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Distributed Receding Horizon Coverage Control by Multiple Mobile Robots

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ABSTRACT

This paper presents a distributed receding horizon coverage control algorithm to control a group of mobile robots having linear dynamics with the assumption that the robot dynamics are decoupled from each other. The objective of the coverage algorithm considered here is to maximize the detection of the occurrence of the events. First the authors introduce a centralized receding horizon coverage control and then they introduce a distributed version of it. To avoid the common disadvantages that are associated with the centralized approach, the problem is then decomposed into several RHCC problems, each associated with a particular robot, that are solved using distributed techniques. In order to solve each RHCC, each robot needs to know the trajectories of its neighbors during the optimization time interval. Since this information is not available, an algorithm is presented to estimate the trajectory of the neighboring robots. To minimize the estimation error, a compatibility constraint, which is also a key requirement in the closed-loop stability analysis, is considered. Moreover, the proof of the close-loop stability of this distributed version is provided and shows that the location of the robots will indeed converge to the centroids of a Voronoi partition. Simulation results validate the algorithm and the convergence of the robots to the centroidal Voronoi configuration.

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1. INTRODUCTION

Application of multi-agent systems for controlling a group of robots has gained a significant attention in recent years due to considerable advancement in computer technology. Researchers have shown that multiple robots could potentially accomplish a task more efficiently than a single robot. In a multi-agent system, several autonomous agents are simultaneously coordinated and controlled in order to achieve a common system objective. The underlying assumption is that in multi-agent systems, the agents are distributed in a predetermined fashion and each agent will act autonomously while exchanging local information with neighboring agents [1], [2]. Furthermore, it has been established that the distributed control approach among autonomous agents provides a better scalability and improved tractability than centralized approaches.

With the progresses made in real-time optimization-based control, some researchers have suggested new distributed control algorithms in an attempt to manipulate constraints in real-time [3], [4]. One major factor for consideration in developing reliable distributed control algorithms is location of nodes for the robot network in the mission space. This is referred to as the coverage control or active sensing problem [5], [6].

The deployment location of the mobile robot must provide for maximum information retrieval, satisfactory communication level, and effective energy consumption [7]. Similar to challenges in facility location optimization such as static problems, an offline scheme can be implemented to determine coverage control by deploying the robots in an optimal location that will not require mobility. As an alternative, in a coordinated-movement dynamic scheme, the mobile robots can be deployed into a geographical area with the highest information density. However, due to similarities between facility location and coverage control optimization, issues regarding robot deployment have been studied using facility location optimization [5].

It has been determined that the level of sensitivity and the domain of coverage of mobile robots in their deployment location is essential to the overall efficiency of the system network. It involves a comprehensive coverage metric encompassing an optimized sensing performance and placements of mobile robots [5], [8]. Researchers have used Voronoi partitioning of the region model to reduce challenges of the locational optimization [9]. The focus of the original algorithm, for an optimal mobile robot placement, was on coordination and control of mobile robots, leading to the development of more enhanced formulations and coordination algorithms by other scholars [5], [8]. As such, during recent years, formulating a cooperative control design among distributed agents assigned to a specific task that can navigate autonomously without collision has received significant attentions [1], [10].Consequently, the concept of coordination and control algorithms for networked dynamic systems has become a central focus for researchers in systems and control arena, drawing overwhelming attentions [5], [11]. For example, Dunbara and his colleagues have suggested a design for formation pattern in a multi-agent system based on receding horizon control [12]. Meguerdchian and his colleagues have purported centroidal Voronoi configuration as a solution to problems associated with area of coverage in a way that clarifies the issue of coverage control. They have presented their algorithms in a centralized manner as a practical approach and as having a possibility for application [16]. Cortes and his associates [5] have suggested a decentralized coverage control algorithm for multi-robots in an area in a way that the mission space is partitioned in Voronoi cells. From this perspective, which is considered in this paper, they have discussed sensory control issue which in fact is the problem of locational optimization for sensors.

While significant results have been achieved, there is still room for new ideas and further improvements. This paper presents a distributed receding horizon coverage control (DRHCC) algorithm for controlling a group of mobile robots having linear dynamics with the assumption that the robot dynamics are decoupled from each other. The algorithm will provide for maximum event detection through confluence of robots position to a centroidal Voronoi Configuration. The proposed algorithm ensures enhanced coverage and stability.

Concepts exploited for theoretical framework include Locational optimization, receding horizon control, distributed coverage control, centroidal Voronoi partitions and are briefly discussed in the next section. Centralized receding horizon coverage control (CRHCC) approach for a group of linear mobile robots is presented in section 3. Using the results of this section, the distributed receding horizon coverage control (DRHCC) algorithm is given in section 4. In section 5, stability analysis of closed-loop system is studied and it is proved that by using suggested DRHCC algorithm, the closed-loop system is stable and will converge to centroidal Voronoi configuration. Section 6 presents simulation results that validate the algorithm and the convergence of the agents to the desired configuration, and finally, section 7 summarizes the main results of this paper.

2. BACKGROUNDS

2.1. Locational Optimization

This section presents some facts regarding the method used to describe *coverage control* for mobile sensing network in [5] and in the framework of locational optimization presented in [9] which underpins coverage algorithms depicted in Voronoi diagram.

Assume that S be a convex space in $\Re^2$ and $P = (p_1, ..., p_n)$ be the location of *n* mobile robots, i.e. $p_i \in S$ denotes i^{th} robot position. Furthermore, assume that movement of each robot is confined in S and $W = \{W_1, ..., W_n\}$ is a tessellation of S such that $I(W_i) \bigcap I(W_j) = \varnothing$. $I(.)$ denotes interior space of each W_i and $\bigcup_{i=1}^{n} W_i = S$. So, it is supposed that each agent *i* is only responsible to cover its domain W_i. To obtain the probability of an event occurring at a point in S, the mapping $\phi : S \rightarrow \Re^+$ is defined. Note that in this sense, ϕ is the distribution density function. As robot *i* moves further away from any given point s inside the mission space S, its sensing performance at point s taken from i^{th} sensor located at $p_i \in W_i$ reduces with the distance $d(s,p_i)=\|s-p_i\|$ because of noise and loss of resolution. This reduction is defined by function $g:\Re^+ \rightarrow \Re^+$. As a measurement for system's performance, coverage cost function is described as

$$J(P,W)=\sum_{i=1}^{n} J(p_i,W_i)=\sum_{i=1}^{n}\int_{W_i} g(d(s,p_i))\phi(s)ds, \tag{1}$$

where J is a differentiable function. Note that the cost function J must be minimized in regards to location of robots and partition of the space.

2.2. Centroidal Voronoi Configuration

A collection of points $P=\{p_1,...,p_n\}$ generate Voronoi Diagram which is defined as $V=\{V_1,...,V_n\}$ and V_i that commonly is referred to as Voronoi domain or Voronoi cell associated with point p_i are defined by

$$V_i=\{s\in S: d(s,p_i)\le d(s,p_j), \forall j\ne i\}$$

The above definition is commonly used to describe Voronoi partition [5], [9]. Voronoi partitioning is one of the important tools in localization optimization theory.

Definition 1 [13]. For robot i all neighboring Voronoi robots (meaning N_i) are described as collection of robots with a shared Voronoi cell border. Based on definition of Voronoi partitioning we have

$$\min_{i\in 1,...,n} g(d(s,p_i))=g(d(s,p_i))$$

For each $s\in V_j$ accordingly,

$$J(P,V(P))=\int_S \min_{i\in 1,...,n} g(d(s,p_i))\phi(s)ds \tag{2}$$

To continue, the two results presented in [5] are reviewed.

Proposition 1 [5]. One of the necessary conditions to minimize (1) is that W partitioning must be equal to Voronoi configuration $V(P)$.
According to (2)

$$\frac{\partial J_V(P)}{\partial p_i}=\frac{\partial J(p_i,V_i)}{\partial p_i}=\int_{V_i}\frac{\partial}{\partial p_i} g(d(s,p_i))\phi(s)ds$$

So, partial derivative of J_V with respect to i th robot is only associated with position of the robot itself and its neighbors. Next, we discuss some of the concepts associated with Voronoi diagram. In [5], the (generalized) mass and first moment (not normalized) and center of Voronoi cell are defined as

$$M_{V_i}=\int_{V_i}\phi(s)ds,\ \ L_{V_i}=\int_{V_i} s\phi(s)ds,\ \ C_{V_i}=\frac{L_{V_i}}{M_{V_i}}=\frac{\int_{V_i} s\phi(s)ds}{\int_{V_i}\phi(s)ds} \tag{3}$$

Using the above definition and proposition and letting $g=\frac{1}{2}\|s-p_i\|$, we have

$$\frac{\partial J_V(P)}{\partial p_i} = \int_{V_i} \frac{\partial}{\partial p_i} g(d(s,p_i))\phi(s)ds = M_{V_i}(p_i - C_{V_i}) \quad (4)$$

Thus, the local minimum points for the locational optimization cost function J_V are centroids of Voronoi cells. In other words, to minimize J_V , each robot must not only be a generator point of its own Voronoi cell but also must be at the center of the cell [5]. Accordingly, the critical partitions and points for J are called centroidal Voronoi partitions. We will refer to a robots' configuration as a centroidal Voronoi configuration if it gives rise to a centroidal Voronoi partition [5].

