	0.004
0.2	0.49	0.16	1.99	0.02	0.65	-0.16	1.99	0.02
0.3	1.25	0.67	2	0.05	1.92	-0.67	2	0.05
0.4	2.49	1.92	1.996	0.12	4.41	-1.92	1.996	0.12
0.5	4.37	4.37	1.98	0.21	8.75	-4.37	1.98	0.21
0.6	7.05	8.64	1.95	0.39	15.69	-8.67	1.95	0.39
0.7	10.73	15.43	1.88	0.62	26.16	- 15.43	1.88	0.62
0.8	15.61	25.6	1.71	0.927	41.21	-25.6	1.71	0.927
0.9	20	40.09	1.44	1.20	59.04	-40.09	1.44	1.20
1	30	60	0.86	1.5	90	-60	0.86	1.5

5.3 Checking the results of the table by using SolidWorks andMatlab/Simulink

t=0.5s

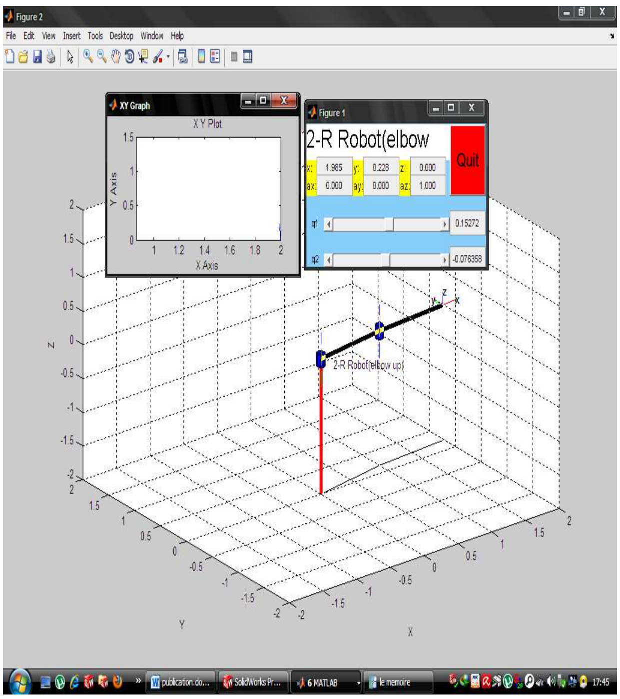




Elbow down;

Figure 9. Elbow down(Matlab/Simulink) Figure 10. Elbow down(Solidworks)

Elbow up :



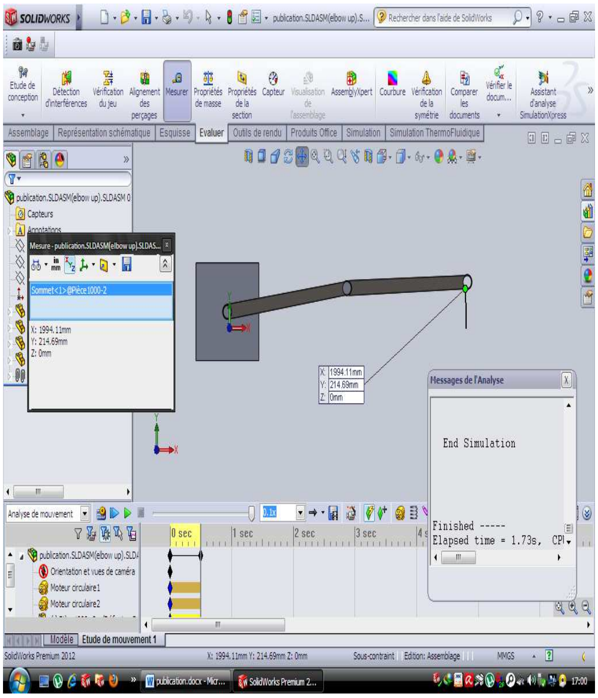
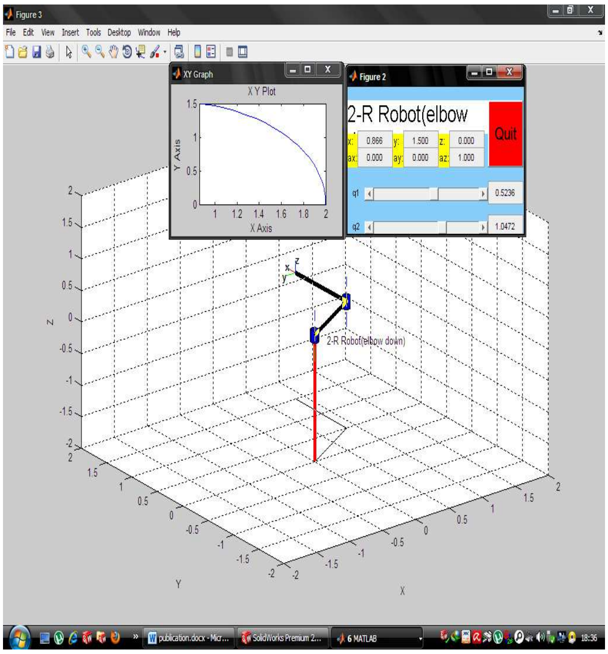


