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A New Method for Time-Jerk Optimal Trajectory Planning Under Kino-dynamic Constraint of Robot Manipulators in Pick-and-Place Operations

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ABSTRACT

A new method for time-jerk optimal planning under Kino-dynamic constraints of robot manipulators in pick-and-place operations is described in this paper. In order to ensure that the resulting trajectory is smooth enough, a cost function containing a term proportional to the integral of the squared jerk (defined as the derivative of the acceleration) along the trajectory is considered. Moreover, a second term, proportional to the total execution time, is added to the expression of the cost function. A Cubic Spline functions are then used to compose overall trajectory. This method makes it possible to deal with the kinematic constraints as well as the dynamic constraints imposed on the robot manipulator. The algorithm has been tested in simulation yielding good results.

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1. INTRODUCTION

The determination of the time-jerk optimal trajectory planning for manipulators is an important problem in robot trajectory planning. Limiting the jerk is very important, because high jerk values can wear out the robot structure, and heavily excite its resonance frequencies; vibrations induced by non-smooth trajectories can damage the robot actuators, and introduce large errors while the robot is performing tasks such as trajectory tracking moreover low-jerk trajectories can be executed more rapidly and accurately. Also, decreased the execution time of the task is very important to increase the productivity of the robot manipulators; this can be the case of the handling of objects, the point-to-point welding or the installation of the electronic components. Some of these applications require the use of a trajectory planner that yields, for a given performance criterion, optimal or near-optimal solutions while considering full dynamics [1].

Many work in the field of robotics has been devoted to the study of the problem of motion planning, we cite in this context the work of [2] The authors have treated the problem of trajectory planning of robot manipulator in imposed tasks by considering the kinematic constraints, and to optimize the cost function which represents a weighting between the execution time of the task and the interval squared jerk they were used the sequential quadratic programming function. In [3] a new approach called interval analysis is used to develop an algorithm that minimizes the maximum absolute value of jerk along the trajectory, the cubic splines were used to represent the trajectory imposed tasks; this problem is solved without considered the dynamics of the robot. In [4] the authors proposed a method based on PSO to optimize the cost function used in [3] cubic splines were used to interpolate between the nodes of the trajectory in an imposed task of the robot. In [5] the authors used the principle of Pontryagin to optimize the cost function.

This work is structured as follows: Section 2 presents the approach taken in [1] to reformulate the various constraints. Section 3 presents the reformulation of the cost function. Section 4 considers the Cubic Spline functions used to model the different segments. Section 5 presents the technique of genetic algorithms. The Section 6 presents the recapitulation of this method. The different results obtained and the conclusions are presented in Section 7 and 8 using a planar robot manipulator.

2. TREATMENT OF THE KINO-DYNAMIC CONSTRAINTS

To solve the problem of trajectory planning in the free tasks for an optimal trajectory Q(T), we carried out a standardization of the time scale that transforms the problem of a research on an interval of an indefinite terminal [0,T] towards another being equivalent whose terminal of research is known [0,1] and who will be easier to solve.

$$q(t) = Q(\xi(t)) \quad \text{Or: } \xi(t) = t/T \quad \text{with: } \xi \in [0,1] \tag{1}$$

We can break up this profile of trajectory into a way $P(\lambda)$ and movement on this way $\lambda(\xi)$, which will be formulated as following:

$$Q(\xi) = P(\lambda(\xi)) \tag{2}$$

Applying the normalization (1) of time scale for a given generalized trajectory $q(t)$, the generalized velocities $\dot{q}(t)$, generalized acceleration $\ddot{q}(t)$ and jerk $\dddot{q}(t)$ of the trajectory can be written as follows:

$$\dot{q}(t) = \frac{1}{T} Q'(\xi), \ \ddot{q}(t) = \frac{1}{T^2} Q''(\xi) \text{ and } \dddot{q}(t) = \frac{1}{T^3} Q'''(\xi) \tag{3}$$