2.3. Receding Horizon Control (RHC)

RHC is an optimization approach that can be used for systems, even if some constraints on states and inputs exist. In RHC, the current control law is obtained by solving a finite horizon optimal problem at each sampling instant. Each optimization generates an open-loop optimal control trajectory, and the first portion of this optimal control trajectory is applied to system until next sampling time [4], [15].

The contribution of this paper is using RHC (the state space-based model predictive control) in order to coverage an environment. Therefore, first we suggest centralized receding horizon coverage control and then this centralized approach will be extended to a distributed approach.

In the sections that follow, the RHC approach is used to drive a group of n mobile robots at centroidal Voronoi configuration.

3. CENTRALIZED RECEDING HORIZON COVERAGE CONTROL (CRHCC)

The cooperative receding horizon coverage control approach for multiple linear mobile robots is proposed in this section. The objective is to asymptotically force a group of *n* linear mobile robots toward centroidal Voronoi configuration in a cooperative manner using receding horizon control. To do so, let $P(t) = (p_1,...,p_n)$ be a *n*-vector whose elements are robots' position, i.e. $p_i(t) = (x_i(t), y_i(t))$, and $C_V = (C_{V_1},...,C_{V_n})$ be a vector of Voronoi cells centroids. The overall system dynamic can be described as

$$\dot{P}(t) = u(t), t \geq 0, \quad (5)$$

where $P(0)$ is known and $P(t) \in \Re^{2n}$ and $u(t) \in \Re^{2n}$ are state and input vectors respectively. It is assumed that there exist some constraints on state and input, i.e. $P(t) \in \aleph^n$ and $u(t) \in \mathcal{U}^n$ where $\aleph^n$ and $\mathcal{U}^n$ are the state and input constraints sets respectively.

Assumption 1.
$\mathcal{U}$ is a compact and a connected set that contains origin in its interior
Each robot can measure all of its states.
The computational time is negligible

The coverage algorithm proposed in this paper is based on Voronoi diagram. Aurenhammer has shown that the dual of Voronoi diagram is Delaunay triangulation which lies under graph theory concept [13]. To proceed, the coverage problem is investigated using graph theory.

Lemma 1 ([13] Lemma 2.4). Two points of P in Voronoi diagram are connected with a Delaunay edge, iff their corresponding Voronoi cells are adjacent.

These two points (or robots) are called neighbors.
By drawing robots' Voronoi diagram and its corresponding Delaunay graph, the set of robots' positions can be shown with a graph where its vertexes are robots position and its edges are connecting segment between any two neighboring robots. We denote the coverage graph topology by $G = (V, E), V = \{1,...,n\}, E \subset V \times V$. Each edge in graph is illustrated with an ordered pair $(i, j) \in E$, where i, j are any two neighboring robots. Our coverage graph is assumed to be undirected. Hence, $(i, j) \equiv (j,i)$. Robots i, j are called neighbors if in the coverage graph $(i, j) \in E$. The set of neighbors of i^{th} robot is denoted by $\mathrm{N}_i \subset V$. Each element of E is

denoted by e_i. Accordingly, $E = \{e_1,...,e_M\}$, where M is the number of Delaunay edges.
To proceed, we need to define the proposed notion of "*coverage vector" and "coverage matrix"*. Before that we define the desired connecting vector between any two neighbors in a coverage graph, denoted by $d_{ij} \in \Re^2$ as

$$d_{ij} = C_{V_j} - C_{V_i} \quad (6)$$

This vector has the following property

$$d_{ij} = -d_{ji} \quad (7)$$

Definition 2. "*coverage vector" and "coverage matrix"*: the *"coverage vector*" denoted by COV is defined as

$$COV = (cov_1,...,cov_l,...,cov_M, cov_{M+1},...,cov_{M+n}) \in \Re^{2(M+n)},$$

where

$$cov_l = p_i - p_j + d_{ij}, \quad l = 1,...,M, \quad (i,j) \in E \quad (8)$$

and

$$cov_{M+k} = p_k - C_{V_k} \quad k = 1,...,n \quad (9)$$

The robots will be in centroidal Voronoi configuration, namely $P = C_V$, when $COV \equiv 0$. Hence, we can write the linear mapping from P to COV as:

$$COV = TP + \bar{d}, \quad (10)$$

where $\bar{d} = (...,d_{ij},...,-C_{V_k},...)$,. $k = 1,...,n,$ for all $(i,j) \in E$.
We call T as *" coverage matrix"*.
From definition of the coverage vector, we know that

$$COV = TP + \bar{d} \rightarrow \text{if} \quad P = C_V \quad \text{then} \quad COV \equiv 0$$

Therefore

$$TC_V + \bar{d} = 0 \Rightarrow \bar{d} = -TC_V \quad (11)$$

Substitution of (11) into (10) yields:

$$COV = TP - TC_V = T(P - C_V) \quad (12)$$

Lemma 2. The coverage matrix T used in (10) has full rank and it is equal to $\dim(P) = 2n$.

Proof. Using the definition of matrix T, one can verify that it can be written as $T = \begin{bmatrix} T' \\ T'' \end{bmatrix}$, where T'' is an identity matrix of size $2n$. Therefore the coverage matrix T in (10) has full rank equal to $\dim(P) = 2n$. □
Matrix T' given in the above formulation is a generalized incidence matrix of the coverage graph and can be obtained from the incidence matrix of the coverage graph by multiplying every element of that matrix by I_2

where I_2 is an identity matrix of size 2. Furthermore, since our coverage graph is a Delaunay graph, it is connected [13], and its generalized incidence matrix has a rank equal to $2(n-1)$.

Definition 3. The centralized receding horizon coverage control cost function is defined as:

$$H(P(t),u(.),h_p)=\int_t^{t+h_p}[\sum_{(i,j)\in E}\xi\left\|p_i(\tau)-p_j(\tau)+d_{ij}\right\|^2+\xi\left\|P(\tau)-C_V\right\|^2+$$

$$\eta\left\|u(\tau)\right\|^2]d\tau+\sigma\left\|P(t+h_p,P(t))-C_V\right\|^2, \tag{13}$$

where ξ,η,σ are positive weighting constants. The first and second terms in (13) are tracking terms, the third term is a term for minimizing control effort, and the last term is called terminal control cost.

Proposition 2. Using the above definitions and Lemma, we can describe the CRHCC cost function (13) designed to drive a group of n robots to a centroidal Voronoi configuration by a cost function given by:

$$H(P(t),u(.),h_p)=\int_t^{t+h_p}\left\|P(\tau;P(t))-C_V\right\|_Q^2+\left\|u(\tau)\right\|_R^2 d\tau+\left\|P(t+h_p;P(t))-C_V\right\|_G^2 \tag{14}$$

Proof. As it has been proved in [13], a Delaunay graph is connected. Furthermore, as stated before, the coverage graph considered in this paper is Delaunay and thus it is a connected graph. If the coverage graph was not connected, it could be separated to at least two sub-graphs. Moreover, the cost function would be separated in to more than one coupled cost function. Since by Lemma 2 the coverage matrix has full rank, $T^T T$ is a positive definite matrix and hence using (12) one can get

$$\left\|COV\right\|^2=((P-C_V)^T T^T T(P-C_V))=\left\|P-C_V\right\|_{T^T T}^2 \tag{15}$$

We denote $Q=\xi T^T T, G=\sigma I$ and $R=\eta I$ (where I is an identity matrix). Since ξ,η,σ are positive, the matrices *Q*, *G*, and *R* are positive definite matrices, and considering $C_V=C_{V_1},...,C_{V_n}$, (13) can be rewritten as:

$$H(P(t),u(.),h_p)=\int_t^{t+h_p}\left\|P(\tau;P(t))-C_V\right\|_Q^2+\left\|u(\tau)\right\|_R^2 d\tau+\left\|P(t+h_p;P(t))-C_V\right\|_G^2$$

which is indeed equal to (14).
Now by using the above concepts, the CRHCC problem can be stated as follows:

Problem 1. *CRHCC problem:*
Find $H^*(P(t),h_p)=\min_{u(.)}H(P(t),u(.),h_p)$, with

$$H(P(t),u(.),h_p)=\int_t^{t+h_p}\left\|P(\tau;P(t))-C_V\right\|_Q^2+\left\|u(\tau)\right\|_R^2 d\tau+\left\|P(t+h_p;P(t))-C_V\right\|_G^2$$

subject to:

$$\left.\begin{aligned}&\dot{P}(\beta)=u(\beta)\\&u(\beta)\in\mathcal{U}\\&P(\beta;P(t))\in\aleph=S\end{aligned}\right\}\beta\in\left[t,t+h_p\right],$$

$$P(t+h_p;P(t)) \in \Psi(\omega) := \left\{ P \in \Re^{2n} : \left\| P - C_V \right\|_G^2 \le \omega, \omega \ge 0 \right\} \quad (16)$$

Note that (16) represents the terminal constraint [10], [12]. Assume that the first segment of the optimal control problem is solved at time instant $t_0 \in \Re$, h_c is the receding horizon update period, and the closed loop system that we wish to stabilize at C_V is

$$\dot{P}(\tau) = u^*(\tau), \tau \ge t_0, \quad (17)$$

where $u^*(\beta;P(t))$, $\beta \in \left[t, t+h_p\right]$, is the open-loop optimal solution of Problem 1. This optimal control solution is applied to the system until $t+h_c$, i.e. the applied control to the system in the time interval $\tau \in \left[t, t+h_c\right)$, $0 < h_c \le h_p$ is $u^*(\tau) = u^*(\tau;P(t)), \tau \in \left[t, t+h_c\right)$. The open-loop optimal state trajectory is denoted as $P^*(\tau;P(t))$.