Figure 11. Elbow up(Matlab/Simulink) Figure 12. Elbow up(solidworks)

t=1s

<u>*Elbow down;*</u>



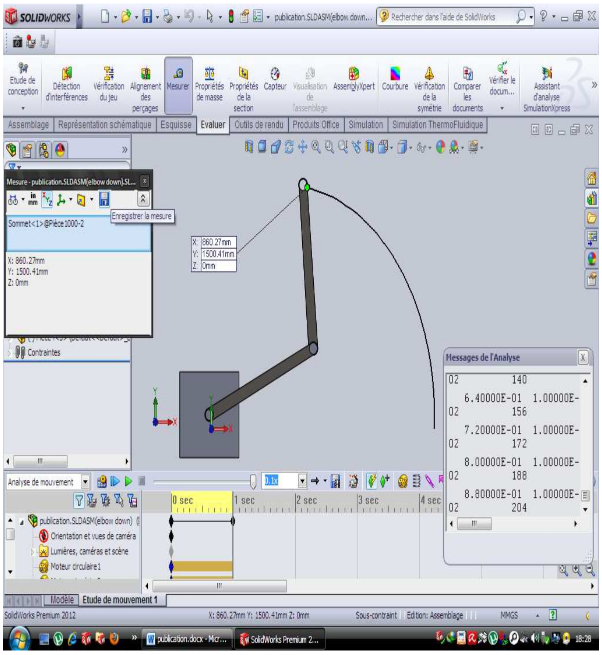
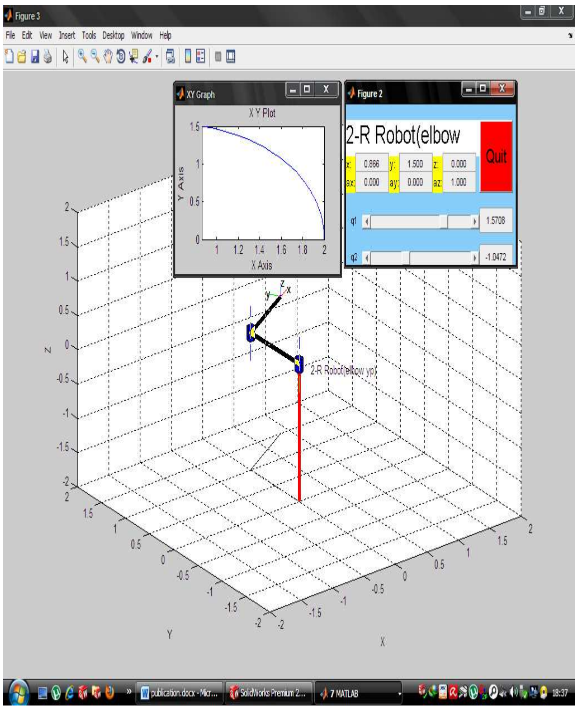


Figure 13. Elbow down(Matlab/Simulink) Figure 14. Elbow down(Solidworks)