2.1. Treatment of Kinematic Constraints

From the equation (3) the kinematic constraints can be formulated as following:

$$\textbf{Velocity. } \forall t \in [0,T]; |\dot{q}_i(t)| \le \dot{q}_i^{max} \Rightarrow T \ge \max_{i=1,\dots,n}\left[\max_{\xi\in[0,1]}\left(\frac{|Q_i'(\xi)|}{\dot{q}_i^{max}}\right)\right] \quad \text{Or: } T \ge T_V \tag{4}$$

$$\textbf{Acceleration. } \forall t \in [0,T]; |\ddot{q}_i(t)| \le \ddot{q}_i^{max} \Rightarrow T \ge \max_{i=1,\dots,n}\left[\max_{\xi\in[0,1]}\left(\frac{|Q_i''(\xi)|}{\ddot{q}_i^{max}}\right)^{1/2}\right] \quad \text{Or: } T \ge T_A \tag{5}$$

$$\textbf{Jerk. } \forall t \in [0,T]; |\dddot{q}_i(t)| \le \dddot{q}_i^{max} \Rightarrow T \ge \max_{i=1,\dots,n}\left[\max_{\xi\in[0,1]}\left(\frac{|Q_i'''(\xi)|}{\dddot{q}_i^{max}}\right)^{1/2}\right] \quad \text{Or: } T \ge T_J \tag{6}$$

For a given trajectory profile $Q(\xi)$, the optimal time T_Q must satisfy the kinematic constraints is:

$$T_Q \ge T^* \text{ With: } T^* = \max_{i=1,\dots,n}\left(\max_{\xi\in[0,1]}\left[\left(\frac{Q_i'(\xi)}{\dot{q}_i^{max}}\right), \left(\frac{Q_i''(\xi)}{\ddot{q}_i^{max}}\right)^{1/2}, \left(\frac{Q_i'''(\xi)}{\dddot{q}_i^{max}}\right)^{1/3}\right]\right) \tag{7}$$

2.2. Treatment of the Dynamic Constraints

The equation of the dynamic model of robot is written:

$$\tau_i(t) = \sum_{j=1}^{n} M_{ij}(q(t))\ddot{q}_j(t) + C_i(q(t),\dot{q}(t)) + G_i(q(t)) \tag{8}$$

Where M_{ij} is the inertia matrix, C_i is the vector of Coriolis and centrifugal forces, G_i is the vector of potential forces and τ_i is the vector of actuator efforts.

By using the equation (1) and the equations of the velocity and the accelerations (3) in the equations of the dynamic model (8) we obtain:

$$\bar{\tau}_i(\xi) = \frac{1}{T^2}\bar{h}_i(\xi) + \bar{G}_i(\xi) \quad \text{With: } \bar{\tau}_i(\xi) = \frac{\tau_i(\xi)}{\tau_i^{max}}; \bar{h}_i(\xi) = \frac{h_i(\xi)}{\tau_i^{max}}; \bar{G}_i(\xi) = \frac{G_i(\xi)}{\tau_i^{max}} \tag{9}$$

The solution of the equation (9) compared to Tfor a given value ξ gives an acceptable interval for T of lower limit T_{L_i} and higher limit T_{R_i} or $T \in [T_{L_i}, T_{R_i}]$, and the intersection of the intervals $[T_{L_i}, T_{R_i}]$ for $i = 1,...,n$ along the trajectory give acceptable interval for the time of displacement respecting dynamic constraints due to the torques:

$$T \in [T_g, T_d] \quad \text{With: } T_g = \max_{i=1,...,n}\left[\max_{\xi\in[0,1]} T_{L_i}(\xi)\right]; \quad T_d = \min_{i=1,...,n}\left[\min_{\xi\in[0,1]} T_{R_i}(\xi)\right] \tag{10}$$

If we denote I_{ad} the interval durations T which satisfy all kino-dynamic constraints:

$$I_{ad} = [T^*, \infty[\cap [T_g, T_d] = [I_{ad}^{inf}, I_{ad}^{sup}] \tag{11}$$

3. REFORMULATION OF THE COST FUNCTION

The cost function in our case represents a weighting between the time transfer and the Jerk, its formula is written:

$$J = \alpha\, T_f + (1-\alpha)\int_0^{T_f}\sum_{i=1}^{n}\left(\dddot{q}_i(t)\right)^2 dt \tag{12}$$

With α is a weight coefficient change between 0 and 1 according to the user needs can favor either the execution time of the task is the jerk. Using equation (3), the equation (12) becomes:

$$J = \alpha\, T_f + (1-\alpha)\int_0^{1}\sum_{i=1}^{n}\left(\frac{1}{T_f^3}Q_i'''(\xi)\right)^2 d\xi \tag{13}$$

With: $Q_i'''(\xi) = \frac{d^3Q}{d\lambda^3}\left(\frac{d\lambda}{d\xi}\right)^2 + 3\frac{d^2Q}{d\lambda^2}\frac{d^2\lambda}{d\xi^2}\frac{d\lambda}{d\xi} + \frac{dQ}{d\lambda}\frac{d^3\lambda}{d\xi^3}$