Based on the results of CRHCC obtained in this section, a DRHCC algorithm is proposed in the next section.

4. DISTRIBUTED RECEDINH HORIZON COVERAGE CONTROL

In DRHCC approach the objective is to force a group of n robots to centroidal Voronoi configuration in a distributed manner using RHC. In CRHCC approach, the control law requires centralized information and computations. The DRHCC approach proposed in this section, avoids the disadvantages associated with CRHCC approach.

Let $p_i \in \Re^2$ and $u_i \in \Re^2$ be state and control input of the i^{th} robot, where $i = 1,...,n$. It is assumed that robots' dynamics are decoupled from each other and hence their dynamics can be written as:

$$\dot{p}_i(t) = u_i(t), t \ge 0, \quad p_i(0) \text{ given} \quad (18)$$

To achieve the desired cost function, the coupling that is inherent with the centralized approach is eliminated by defining n different costs, one for each robot, and only the connections between any given robot and its neighbors are present. To facilitate the results, the terminal constraint and the terminal cost are assumed to be decoupled, i.e. $G = diag(G_1,...,G_n)$. Furthermore, in addition to previous constraints, a compatibility constraint is added to ensure that each robot does not move away too far from the trajectory expected by its neighbors [12]. It will be explained later. It is also assumed that h_p, h_c are identical for all robots. Considering (5) and defining $P(t) = (p_1,..., p_n)$ and $u = (u_1,...,u_n)$, the overall system dynamic can be decomposed into n sub-systems having the dynamics given by (18). Accordingly, the objective is to design a DRHCC for each robot that drives the robot to the centroid of its own cell in centroidal Voronoi configuration, while cooperating with its neighbors.

Definition 4. The DRHCC cost function for each robot with the objective of reaching its cell's centroid in centroidal Voronoi configuration in a cooperative way with its neighbors, is defined as:

$$H_i(p_i(t), p_j(t), u_i(.), h_p) = \int_t^{t+h_p} \sum_{j \in N_i} \frac{\xi}{2} \left\| p_i(\tau) - p_j(\tau) + d_{ij} \right\|^2 + \xi \left\| p_i(\tau) - C_{V_i} \right\|^2 +$$
$$\eta \left\| u_i(\tau) \right\|^2 d\tau + \left\| P_i(t+h_p, p_i(t)) - C_{V_i} \right\|_{G_i}^2 \quad (19)$$

In the newly considered system, the state and the control constraints are separated for each robot, i.e. $p_i(t) \in \aleph \subseteq \Re^2$ and $u_i(t) \in \boldsymbol{U} \subseteq \Re^2$.

Given $R = diag(R_1,...,R_n)$, control cost can be rewritten as $\left\| u \right\|_R^2 = \sum_{i=1}^{n} \left\| u_i \right\|_{R_i}^2$, where each $R_i = \sigma I$ is a positive definite matrix. To proceed further, the notions of “*distributed coverage vector*” and “*distributed coverage matrix*” are needed. Before that, some notations must be defined.

As stated in section 3, N_i is the set that contains the neighbors of the i^{th} robot neighbors. Therefore, there exists a Delaunay edge between the robot and its neighbors. Let $p_{-i} = (..., p_j, ...)$ and $C_{V_{-i}} = (..., C_{V_j}, ...)$ where $j \in N_i$, denote the state and centroid vectors of the neighbors of robot *i* respectively. Now for each robot, *i*, define the following vector:

$$\overline{cov}^i = (..., \overline{cov}_l^i, ..., \overline{cov}_{|N_i|+1}^i),$$

where

$$\overline{cov}_l^i = p_i - p_j + d_{ij} \qquad l = 1, ..., |N_i|, \forall j \in N_i \tag{20}$$

$$\overline{cov}_{|N_i|+1}^i = p_i - C_{V_i} \tag{21}$$

and $|N_i|$ is the number of elements in N_i. Let the linear mapping from $P^i = (p_i, p_{-i})$ to $\overline{cov}^i$ be written as

$$\overline{cov}^i = \overline{T}^i P^i + \overline{d}^i, \tag{22}$$

where

$$\overline{d}^i = (..., d_{ij}, ..., -C_{V_i}), \; j \in N_i \tag{23}$$

We can now state the following definitions:

Definition 5. *"Distributed coverage vector" and "distributed coverage matrix":*
For each i^{th} robot, the "*distributed coverage vector*" is defined as:

$$cov^i = (..., cov_l^i, ..., cov_{|N_i|+1}^i),$$

where

$$cov_l^i = \frac{1}{2}\overline{cov}_l^i, \;\; l = 1, ..., |N_i| \tag{24}$$

and

$$cov_{|N_i|+1}^i = \overline{cov}_{|N_i|+1}^i \tag{25}$$

The "*distributed coverage matrix*" is defined as matrix T^i in the following equation

$$cov^i = T^i P^i + d^i, \tag{26}$$

where $d^i = \left(..., \frac{1}{2} d_{ij}, ..., C_{V_i}\right) \forall j \in N_i$ and $P^i = (p_i, p_{-i})$..

Since $Q \neq diag(Q_1, ..., Q_n)$, the term ½ is added in (24) in order to satisfy the following equation:

$$\|P - C_V\|_Q^2 = \sum_{i=1}^{n} \left\| \begin{matrix} p_i - C_{V_i} \\ p_{-i} - C_{V_{-i}} \end{matrix} \right\|_{Q_i}^2 = \sum_{i=1}^{n} \left\| P^i - C_V^i \right\|_{Q_i}^2, \quad C_V^i = (C_{V_i}, C_{V_{-i}})$$

Note that if robot *i* and its neighbors be located at their centroids in centroidal Voronoi configuration, i.e. $P^i = C_V^i$, then (20), (21) and therefore (24), (25) will be equal to zero. Hence

$$T^i C_V^i + d^i = 0 \Rightarrow d^i = -T^i C_V^i \Rightarrow cov^i = T^i P^i - T^i C_V^i \Rightarrow \; cov^i = T^i (P^i - C_V^i) \tag{27}$$

Similar to centralized case, it can be proved that The distributed coverage matrix T^i in (27) has full rank and it is equal to $\dim(p) = 2$.

Proposition 3. The cost function given by (19) can be rewritten as

$$H_i(P^i(t), u(.), h_p) = \int_t^{t+h_p} \left(\left\| p^i(\tau) - C_V^i \right\|_{Q_i}^2 + \left\| u_i(\tau) \right\|_{R_i}^2 \right) d\tau + \left\| p_i(t+h_p; p(t)) - C_{V_i} \right\|_{G_i}^2 \tag{28}$$

and

$$\sum_{i=1}^{n} \bar{H}^i(P^i(t), u_i(.), h_p) = \bar{H}(P(t), u(.).h_p),$$

where $\bar{H}$ is the CRHCC cost function.
Proof. Using (27), it can be seen that

$$\left\| cov^i \right\|^2 = \left\| P^i - C_V^i \right\|_{T^i T^{iT}}^2 .$$

Since $R = diag(R_1, ..., R_n)$ and $G = diag(G_1, ..., G_n)$, then by defining $Q_i = \xi T^i T^{iT}$ one can rewrite (19) as:

$$H_i(P^i(t), u(.), h_p) = \int_t^{t+h_p} \left(\left\| p^i(\tau) - C_V^i \right\|_{Q_i}^2 + \left\| u_i(\tau) \right\|_{R_i}^2 \right) d\tau + \left\| p_i(t+h_p; p(t)) - C_{V_i} \right\|_{G_i}^2$$

This is indeed (28) which is useful in stability analysis. Now, according to definitions 2 -5, it is concluded that

$$\sum_{i=1}^{n} H^i(P^i(t), u_i(.), h_p) = H(P(t), u(.).h_p),$$

i.e. the sum of n distributed cost functions is equivalent to centralized cost function.

Now suppose that *n* DRHCC optimal problems, one corresponding to each robot, are all solved at a common time instant called "update time", denoted by $t_k = t_0 + h_c k$, $k \in \{0,1,...\}$. As stated in (19), (28), for each cost function, there is a term that contains connection between the corresponding robot and its neighbors. So, in every update time, when the local optimal problems are solved, each robot requires to know the state trajectories of all its neighbors over time interval $[t_k, t_k + h_p]$. But, such information doesn't exist at instant t_k .Therefore each robot must estimate some state trajectories for its neighbors at $[t_k, t_k + h_p]$ and then solves its optimal control problem. The trajectories that each robot estimates for its neighbors are called *estimated trajectories*. Since each robot is assumed to have the information about the dynamics of its neighbors, an estimated control (defined shortly) is obtained from which the state trajectories are derived. To ensure compatibility between the actual and the estimated trajectories, an additional constraint called "compatibility constraint" is added to DRHCC problem of each robot.

Definition 6. *Estimated control*
At every time interval $\tau \in \left[t_k, t_k + h_p\right]$, the estimated control for each robot is defined as

$$\begin{cases} \hat{u}_i(\tau; p_i(t_k)) = 0; & \textit{if} \quad P(t_k) = C_V \\ \hat{u}_i(\tau; p_i(t_k)) = u_i^*(\tau; p_i(t_{k-1})); & \textit{Otherwise} \end{cases}$$

The actual and the estimated state trajectories are denoted by $p_i(.; p_i(t_k))$ and $\hat{p}_i(.; p_i(t_k))$ respectively. Note that $\hat{p}_i(t_k; p_i(t_k)) = p_i(t_k; p_i(t_k)) = p_i(t_k)$, $i = 1,...,n$.
The DRHCC problem can now be stated as follows.