<u>*Elbow up;*</u>



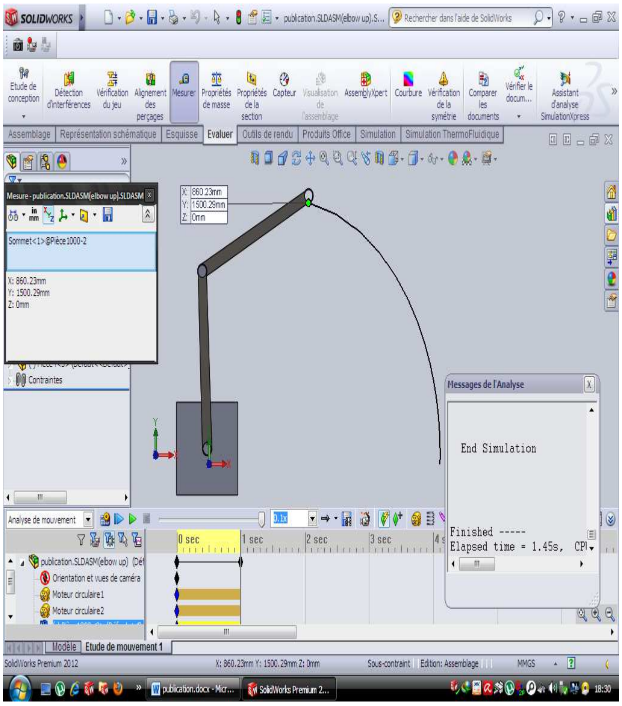


Figure 15. Elbow up (Matlab/Simulink) Figure 16. Elbow up (Solidworks)

PS: the joint variables expressed in Matlab/Simulink are in radians

5.4 Arbitray change of the two links of the two postures during 1s:

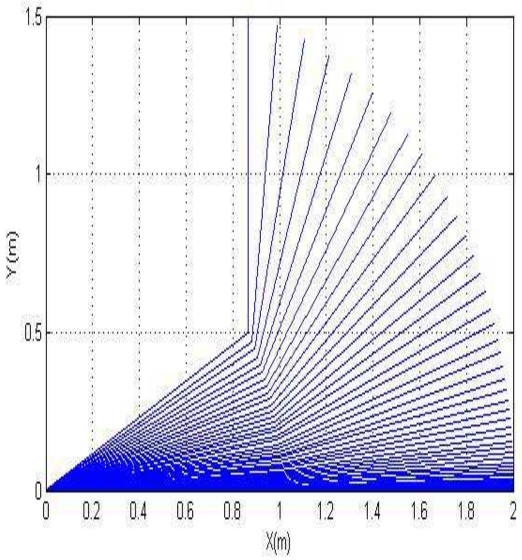


Figure 17. Elbow down

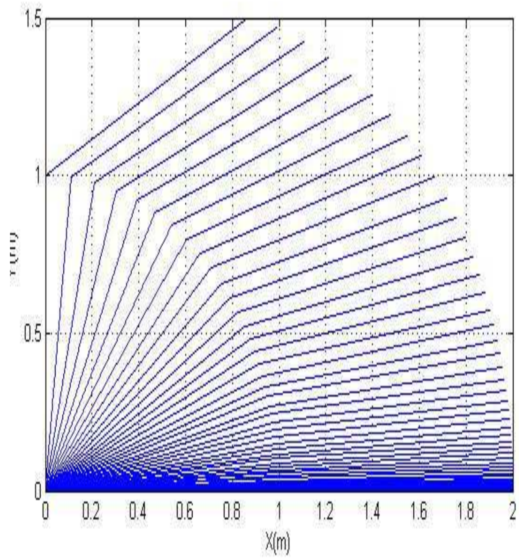


Figure 18. Elbow up

The results obtained whether by using the software Solid Works or calculated by the software MATLAB/Simulink are exactly the same. This similarity of results confirms the reliability of the kinematic model.

We can take advantages from this analysis using the software Solid works to study the behavior of the robot in the two postures to establish an energetic balance comparative to these postures.