The time of displacement T_m which minimizes the cost function for the profile $Q(\xi)$ is:

$$T_m = \sqrt[7]{6\,\frac{S_2}{S_1}} \quad \text{With: } S_1 = \alpha; \quad S_2 = (1-\alpha)\int_0^1\sum_{i=1}^{n}\left(Q_i'''(\xi)\right)^2 d\xi \tag{14}$$

4. MODELING THE FUNCTIONS OF THE WAY AND THE MOVEMENT

We chose a model by Cubic Spline functions, these functions are composed by pieces of polynomials of three degree, and which will be written using the standardization of the time of equation (1) as following:

$$Q_i(\xi) = a_{0i} + a_{1i}(\xi - \xi_{i-1}) + a_{2i}(\xi - \xi_{i-1})^2 + a_{3i}(\xi - \xi_{i-1})^3 \quad \text{For: } \xi_{i-1} \le \xi \le \xi_i \tag{15}$$

And the derivatives of the joint variation $q_i(t)$ are:

$$\begin{cases} \dot{q}_i(t) = \frac{1}{T}\left(a_{1i} + 2a_{2i}(\xi - \xi_{i-1}) + 3a_{3i}(\xi - \xi_{i-1})^2\right) \\ \ddot{q}_i(t) = \frac{1}{T^2}\left(2a_{2i} + 6a_{3i}(\xi - \xi_{i-1})\right) \\ \dddot{q}_i(t) = \frac{1}{T^3}(6a_{3i}) \end{cases}$$

4.1. Function of the Way

This function is modeled by Cubic Splines called Natural (Figure 1), the boundary conditions imposed on the joint positions of the robot manipulator are:

$$P(\lambda = 0) = q^{ini} \quad \text{and} \quad P(\lambda = 1) = q^{fin} \tag{16}$$

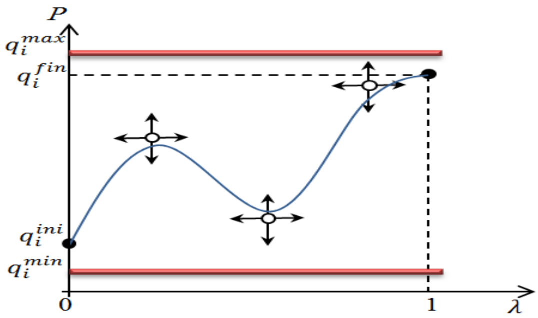

Figure 1. Representation on the profile of the path with $N_C + 2$ points of control

4.2. Function of the Movement

We have adopted to represent the profiles of this function by Cubic Splines called Clamped this model is well adopted to take the boundary conditions of velocities (Figure 2).

$$\lambda'(\xi = 0) = \lambda'(\xi = 1) = 0 \tag{17}$$

In addition, this profile of movement is composed by points of control placed in a standardized plan so that the first and the last point are fixed according to (18), while the interior points are placed freely according to the conditions (17) and (19):

$$\lambda(\xi = 0) = 0 \quad \text{and} \quad \lambda(\xi = 1) = 1 \tag{18}$$

$$\lambda'(\xi) \geq 0 \tag{19}$$

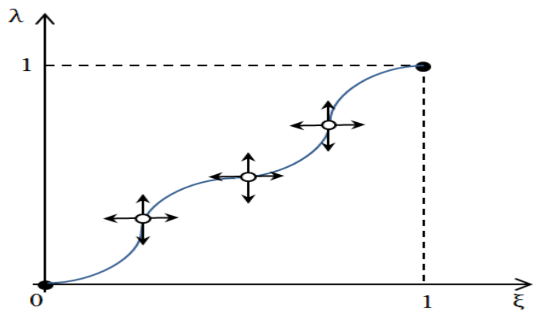

Figure 2. Representation on the profile of the movement with $N_C + 2$ points of control

5. OPTIMIZATION USING ALGORITHMS

The use of a genetic algorithm starts with the creation of an initial population or chromosome in genetics, this chromosome is composed by genes or their number is defined according to the number of the points of control used to generate the function way and at the same time the function movement. The steps used in our calculation code are as following:

1) Beginning. Selected chromosome number, chromosome size, the probability of crossover and the probability of mutation.

2) Initialization. Generate a random population of *n* chromosomes.