Problem 2. With a given fixed update period time $h_c \in \left(0, h_p\right)$ and a optimization period time h_p , for every $i = 1,...,n$ and at any sampling time t_k and with given $\hat{u}_{-i}(\beta, p_{-i}(t_k)), p_i(t_k), p_{-i}(t_k), \hat{u}_i(\beta; p_i(t_k))$ at $\beta \in [t_k, t_k + h_p]$ find

$$H_i^*(p_i(t_k), p_{-i}(t_k), h_p) = \min_{u(.)} H_i(p^i(t_k), u_i(.; p_i(t_k), h_p),$$

where

$$H_i(p^i(t), u_i(.), h_p) = \int_t^{t+h_p} (\sum_{j \in \mathrm{N}_i} \frac{\xi}{2} \left\| p_i(\tau) - p_j(\tau) + d_{ij} \right\|^2 + \xi \left\| p_i(\tau) - C_{V_i} \right\|^2 + \eta \left\| u_i(\tau) \right\|^2) dt +$$
$$\left\| P_i(t + h_p, p_i(t)) - C_{V_i} \right\|_{G_i}^2 ,$$

subject to the following
$\dot{p}_i(\beta; p_i(t_k)) = u_i(\beta)$,
$\dot{\hat{p}}_j(\beta; p_j(t_k)) = \hat{u}_j(\beta), j \in N_i$,
$u_i(\beta; p_i(t_k)) \in \boldsymbol{U}$, $p_i(\beta; p_i(t_k)) \in \aleph$,

$$\left\| p_i(\beta; p_i(t_k)) - \hat{p}_i(\beta; p_i(t_k)) \right\| \le h_c{}^2 \kappa, \ \kappa \in (0, \infty) \tag{29}$$

$p_i(t_k + h_p; p_i(t_k)) \in \Psi_i(\varepsilon_i)$,
where $\beta \in \left[t_k, t_k + h_p\right]$,
and

$$\Psi_i(\in_i) =: \left\{ p_i \in R^2 : \left\| p_i - C_{V_i} \right\|_{G_i}^2 \le \varepsilon_i, \varepsilon_i \ge 0 \right\} \tag{30}$$

where $h_c{}^2 \kappa \approx 0$.
(29) is called compatibility constraint and (30) is *target or terminal set*. The optimal solution for each DRHCC problem is denoted by $u_i^*(\tau; p_i(t_k)), \tau \in \left[t_k, t_k + h_p\right]$ and the closed-loop system where we wish to stabilize it, is

$$\dot{P}(\tau) = u^*(\tau)), \qquad \tau \ge 0, \tag{31}$$

where $u^*(\tau; P(t_k)) = (u_1^*(\tau; p_1(t_k)),...,u_n^*(\tau; p_n(t_k)))$. The optimal state trajectory for i^{th} robot is denoted by

$p_i^*(\tau; p_i(t_k)),\ \tau \in \left[t_k, t_k + h_p\right]$.

The augmented optimal state trajectory for $\tau \in \left[t_k, t_k + h_p\right]$ is $P^*(\tau; P(t_k)) = (p_1^*(\tau; p_1(t_k)), ..., p_n^*(\tau; p_n(t_k)))$. Note that the i^{th} robot's optimal control, i.e. $u_i^*(\tau; p_i(t_k))$, is dependent on its initial state, $p_i(t_k)$, and the initial states of its neighbors. In DRHCC problem, initialization is more difficult. As stated before, each robot has an estimated trajectory at t_0. To solve the optimal problem corresponding to a robot, the estimated control information of its neighbors is needed. Since the estimated control of each step is assumed to be the optimal control that is obtained in the previous step, and since prior to t_0 no optimal problem has been solved, one must define an initialization method to obtain the estimated control. The time instant that this initialization occurs is denoted by $t_0 - h_c$.

Algorithm 1. (*Initial setting method*): at time instant $t_0 - h_c$, we solve Problem 2 with initial state $p^i(t_0 - h_c)$ and with $\hat{u}_i(\tau; p_i(t_0 - h_c)) = 0$ for all $\tau \in \left[t_0 - h_c, t_0 - h_c + h_p\right]$ and $\kappa = +\infty$.

The optimal control that is obtained by solving this problem with the above conditions is the estimated control for the time interval $\left[t_0, t_0 + h_p\right]$. $\kappa = +\infty$ implies that the compatibility constraint is not important prior to t_0. State and control trajectories that are obtained at $t_0 - h_c$ over interval $\tau \in \left[t_0 - h_c, t_0 - h_c + h_p\right]$ are denoted by $p_i^*(\tau; p_i(t_0 - h_c))$ and $u_i^*(\tau; p_i(t_0 - h_c))$. This optimal control is applied to i^{th} robot over $\left[t_0 - h_c, t_0\right)$.

The proposed DRHCC algorithm is given in Table 1. Some of the advantages of this algorithm can be listed as follows:

General: As a general rule, the RHC is the only approach in control theory that can employ generic dynamics and generic constraints [4]. As a result, the algorithm can be extended to other systems such as nonlinear systems (holonomic and non-holonomic) and it can handle any constraint imposed on the system.

Near optimal: The optimal position for robots, when maximizing the event detections in Voronoi based coverage, are centroids of Voronoi cells in centroidal Voronoi configuration. Since, RHC is an optimization approach that yields an optimal control, the suggested algorithm is near optimal in a sense that it will force the robots to converge to centroids of Voronoi cells using an optimal control input.

Adaptive: Since the algorithm determines the coverage graph at each update instant, it can accept any possible switching in coverage graph at that particular update instant. As a result, it can address any possible change in the network topology such as robot failure or departure.

Distributed: The DRHCC algorithm is distributed in the sense that each robot computes a control trajectory for itself based on its own and its neighbors information.

Scalable: Scalability is one of the main advantages of distributed approach over centralized case [4], [5]. In a distributed approach, every robot just needs the information about its neighbors and the average number of Voronoi neighbors in Voronoi diagram is less than 6 [13]. This makes the DRHCC algorithm scalable. This property of the DRHCC algorithm will be shown by an example in section 6.

Robust: Robustness is one of the main advantages of distributed approaches [5].

Collision avoidance: The mission space is assumed to be convex. Therefore each Voronoi cell is convex and contains its own center. Since in DRHCC approach each robot moves towards its own center, no robot can leave its Voronoi cell and we know that $I(V_i) \cap I(V_j) = \emptyset$. Hence assuming that the robots sizes are small, there is no collision between robots.

Stable: Section 5

Table 1. Distributed receding horizon coverage control algorithm

Name: DRHCC algorithm **Goal:** Asymptotically drive a group of n mobile robots toward centroidal Voronoi configuration.
At instant $t_0 - h_c$ **every robot:**
A1- senses its position and transmits the information about its position to the neighbors and receives its neighbor's position
A2- computes its Voronoi region $V_i(t_0 - h_c)$ and centroid of that according to (3)
A3- follows the initial setting procedure given in Algorithm 1
At every update instant $t_k, k \in N$ **each robot:**
B1- senses its own and its neighbors' position (or receives neighbors position).
B2- computes its Voronoi region $V_i(t_k)$.
B3- computes centroid of its cell according to (3)
B4- transmits the information about its Voronoi center to each of its neighbor in the system and retrieves the same information from its neighbors
B5- computes its own and its neighbors estimated trajectory using (18)
B6- computes distributed optimal control trajectory $u_i^*(\tau; p_i(t_k))$ over interval $\tau \in [t_k, t_k + h_p]$ using Problem 2
Over every interval $[t_{k-1}, t_k)$**, each robot:**
C1- applies the distributed optimal control trajectory that has been obtained at t_{k-1}
C2- computes its estimated control for $[t_k, t_k + h_p]$ according to Definition 6
C3- transmits its estimated control that was computed in C2 to every neighbors and receives their estimated control

5. STABILITY ANALYSIS

The stabilization of the closed-loop system (31) is investigated in this section. As stated in section 3, the overall cost function for the system is given by (14) where

$$H^*(P(t_k), h_p) = \sum_{i=1}^{n} H_i^*(p_i^*(t_k), p_{-i}(t_k), h_p) \tag{32}$$

Proposition 4. All solutions to the equation $\dot{P}^*(\tau) = u^*(\tau)), \quad \tau \geq 0$ which $P^* = C_V$ are equilibriums of the system (31).

Proof. If each robot is located at C_V at time $t_0 - h_c$, i.e. $P(t_0 - h_c) = C_V$, then the optimal solution for Problem 2 over time interval $\tau \in [t_0 - h_c, t_0 - h_c + h_p]$ is $u^*(\tau, C_V) = 0$. On the other hand, using systems dynamics given by (5), one can write $\dot{P}(t) = 0$. Furthermore, since every optimal control is applied to the system until next update time, the estimated control is equal to zero at t_0, i.e. $\hat{u}_i(.; p_i(t_0)) = 0$ and hence $P(t_0) = C_V$. Therefore, over interval $\tau \in [t_k, t_k + h_p]$, $u^*(\tau, P(t_k)) = 0$ and hence all solutions to the equation $\dot{P}^*(\tau) = u^*(\tau)), \quad \tau \geq 0$ which $P^* = C_V$ are equilibriums of the system (31).