5.5 The displacement of center of gravity of two postures:

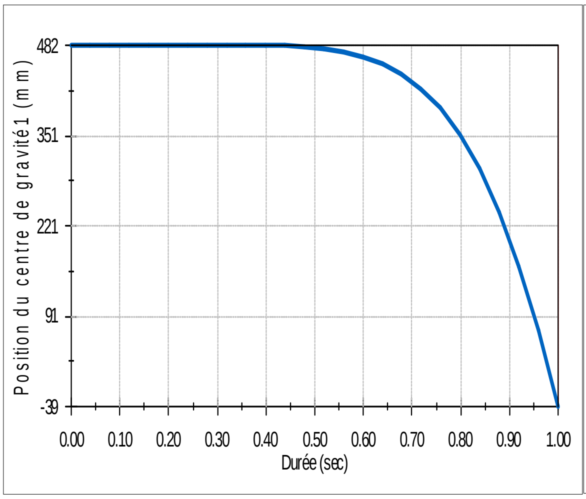


Figure 21. Link 1(x) elbow up

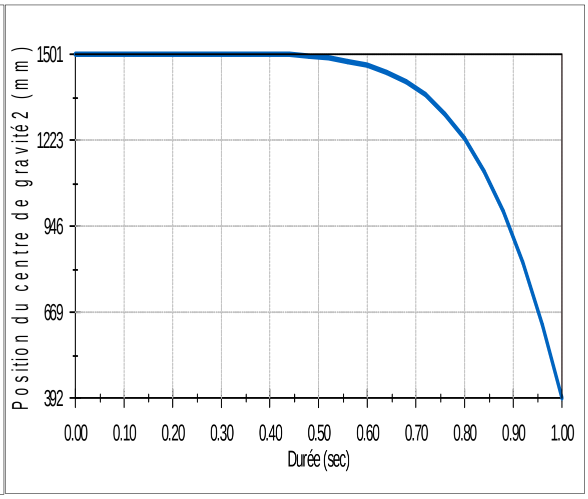


Figure 22. Link 2(x) elbow up

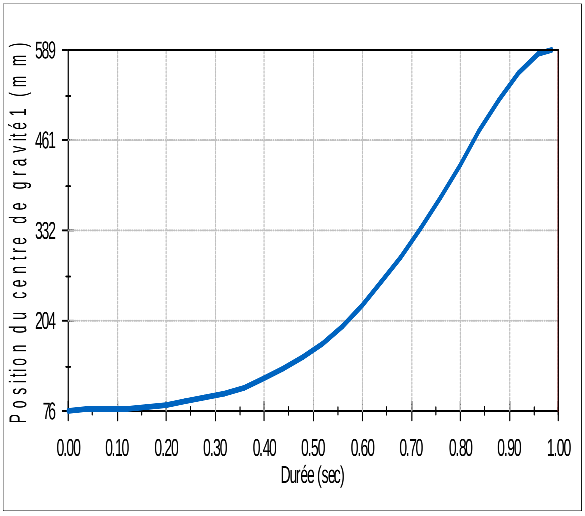


Figure 23. Link 1(y) elbow up

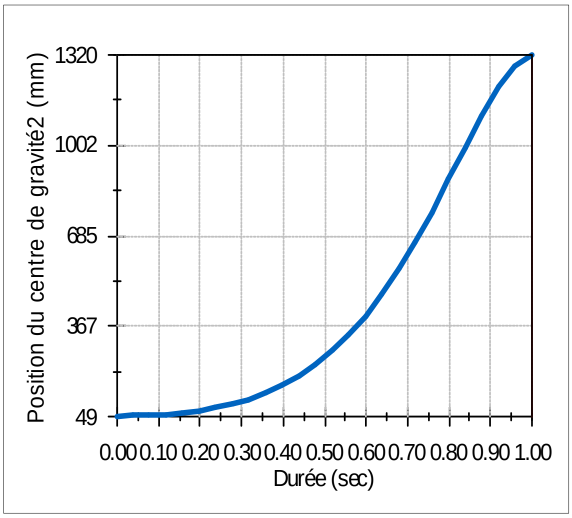


Figure 24. Link 2(y) elbow up

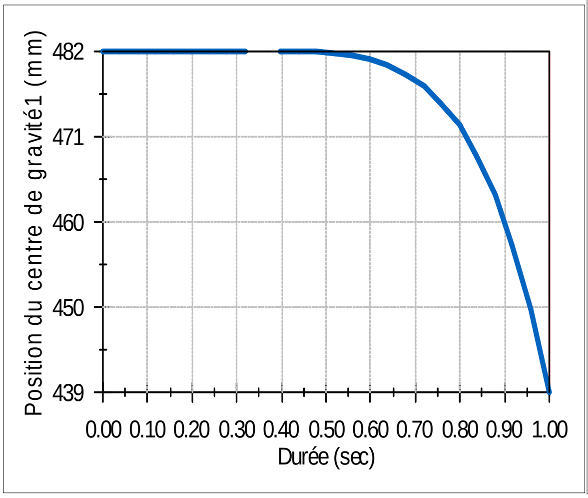


Figure 25. Link 1(x) elbow down

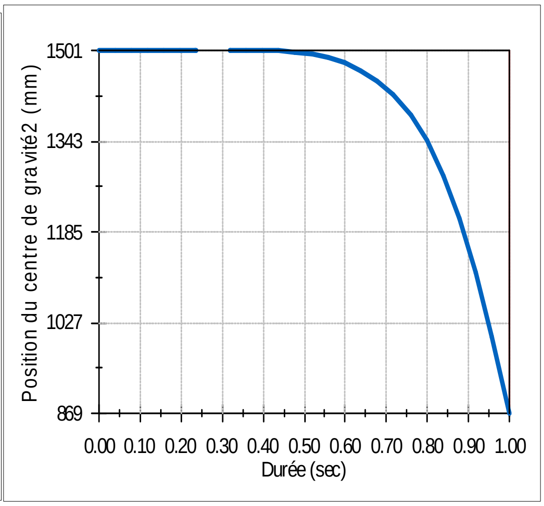


Figure 26. Link 2(x) elbow down