3) Cost function. Calculate $J_Q(T)$ ofeachchromosome.

4) New generation. Create a new population by repeating the following genetic operators:

Selection. Chromosomes with best cost function have the ability to select more.

Crossing. Create two children by making a mixture of chromosomes from both parents.

Mutation. This operator allows the emergence of new genes by exploring areas of the search space that could not be visited by a simple application of the crossing operator.

5) Test. If the initial conditions are satisfied, stop go to step 6, if not go to step 3.

6) Goal. Obtain the minimal value of $J_Q(T_Q)$.

6. RECAPITULATION OF THE METHOD OF RESOLUTION

To seek the optimal trajectory, we must generate by chance according to the genetic technique of optimization of the algorithms a profile of way and a profile of movement which will give us thereafter a profile of trajectory (Figure 3), candidate the latter will be evaluated thereafter and compared with other, this operation is repeated for all the introduced chromosomes, and the best result, it is that which satisfies the given criterion convergence. It should be noted that any profile of way which would violate one of the geometrical constraints, as any profile of trajectory which would violate one of the constraints kinematics or dynamics will be automatically rejected.

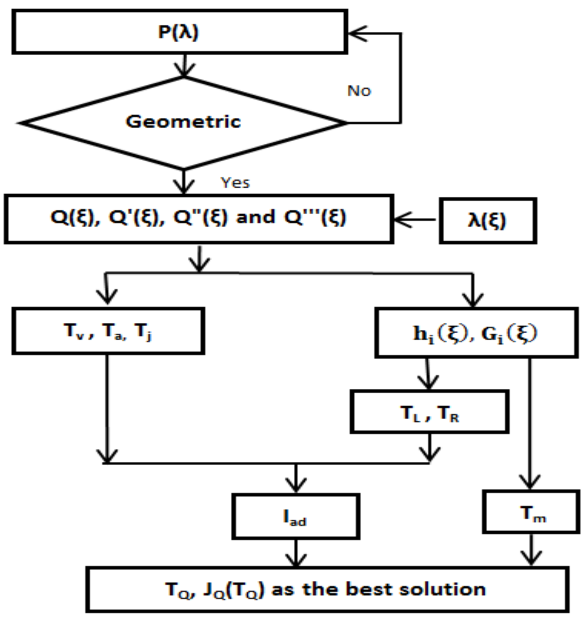

Figure 3. Flow chart of Resolution

7. RESULTS OF SIMULATION

We will consider a planar robot 3R, we ask him to carry out a displacement between the initial configurations $q_{init} = [0, -\pi/3, -\pi/10]^T$ to the final configurations $q_{fin} = [\pi, 0, 0]^T$ .We will fix the rate or the probability of crossing equal to 65% and the probability of mutation equal 4%. The collected results and the optimal aspects for the movement of a planar robot 3R are represented respectively in Figure 4 and Figure 5.

Table 1. The Parameters of Planar Robot 3R

Segment i	σ_i	L_i	α_i	d_i	θ_i	m_i[Kg]	x_{gi}[m]	I_{zi} [Kg.m^2]	q_i^{max} [rad]	$\dot{q}_i^{max}$ [rad/s]	$\ddot{q}_i^{max}$ [rad/s^2]	$\dddot{q}_i^{max}$ [rad/s^3]	τ_i^{max} [N.m]
1	0	0.7	0	0	q_1	7	0.35	0.8	π	3	8	10	30
2	0	0.5	0	0	q_2	5	0.5	0.5	3π/4	3	8	15	25
3	0	0.5	0	0	q_3	5	0.5	0.5	3π/4	3	8	20	25