□

Theorem 1. Based on DRHCC algorithm given in Table 1, the closed loop system (31) converges to centroidal Voronoi configuration and C_V is an asymptotically stable equilibrium point for the closed-loop system, with $\aleph$ as its region of attraction.

Proof. Since the mission space is assumed to be convex, each Voronoi cell is also convex and contains its centroid in its interior. Consequently, each robot always moves inside its cell and therefore never leaves the mission space S . Assuming Problem 2 is feasible at initialization, $u_i(\tau; p_i(t_0 - h_c))$ is an admissible control over $\tau \in [t_0 - h_c, t_0 - h_c + h_p]$. Presuming that all robots are located inside S at $t_0 - h_c$, then at times $t_0 - h_c + h_c \leq \tau \leq t_0 - h_c + h_p$ the state and control trajectories are admissible and as a result for all time instants after initialization, they are admissible as well. Therefore, $\aleph$ is a positively invariant set meaning that the closed-loop state trajectory for all $t \ge t_0 - h_c$ is contained in the interior of $\aleph$.

Now let $V = H^*(P(t_k), h_p)$ where V is a Lyapunov function. From (28) and (32) and given the fact that $G_i \ge 0$, one can deduce that $H^*(P(t_k), h_p)$ is a non-negative function. If $H^*(P(t_k), h_p) = 0$, then based on (28) and (32) for every robot $\left\| p_i^*(t_k + h_p, p_i(t_k)) - C_{V_i} \right\|_{G_i}^2 = 0$

and

$$\int_{t_k}^{t_k+h_p} \frac{\xi}{2} \sum_{j \in N_i} \left\| p_i^*(\tau, p_i(t_k)) - \hat{p}_j(\tau, p_j(t_k)) + d_{ij} \right\|^2 + \xi \left\| p_i^*(\tau, p_i(t_k)) - C_{V_i} \right\|^2 + \eta \left\| u_i^*(\tau, p_i(t_k)) \right\|^2 d\tau = 0$$

Since the function under the integral in (28) is piecewise continuous and non-negative over time interval $[t_k, t_k + h_p]$, one can write

$$\sum_{j \in N_i} \left\| p_i^*(\tau, p_i(t_k)) - \hat{p}_j(\tau, p_j(t_k)) + d_{ij} \right\|^2 + \xi \left\| p_i^*(\tau, p_i(t_k)) - C_{V_i} \right\|^2 + \eta \left\| u_i^*(\tau, p_i(t_k)) \right\|^2 = 0, \ \forall \tau \in \left[t_k, t_k + h_p\right]$$

Hence, $u^*(\tau, P(t_k)) = 0$ over the interval $\tau \in [t_k, t_k + h_p]$, and given the fact that G_i is a positive definite matrix then $p_i^*(t_k + h_p, p_i(t_k)) = C_{V_i}$. Note that the system dynamics given by (5) is time invariant and therefore with initial state $P(t_k + h_p) = C_V$ and control $u(\beta) = 0,\ \beta \in \left[t_k + h_p, t_k\right]$, one can write $P(\beta) = C_V$. Therefore, over interval $\tau \in [t_k, t_k + h_p]$, the optimal closed-loop state is $P^*(\tau; P(t_k)) = C_V$. Furthermore, with $P(t_k) = C_V$, using Definition 6 over time interval $\tau \in [t_k, t_k + h_p]$, the estimated control is $\hat{u}_i(\tau, p_i(t_k)) = 0$, and hence $\hat{p}_i(\tau; p_i(t_k)) = C_{V_i}$. Since, the DRHCC cost function is denoted as

$$\int_{t_k}^{t_k+h_p} \frac{\xi}{2} \sum_{j \in N_i} \left\| p_i^*(\tau, p_i(t_k)) - \hat{q}_j(\tau, q_j(t_k)) + d_{ij} \right\|^2 + \xi \left\| p_i^*(\tau, p_i(t_k)) - C_{V_i} \right\|^2 + \eta \left\| u_i^*(\tau, p_i(t_k)) \right\|^2 d\tau + \sigma \left\| p_i^*(t_k + h_p, p_i(t_k)) - C_{V_i} \right\|^2$$

and because $p_i(t_k) = C_{V_i}$, the optimal solution to Problem 2, over time interval $\tau \in [t_k, t_k + h_p]$, and for every robot are $u_i^*(\tau; p_i(t_k)) = 0$ and $p_i^*(\tau; p_i(t_k)) = C_{V_i}$. Hence, since for every robot $H_i^* = 0$, based on Proposition 3 the total cost is $H^*(P(t_k), h_p) = 0$. Consequently, $H^*(P(t_k), h_p) > 0$ everywhere except at $P(t_k) = C_V$ where $H^*(P(t_k), h_p) = 0$. Therefore H^* is positive definite. Since H^* satisfies the following

$$H^*(P(t_k), h_p) = \sum_{i=1}^{n} \int_{t_k}^{t_k+h_p} \frac{\xi}{2} \sum_{j \in N_i} \left\| p_i^*(\tau, p_i(t_k)) - \hat{p}_j(\tau, p_j(t_k)) + d_{ij} \right\|^2 + \xi \left\| p_i^*(\tau, p_i(t_k)) - C_{V_i} \right\|^2 + \eta \left\| u_i^*(\tau, p_i(t_k)) \right\|^2 d\tau + \left\| p_i^*(t_k + h_p, p_i(t_k)) - C_{V_i} \right\|_{G_i}^2 \quad (33)$$

Therefore

$$\Delta V = V(t_{k+1}) - V(t_k) = H^*(P(t_{k+1}), h_p) - H^*(P(t_k), h_p) =$$

$$[\sum_{i=1}^{n} \int_{t_{k+1}}^{t_{k+1}+h_p} \frac{\xi}{2} \sum_{j\in N_i} \left\| p_i^*(\tau, p_i(t_{k+1})) - \hat{p}_j(\tau, p_j(t_{k+1})) + d_{ij} \right\|^2 +$$

$$\xi \left\| p_i^*(\tau, p_i(t_{k+1})) - C_{V_i} \right\|^2 + \eta \left\| u_i^*(\tau, p_i(t_{k+1})) \right\|^2 d\tau +$$

$$\sigma \left\| p_i^*(t_{k+1} + h_p, p_i(t_{k+1})) - C_{V_i} \right\|^2] -$$

$$[\sum_{i=1}^{n} \int_{t_k}^{t_k+h_p} \frac{\xi}{2} \sum_{j\in N_i} \left\| p_i^*(\tau, p_i(t_k)) - \hat{p}_j(\tau, p_j(t_k)) + d_{ij} \right\|^2 +$$

$$\xi \left\| p_i^*(\tau, p_i(t_k)) - C_{V_i} \right\|^2 + \eta \left\| u_i^*(\tau, p_i(t_k)) \right\|^2 d\tau +$$

$$\sigma \left\| p_i^*(t_k + h_p, p_i(t_k)) - C_{V_i} \right\|^2] \qquad (34)$$

Since $u_i^*(\tau, p_i(t_{k+1}))$ is an optimal control that minimizes $H_i(P(t_k), h_p)$ one can write

$$H^*(P(t_{k+1}), h_p) \le \sum_{i=1}^{n} \int_{t_{k+1}}^{t_{k+1}+h_p} \frac{\xi}{2} \sum_{j\in N_i} \left\| \hat{p}_i(\tau, p_i(t_{k+1})) - \hat{p}_j(\tau, p_j(t_{k+1})) + d_{ij} \right\|^2 +$$

$$\xi \left\| \hat{p}_i(\tau, p_i(t_{k+1})) - C_{V_i} \right\|^2 \eta \left\| \hat{u}_i(\tau, p_i(t_{k+1})) \right\|^2 d\tau +$$

$$\sigma \left\| \hat{p}_i(t_{k+1} + h_p, p_i(t_{k+1})) - C_{V_i} \right\|^2$$

According to (34)

$$H^*(P(t_{k+1}), h_p) - H^*(P(t_k), h_p) \le$$

$$\sum_{i=1}^{n} \int_{t_{k+1}}^{t_{k+1}+h_p} \frac{\xi}{2} \sum_{j\in N_i} \left\| \hat{p}_i(\tau, p_i(t_{k+1})) - \hat{p}_j(\tau, p_j(t_{k+1})) + d_{ij} \right\|^2 +$$

$$\xi \left\| \hat{p}_i(\tau, p_i(t_{k+1})) - C_{V_i} \right\|^2 + \eta \left\| \hat{u}_i(\tau, p_i(t_{k+1})) \right\|^2 d\tau +$$

$$\sigma \left\| \hat{p}_i(t_{k+1} + h_p, p_i(t_{k+1})) - C_{V_i} \right\|^2 -$$

$$[\sum_{i=1}^{n} \int_{t_k}^{t_k+h_p} \frac{\xi}{2} \sum_{j\in N_i} \left\| p_i^*(\tau, p_i(t_k)) - \hat{p}_j(\tau, p_j(t_k)) + d_{ij} \right\|^2 +$$

$$\xi \left\| p_i^*(\tau, p_i(t_k)) - C_{V_i} \right\|^2 + \eta \left\| u_i^*(\tau, p_i(t_k)) \right\|^2 d\tau +$$