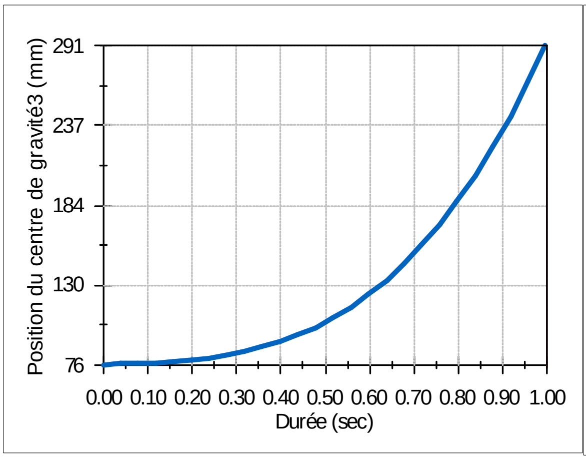


Figure 27; Link 1(y) elbow down

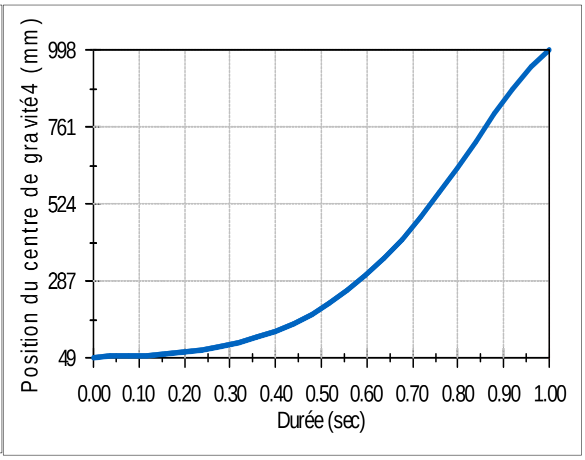


Figure 28. Link 2(y) elbow down

5.6 Conclusion:

According to the results, we notice that the displacement and velocity of the links 1 and 2 of **elbow up** are higher than the displacement velocity of the links 1 and 2 of **elbow down,** this difference can induce a difference in the consumed energy at the two postures, for the same trajectory and the same desired position.