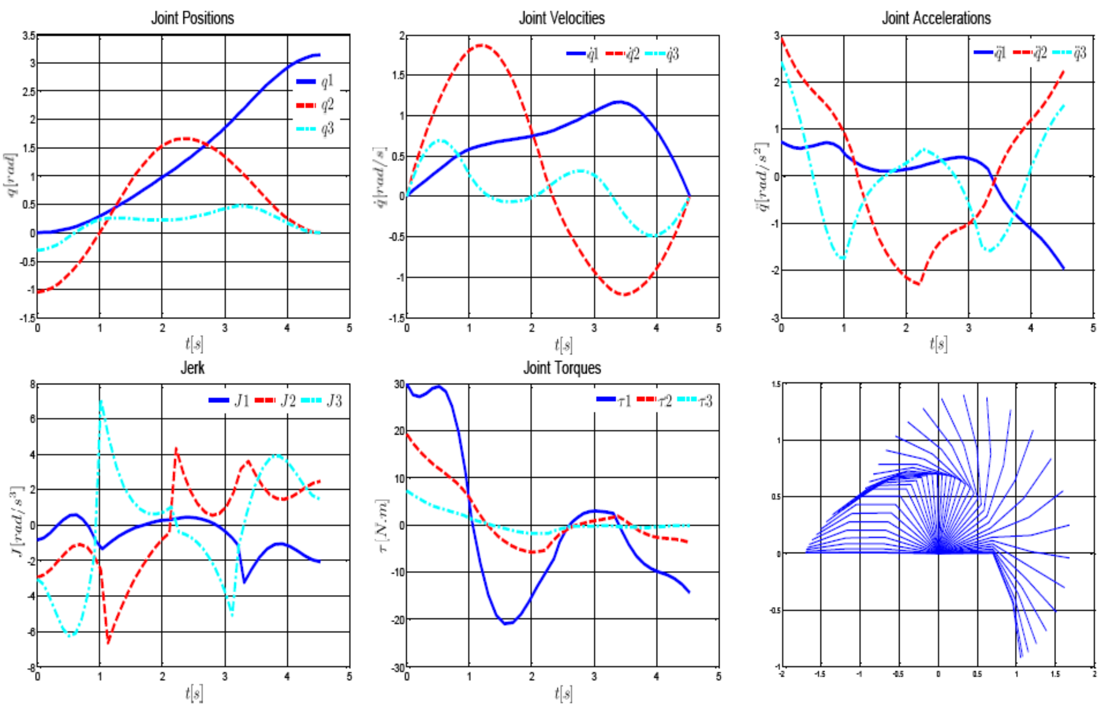

Figure 4. Results Positions, Velocities, Accelerations, Jerk, Torques, and successive configurations for a trajectory optimized of Planar Robot 3R

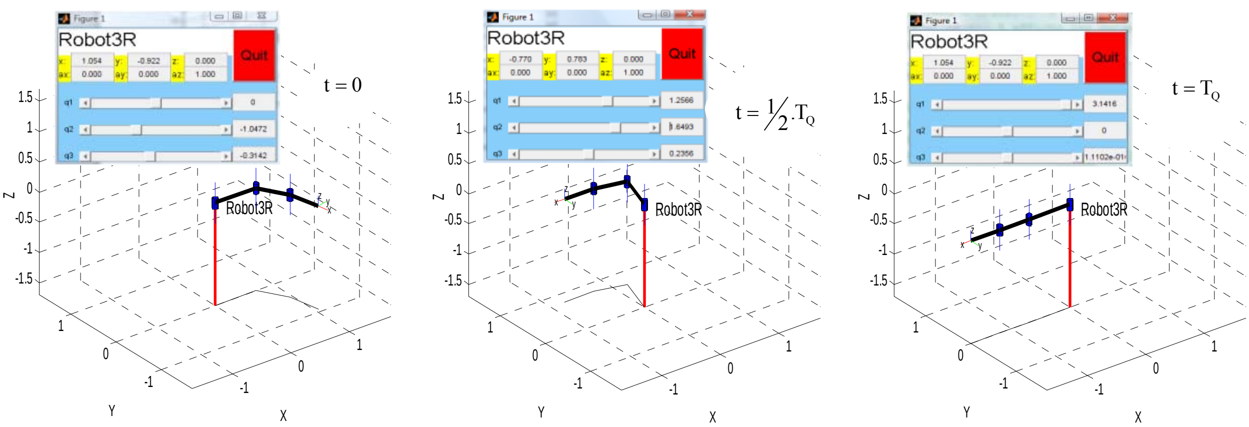

Figure 5. Optimal aspects for a movement of planar robot 3R

8. CONCLUSION

Through this work, we could show the possibility of this approach of giving us the sub-optimal results for the problems of optimal trajectory planning of the robot manipulators by minimizing a cost function which presents Time/Jerk and with imposed Kino-dynamic constraints. We could deal with a problem of trajectory planning in Pick and Place operations by using Cubic Splines functions who allow to guarantee the smoothing of the trajectory and at the same time the continuity of the velocities, the accelerations, and Jerk for a Planar Robot manipulator 3R, noting that the time execution of the code computer requires 34 seconds by using 100 chromosomes in genetic algorithms and on a PC of 2Ghz.

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