$$\sigma \left\| p_i^*(t_k + h_p, p_i(t_k)) - C_{V_i} \right\|^2] \qquad (35)$$

Since $G = diag(G_1, ..., G_n), \hat{P} = (\hat{p}_1, ..., \hat{p}_n)$
it is concluded that

$$\sum_{i=1}^{n}\left\|\hat{p}_i\,(t_{k+1}+h_p;p_i(t_{k+1}))-C_{V_i}\right\|_{G_i}^2=\left\|\hat{P}(t_{k+1}+h_p;P(t_{k+1}))-C_V\right\|_G^2,$$

$$\sum_{i=1}^{n}\left\|\hat{p}_i\,(t_k+h_p;p_i(t_k))-C_{V_i}\right\|_{G_i}^2=\left\|\hat{P}(t_k+h_p;P(t_k))-C_V\right\|_G^2 \tag{36}$$

and using Definition 6, (36) can be rewritten as:

$$\begin{aligned}
&H^*(P(t_{k+1}),h_p)-H^*(P(t_k),h_p)\le\\
&\sum_{i=1}^{n}\int_{t_{k+1}}^{t_k+h_p}\frac{\xi}{2}\sum_{j\in N_i}\left\|\hat{p}_i\,(\tau,p_i(t_{k+1}))-\hat{p}_j(\tau,p_j(t_{k+1}))+d_{ij}\right\|^2+\\
&\qquad\xi\left\|\hat{p}_i\,(\tau,p_i(t_{k+1}))-C_{V_i}\right\|^2+\eta\left\|\hat{u}_i\,(\tau,p_i(t_{k+1}))\right\|^2d\tau+\\
&\sum_{i=1}^{n}\int_{t_k+h_p}^{t_{k+1}+h_p}\frac{\xi}{2}\sum_{j\in N_i}\left\|\hat{p}_i\,(\tau,p_i(t_{k+1}))-\hat{p}_j(\tau,p_j(t_{k+1}))+d_{ij}\right\|^2+\\
&\qquad\xi\left\|\hat{p}_i\,(\tau,p_i(t_{k+1}))-C_{V_i}\right\|^2+\eta\left\|\hat{u}_i\,(\tau,p_i(t_{k+1}))\right\|^2d\tau-\\
&[\sum_{i=1}^{n}\int_{t_k}^{t_{k+1}}\frac{\xi}{2}\sum_{j\in N_i}\left\|p_i^*(\tau,p_i(t_k))-\hat{p}_j(\tau,p_j(t_k))+d_{ij}\right\|^2+\\
&\qquad\xi\left\|p_i^*(\tau,p_i(t_k))-C_{V_i}\right\|^2+\eta\left\|u_i^*(\tau,p_i(t_k))\right\|^2d\tau]-\\
&[\sum_{i=1}^{n}\int_{t_{k+1}}^{t_k+h_p}\frac{\xi}{2}\sum_{j\in N_i}\left\|p_i^*(\tau,p_i(t_k))-\hat{p}_j(\tau,p_j(t_k))+d_{ij}\right\|^2+\\
&\qquad\xi\left\|p_i^*(\tau,p_i(t_k))-C_{V_i}\right\|^2+\eta\left\|u_i^*(\tau,p_i(t_k))\right\|^2d\tau]+\\
&\left\|\hat{P}(t_{k+1}+h_p;P(t_{k+1}))-C_V\right\|_G^2-\left\|P^*(t_k+h_p;P(t_k))-C_V\right\|_G^2=\\
&\sum_{i=1}^{n}\int_{t_{k+1}}^{t_k+h_p}\frac{\xi}{2}\sum_{j\in N_i}\left\|\hat{p}_i\,(\tau,p_i(t_{k+1}))-\hat{p}_j(\tau,p_j(t_{k+1}))+d_{ij}\right\|^2+\\
&\qquad\xi\left\|\hat{p}_i\,(\tau,p_i(t_{k+1}))-C_{V_i}\right\|^2+\eta\left\|\hat{u}_i\,(\tau,p_i(t_{k+1}))\right\|^2d\tau-\\
&[\sum_{i=1}^{n}\int_{t_k}^{t_{k+1}}\frac{\xi}{2}\sum_{j\in N_i}\left\|p_i^*(\tau,p_i(t_k))-\hat{p}_j(\tau,p_j(t_k))+d_{ij}\right\|^2+\\
&\qquad\xi\left\|p_i^*(\tau,p_i(t_k))-C_{V_i}\right\|^2+\eta\left\|u_i^*(\tau,p_i(t_k))\right\|^2d\tau]-\\
&[\sum_{i=1}^{n}\int_{t_{k+1}}^{t_k+h_p}\frac{\xi}{2}\sum_{j\in N_i}\left\|p_i^*(\tau,p_i(t_k))-\hat{p}_j(\tau,p_j(t_k))+d_{ij}\right\|^2+\\
&\qquad\xi\left\|p_i^*(\tau,p_i(t_k))-C_{V_i}\right\|^2+\eta\left\|u_i^*(\tau,p_i(t_k))\right\|^2d\tau]-
\end{aligned}$$

$$\int_{t_k+h_p}^{t_{k+1}+h_p} \left\| \hat{P}(t_{k+1}+h_p; P(t_{k+1})) - C_V \right\|_{Q^*-Q}^2 \quad (37)$$

Q is the weighting matrix in CRHCC and $Q^* = diag(Q_i)$.
Likewise, according to Definition 6

$$\hat{p}_i(\tau, p_i(t_{k+1})) = p^*(\tau, p_i(t_k)), \hat{u}_i(\tau, p_i(t_{k+1})) = u_i^*(\tau, p_i(t_k))$$

Therefore

$$\sum_{i=1}^{n} \int_{t_{k+1}}^{t_k+h_p} \frac{\xi}{2} \sum_{j\in N_i} \left\| \hat{p}_i\,(\tau, p_i(t_{k+1})) - \hat{p}_j(\tau, p_j(t_{k+1})) + d_{ij} \right\|^2 +$$

$$\xi \left\| \hat{p}_i\,(\tau, p_i(t_{k+1})) - C_{V_i} \right\|^2 + \eta \left\| \hat{u}_i\,(\tau, p_i(t_{k+1})) \right\|^2 d\tau -$$

$$[\sum_{i=1}^{n} \int_{t_{k+1}}^{t_k+h_p} \frac{\xi}{2} \sum_{j\in N_i} \left\| p_i^*(\tau, p_i(t_k)) - \hat{p}_j(\tau, p_j(t_k)) + d_{ij} \right\|^2 +$$

$$\xi \left\| p_i^*(\tau, p_i(t_k)) - C_{V_i} \right\|^2 + \eta \left\| u_i^*(\tau, p_i(t_k)) \right\|^2 d\tau] =$$

$$\sum_{i=1}^{n} \int_{t_{k+1}}^{t_k+h_p} \left\| \begin{bmatrix} p_i^*(\tau, p_i(t_k)) - C_{V_i} \\ p_{-i}^*(\tau, p_i(t_k)) - C_{V_{-i}} \end{bmatrix} \right\|_{Q_i} - \left\| \begin{bmatrix} p_i^*(\tau, p_i(t_k)) - C_{V_i} \\ \hat{p}_{-i}(\tau, p_{-i}(t_k)) - C_{V_{-i}} \end{bmatrix} \right\|_{Q_i},$$

where $Q_i = \xi T^{i^T} T^i$. Since $Q^* > Q$,

$$H^*(P(t_{k+1}), h_p) - H^*(P(t_k), h_p) \leq$$

$$-[\sum_{i=1}^{n} \int_{t_k}^{t_{k+1}} \frac{\xi}{2} \sum_{j\in N_i} \left\| p_i^*(\tau, p_i(t_k)) - \hat{p}_j(\tau, p_j(t_k)) + d_{ij} \right\|^2 +$$

$$\xi \left\| p_i^*(\tau, p_i(t_k)) - C_{V_i} \right\|^2 d\tau] +$$

$$\sum_{i=1}^{n} \int_{t_{k+1}}^{t_k+h_p} \left\| \begin{bmatrix} p_i^*(\tau, p_i(t_k)) - C_{V_i} \\ p_{-i}^*(\tau, p_i(t_k)) - C_{V_{-i}} \end{bmatrix} \right\|_{Q_i} - \left\| \begin{bmatrix} p_i^*(\tau, p_i(t_k)) - C_{V_i} \\ \hat{p}_{-i}(\tau, p_{-i}(t_k)) - C_{V_{-i}} \end{bmatrix} \right\|_{Q_i}$$

Now, using the definition of matrix Q_i, one can verify that it can be written as

$$Q_i = \begin{bmatrix} Q_{i1} & Q_{i2} \\ Q_{i2}^T & Q_{i3} \end{bmatrix}, \quad Q_{i1} \in \Re^{2\times 2}, \quad Q_{i2} \in \Re^{2\times 2|N_i|}, \quad Q_{i3} \in \Re^{2|N_i|\times 2|N_i|}, \text{ where } |N_i| = N_i - 1.$$