5.7 Simulation of dynamic model:

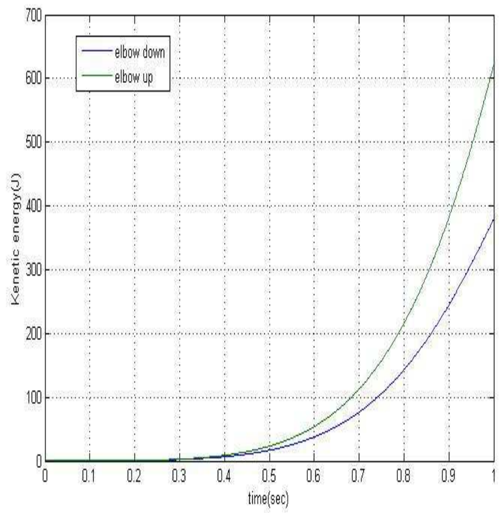


Figure 29: Kinetic energy

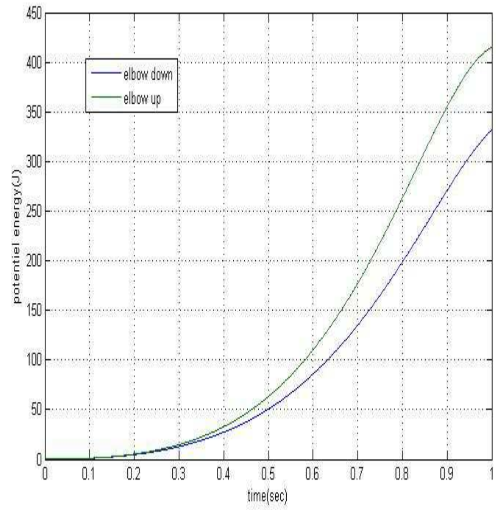


Figure 30: Potential energy

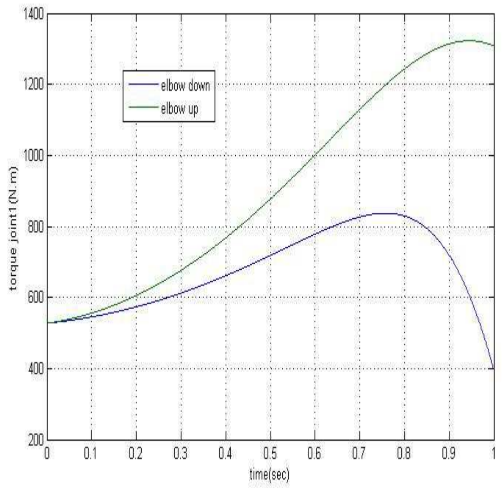


Figure 31: joint torque (1)

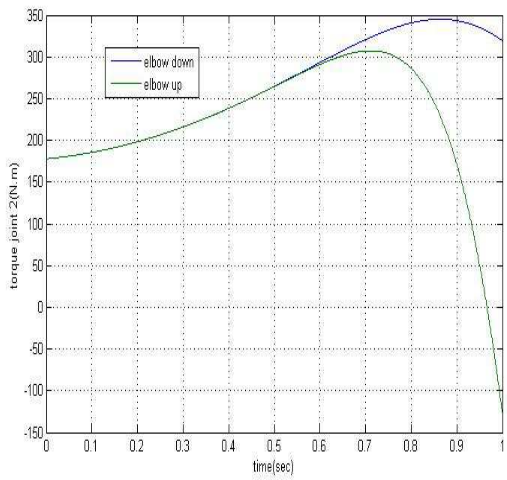


Figure 32: joint torque (2)

5.8 Comparison between the two postures:

We notice from the simulation results that:

- The variation of the total kinetic energy calculated according the posture **elbow up** is higher than the variation of the total kinetic energy calculated according the posture **elbow down** with: $\Delta K = 243.02J$
- The variation of the total potential energy calculated according the posture **elbow up** is higher than the variation of the total potential energy calculated according the posture **elbow down** with: $\Delta V = 82.95J$

This difference in the variation of energies kinetic and potential is due to the differences noted in the conclusion of **conclusion 5.6**

We notice also that the torque of the **joint 1** calculated according the posture **elbow up** at t=1s is T_1 = 446.33N m with a a max T_1 =900N m ,on the other side the torque of **the joint 1** calculated according the posture **elbow down** is T_1 = 395.18N m with a max T_1 =820N m .

Otherwise the torque of the **joint 2** calculated according the posture **elbow up** at t=1s isT_1 = 127N m with a max T_1 = 300N ,while the torque of the **joint 2** calculated according the posture **elbow down** is T_1= 319.65Nm with a max T_1=330N m.