According to the fact that for a given vector $v = (v_1, v_2)$, $\|v\| \leq \|v_1\| + \|v_2\|$ and according to the fact that in Voronoi partition $|p_i| \leq \rho_{\max}$ and using compatibility constraint, the above inequality can be written as

$$H^{*}(p(t_{k+1}),h_p)-H^{*}(p(t_k),h_p)\le$$

$$-[\sum_{i=1}^{n}\int_{t_k}^{t_{k+1}}\frac{\xi}{2}\sum_{j\in N_i}\left\|p_i^{*}(\tau,p_i(t_k))-\hat{p}_j(\tau,p_j(t_k))+d_{ij}\right\|^2+$$

$$\xi\left\|p_i^{*}(\tau,p_i(t_k))-C_{V_i}\right\|^2 d\tau]+$$

$$2\lambda_{\max}(Q)\kappa\rho_{\max}h_p h_c^2\sum_{i=1}^{n}N_i(N_i-1) \tag{38}$$

At time $\tau=t_k$, $p_i^{*}(t_k;p_i(t_k))=p_i(t_k)$ and $\hat{p}_{-i}(t_k;p_{-i}(t_k))=p_{-i}(t_k)$, and according to definition of coverage matrix one can write

$$\sum_{i=1}^{n}[\sum_{j=1}^{N_i}\left\|p_i(\tau)-p_j(\tau)+d_{ij}\right\|^2+\xi\left\|p_i(\tau)-C_{V_i}\right\|^2]=\left\|p(t_k)-C_V\right\|_Q^2$$

So, at any $k\in N$

$$H^{*}(p(s),h_p)-H^{*}(p(t_k),h_p)\le-\int_{t_k}^{s}\left\|p^{*}(\tau;p(t_k))-C_V\right\|_Q^2 d\tau,\ s\in[t_k,t_{k+1}]$$

Applying this recursively gives the result for any $t\ge t_0$ and any $t'\in(t,\infty]$ where

$$H^{*}(p(t'),h_p)-H^{*}(p(t),h_p)\le-\int_{t}^{t'}\left\|p^{*}(\tau)-C_V\right\|_Q^2 d\tau,$$

for any times t, t' with $t_0\le t<t'\le\infty$.
Given $\wp>0$, choose $r\in(0,\alpha]$ such that the closed ball

$$B(C_V,r)=\left\{P\in\Re^{2n}\,|\,\|P-C_V\|\le r\right\}$$

Is a small region around C_V. For facility $V(P)$ is denoted by $H^{*}(P,h_p)$. Since $V(P)$ is continuous at $P=C_V$ and $V(P)>0$ for all $P\ne C_V$, there exists a $\alpha\in(0,\infty)$ such that

$$\alpha<\min_{\|P-C_V\|=r}V(P)$$

Define the level set of $V(P)$, by contradiction it can be shown that

$$Z_\alpha=\left\{P\in B(C_V,r)\,|\,V(P)\le\alpha\right\},$$

which is a subset contained in the interior of $B(C_V,r)$. Because $V(P)$ is monotonic,

$$V(P(t))\le V(P(t_0))\le\beta,\quad \forall t\ge t_0$$

So, Z_α is a positively invariant set for the closed-loop system (31). Since $V(C_V)=0$ and $V(P)$ is continuous at $P=C_V$, there exists a constant $\eta\in(0,r)$ which

$$\|P(t_0)-C_V\|<\eta \Rightarrow V(P(t_0))<\alpha \Rightarrow V(P(t))<\alpha \Rightarrow \|P(t)-C_V\|<\wp$$

Thus, C_V is a stable equilibrium point of the closed-loop system (31). Since $\alpha \geq V(P(t_0))$ and $0 \geq -V(P(\infty))$ by induction, it can be shown that

$$V(P(t_0))-V(P(\infty)) \geq \int_{t_0}^{\infty} \left\|P^*(\tau)-C_V\right\|_Q^2 d\tau \Rightarrow \alpha \geq \int_{t_0}^{\infty} \left\|P^*(\tau)-C_V\right\|_Q^2$$

Therefore, the infinite integral above exists and is bounded. Let $\wp_1 < \wp$ be such that $P(t)$ belongs to the compact set $\{\|P(t)-C_V\| \leq \wp_1\}$ for all $t \in [t_0,\infty)$, known to exist because of the strict inequality bound by $\wp$ shown above. Since $u^*(t)$ is in the compact set U^n for all $t \in [t_0,\infty)$ and from (5), it is clear that $P(t)$ is continuous in P and u, we have that $P(t)$ is bounded for all $t \in [t_0,\infty)$. So, $P(t)$ is uniformly continuous [14] in t on $[t_0,\infty)$. Since $\|P-C_V\|_Q^2$ is uniformly continuous in P on the compact set $\{\|P-C_V\| \leq \wp_1\}$, $\|P(t)-C_V\|_Q^2$ is uniformly continuous in t on $[t_0,\infty)$. Since $Q>0$, from Barbalat's Lemma [14] it is concluded that

$$\|P(t)-C_V\| \to 0 \text{ as } t \to \infty$$

Thus, C_V is an asymptotically stable equilibrium point for closed-loop system (31) with region of attraction Z_α.

Now, for any $P(t_0) \in \aleph$, there exists a finite time T' such that $P(T') \in Z_\alpha$, which can be shown by contradiction as follows. Suppose $P(t) \notin Z_\alpha$ for all $t \geq t_0$. Since $V(P)>0$ and $Q>0$, from equation (40), for all $t \geq t_0$,

$$V(P(t+h_c))-V(P(t)) \leq -\int_{t}^{t+h_c} \left\|P^*(\tau)-C_V\right\|_Q^2 d\tau \leq -h_c.\inf\left\{\|P-C_V\|_Q^2 \mid P \notin Z_\alpha\right\} \leq h_c$$

By induction, $V(P(t)) \to -\infty$ as $t \to \infty$; however, this contradicts $V(P(t)) \geq 0$. Therefore, any trajectory starting in $\aleph$ enters Z_α in finite time. Finally, since $\aleph$ is a positively invariant set it is a region of attraction for the closed-loop system (31). Moreover, for any $P(t) \in \aleph$, by absolute continuity of $P(t')$ in $t' \geq t_0$, it can always be chosen a small neighborhood of $P(t)$ in which the optimization problem is still feasible. Thus, $\aleph$ is open and connected.

6. SIMULATION RESULTS

The proposed DRHCC algorithm has been numerically simulated using three different scenarios for 20 linear mobile robots having the dynamics given by (18). In the first scenario, the event density function is assumed to be uniform denoted as $\phi(s)=1$. It is also assumed that the robots are initially distributed randomly in the mission space as shown in Figure 1-(a). After 0.8 second, the robots converge to a centroidal Voronoi configuration shown in Figure 1 (b). The Robots' paths are shown in Figure (1)-c and Figure 1-(d) shows the gradual reduction of $H^*(P(t_k),h_p)$ towards zero.

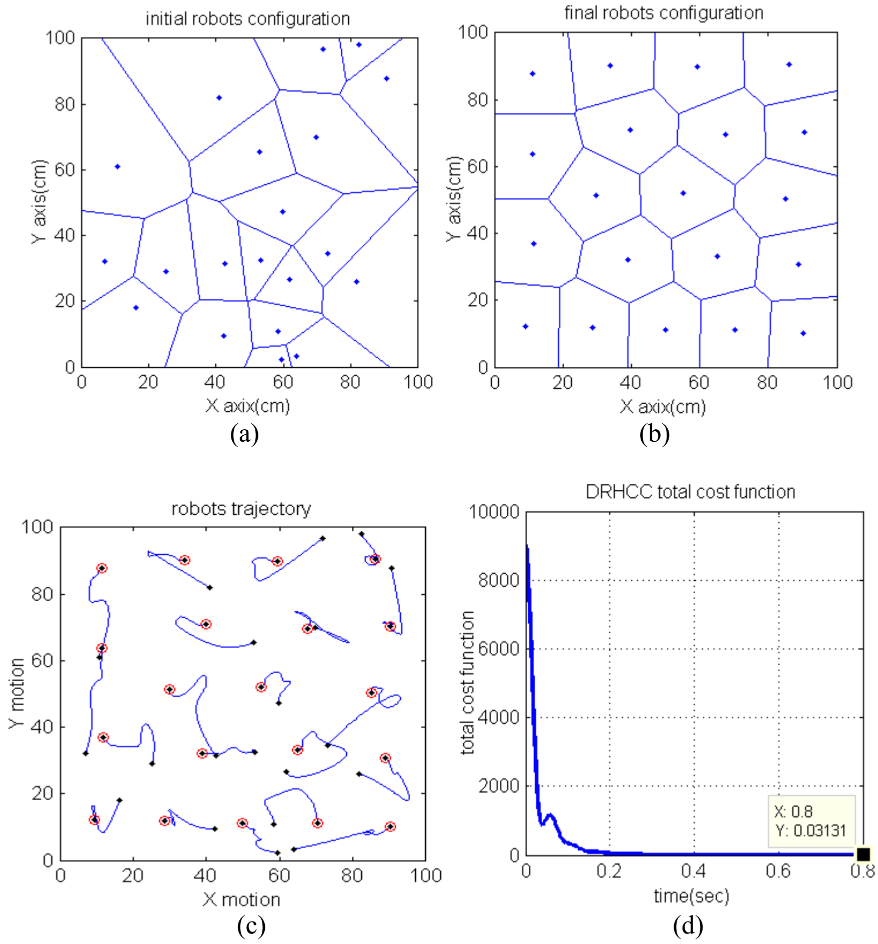

Figure 1. Simulation results by applying DRHCC algorithm to a group of 20 mobile robots in an environment with uniform density of events

In the second scenario, 20 robots are distributed in an environment with a Gaussian events density function equal to $e^{-[(x-0.8)^2+(y-0.8)^2]}$. The simulation results for this scenario are shown in Figure 2. Figure 2-(a) shows the Gaussian density function. The initial random distribution of robots in the mission space is shown in Figure 2-(b). Final configuration is shown in Figure 2-(c). The Robots' paths are shown in Figure 2-(d) and Figure 2-(e) shows that DRHCC algorithm causes the robots converge to a centroidal Voronoi configuration with nearly zero total cost value. As expected, convergence to centroidal Voronoi configuration presented in section 5 has been validated.