<u>This study reveals that:</u>
*The robot is brought to consume much more energy when it is requested to work according to **elbow up,** comparatively to its use, for the same trajectory and the same desired position according to **elbow down**
*The choice of posture is designed in the eventuality of power saving [10-12]

6. CONCLUSION

In this paper we used both software SolidWoks and Matlab/Simulink to verify the theory and the simulation of the robot motion performed by the softwares above. The checking of the results obtained by the two softwaresSolidWorks and Matlab/Simulink permitted us to qualitatively develop and highlight the relevance of the studied model. In our case, we can conclude depending on the obtained results by the comparative analysis of the two postures, that the dissipated energies during the movement of the robot according to the first posture (**elbow down**) are very lower than those which would be dissipated if the robot performs the movement according to the second posture (**elbow up**), and this is for the same trajectory and the same duration according to the presentations listed above. So during the work of the robot, the choice of posture is designed in the eventuality of power saving which is always recommended.

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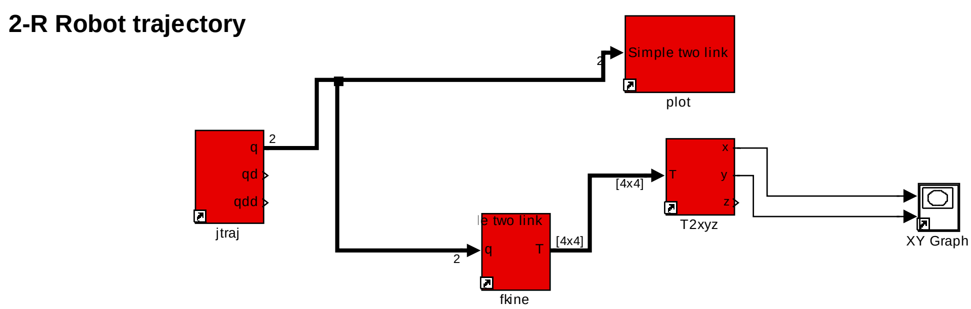
[12] Bruno Siciliano • Lorenzo Sciavicco Luigi Villani • Giuseppe OrioloRobotics Modelling, Planning and Control 123 ISBN 978-1-84628-641-4 e-ISBN -1-84628-642-1 DOI 10.1007/978-1-84628-642-1

APPENDIX:



Trajectory generation of 2-R Robot with Matlab/Simulink

Some trajectories obtained in the displacements of the two postures for the same desired positionduring 1s:

Elbow up:[11]

$$\theta_1 = (180t - 20t^2 - 70t^5)^0$$

$$\theta_2 = (-180t + 20t^2 + 100t^5)^0$$

$$l_1 = l_2 = 1m$$

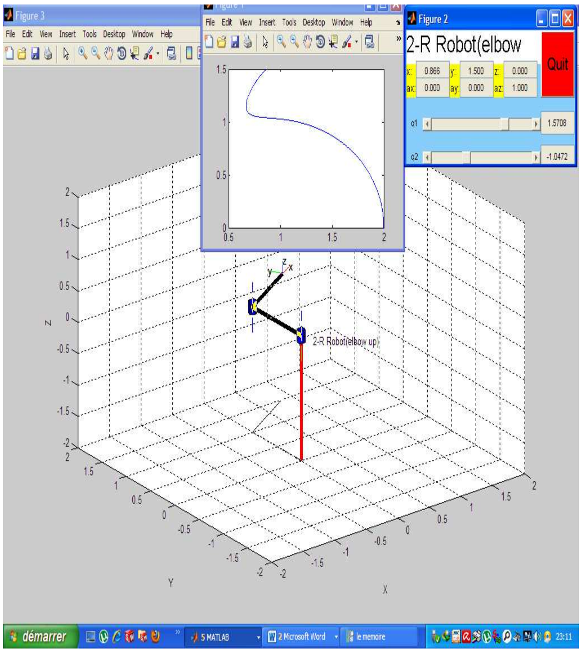
elbow down:[11]

$$\theta_1 = (90t^{10} - 60t^2)^0$$

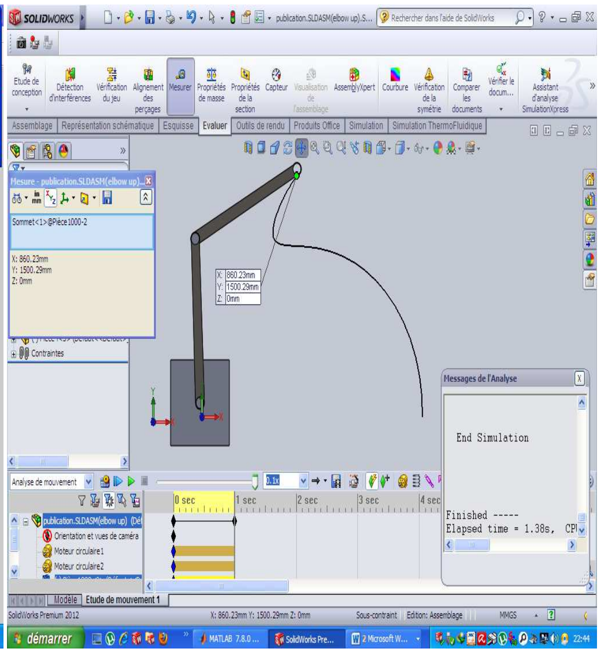
$$\theta_2 = (-90t^{10} + 150t^2)^0$$

$$l_1 = l_2 = 1m$$

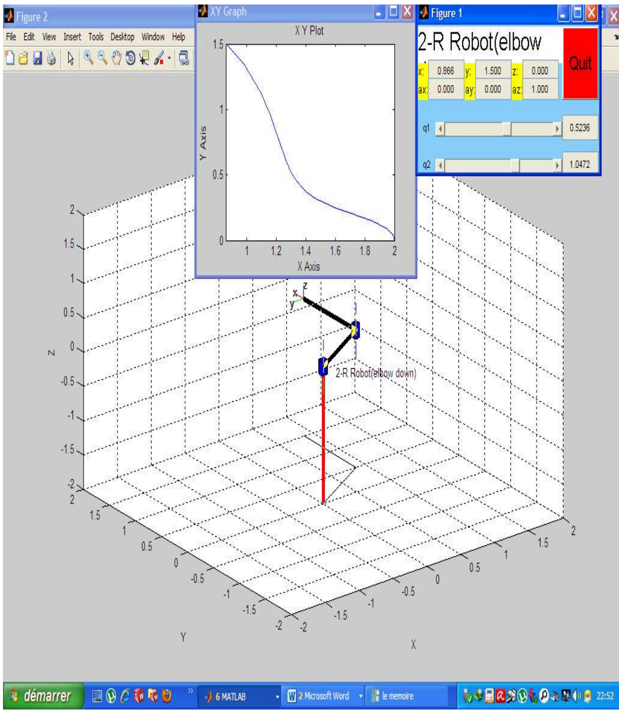
Simulation of the motion:



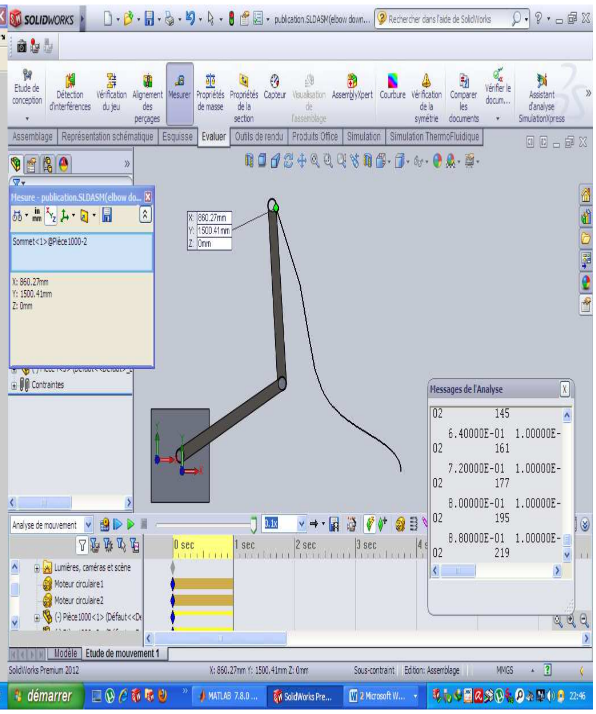
elbow up(Matlab/Simulink)



elbow up(Solidworks)



elbow down(Matlab/Simulink)



elbow down(Solidworks)