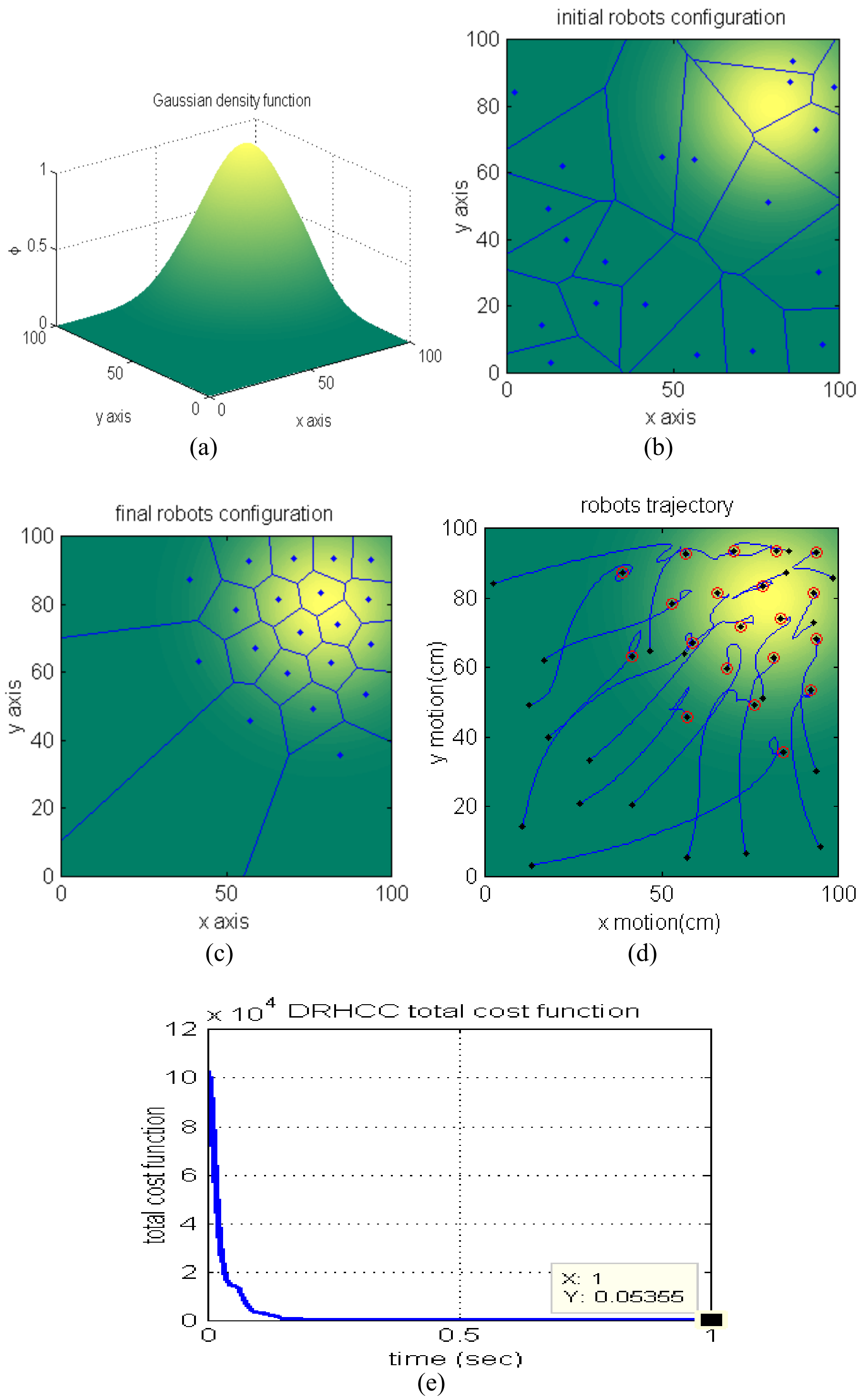

Figure 2. (a) Gaussian density function (b) Initial configuration for 20 linear mobile robots in an environment with Gaussian density of events (c) Final centroidal Voronoi configuration for 20 linear mobile robots in an environment with Gaussian density of events (d) Robots' trajectories (e) DRHCC total cost function.

In the third scenario, the scalability of the DRHCC algorithm is shown. In this scenario, 50 mobile robots are distributed in an environment with a Gaussian events density function equal to $e^{-[(x-0.8)^2+(y-0.8)^2]}$ similar to previous scenario. The simulation results for this scenario are shown in Figure 3.

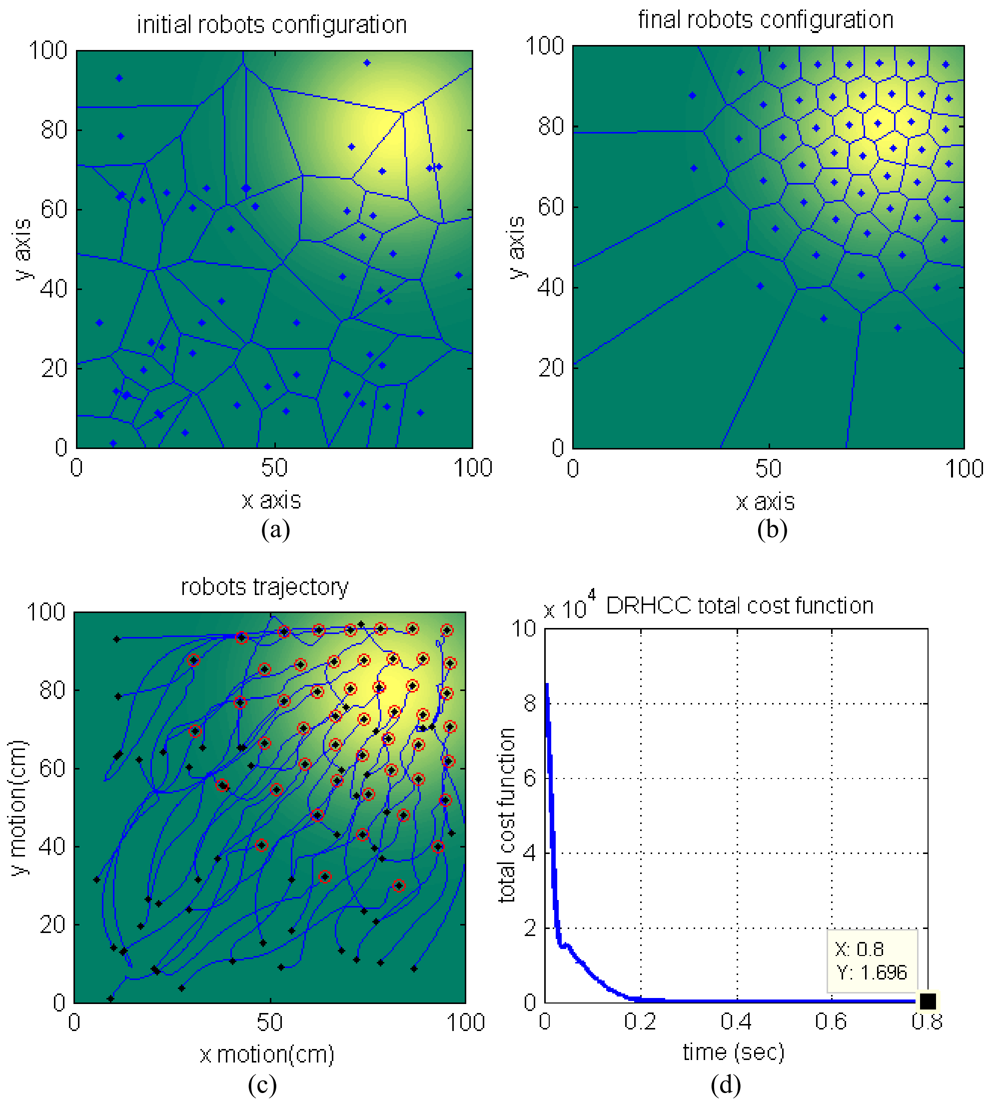

Figure 3. Simulation results by applying DRHCC algorithm to a group of 50 mobile robots in an environment with Gaussian distribution density function of events

7. CONCLUSION

In this paper, the authors proposed a distributed receding horizon coverage control algorithm for controlling a group of linear mobile robots, with a focus on network convergence and stability. In the proposed algorithm, the dynamics of every mobile robot was assumed decoupled from each other and by use of graphs for analysis, the authors proved system's stability. The objective of the coverage algorithm considered here was to maximize the detection of the occurrence of the events. Simulation results validated the algorithm and convergence of the robots to the centroidal Voronoi configuration.

The proposed approach can be extended to time-varying environments (e.g., consider a time-varying distribution density function), systems with non-negligible computational time (time delayed systems), unknown environment, non-isotropic sensors and sensors with nonlinear dynamics and multi-agents coverage based formation control.

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