

## Research and application on fractional-order Darwinian PSO based adaptive extended kalman filtering algorithm\*

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### ABSTRACT

To resolve the difficulty in establishing accurate priori noise model for the extended Kalman filtering algorithm, propose the fractional-order Darwinian particle swarm optimization (PSO) algorithm has been proposed and introduced into the fuzzy adaptive extended Kalman filtering algorithm. The natural selection method has been adopted to improve the standard particle swarm optimization algorithm, which enhanced the diversity of particles and avoided the premature. In addition, the fractional calculus has been used to improve the evolution speed of particles. The PSO algorithm after improved has been applied to train fuzzy adaptive extended Kalman filter and achieve the simultaneous localization and mapping. The simulation results have shown that compared with the geese particle swarm optimization training of fuzzy adaptive extended Kalman filter localization and mapping algorithm, has been greatly improved in terms of localization and mapping.

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### 1. INTRODUCTION

Simultaneous location and mapping (SLAM) means that when a robot moves in an unknown environment, during the movement, the location can be achieved according to the position estimation and sensor observation and the surrounding environment map can be established simultaneously, which makes up the precondition of autonomous operation for mobile robots. Therefore, the SLAM problem remains one of research hotspots of the robot field.

So far, the general algorithms solving SLAM problems include extended Kalman filtering, particle filtering, unscented Kalman filtering, and so on [1, 2]. Among those algorithms, extended Kalman filtering

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and some improved extended Kalman filter are used more commonly. Extended Kalman filter has good nonlinear and mathematical rigor, however it needs to obtain the priori information, such as the system noises and the observation noises, in which the system noises can be obtained through experiments, while because of the great influences for environmental factors, the observation noises are hard to get in advance. Insufficiency and inaccuracy of priori information may lead to lower filtering accuracy and even divergence. Therefore, fuzzy logic has been used to adjust the observation noise of extended Kalman filter algorithm in reference [3], but the fuzzy rules are selected through experience, which may cause a certain deviation in the application of dynamic environment. In reference [4], PSO algorithm has been used to train fuzzy rules to obtain the proper parameters of membership functions, but it is easy to fall into premature convergence and local optimal problem. For the disadvantages of PSO algorithm, a lot of improved PSO algorithms have been put forward, such as the geese particle swarm algorithm [5], chaotic particle swarm algorithm [6], discrete particle swarm algorithm [7]. These improved algorithms increase the diversity of particles to a certain extent, and the problem of premature convergence and local optimal has been improved.

In this paper, the fractional-order Darwinian particle swarm algorithm has been introduced to optimize the key parameters of fuzzy logic. The natural selection method has been adopted to improve the standard particle swarm optimization algorithm, which enhanced the diversity of particles and avoided the premature. In addition, the fractional calculus has been used to improve the evolution speed of particles. The PSO algorithm after improved has been applied to train fuzzy adaptive extended Kalman filter and achieve the simultaneous localization and mapping.

## 2. FRACTIONAL-ORDER DARWINIAN PARTICLE SWARM ALGORITHM

### 2.1. Darwinian Particle Swarm Algorithm

Particle swarm optimization algorithm originates from the research on the group moving behaviors of bird flocks and fish schools, and has been widely used to solve the complex optimization problems. Jason Tillett and T.M. Rao put forward that Darwinian natural selection in biology can be used to improve particle swarm algorithm [8]. The basic idea of Darwinian particle swarm algorithm is: at each moment there are several particle swarms searching simultaneously. Each particle swarm can run in accordance with standard particle swarm algorithm, and some rules are added to simulate natural selection during the running, in which the appropriate swarm would be chosen in the continuous swarm combination.

In the searching process, when one of the swarms tends to fall into the local optimum, the searching in this area would be discarded immediately and turn to the searching in other areas. The particle swarms with better fitness will get rewards, otherwise, those tend to stagnate will be punished. Removing a particle (or a particle swarm) needs to comply with the following rules:

(1) When the particle number of the swarm is less than the specified minimum number of particles, it will be removed.

(2) If the optimal fitness of a particle swarm has not been improved within the predetermined searching number, the particle with the worst fitness will be removed. In addition, the searching will not be reset to 0, but reset to a value close to the maximum searching number. It can be described as the following formula:

$$SC_c(N_{kill}) = SC_c^{\max} \left[ 1 - \frac{1}{N_{kill} + 1} \right] \quad (1)$$

The generation of a new particle swarm must meet with two conditions: No particle has been removed from this particle swarm and particle swarm number has not reached the maximum. Even if these two conditions are met, the emerging probability of a new particle swarm is  $p = f/NS$ ,  $f$  is a random number between 0 and 1, NS is particle swarm number. The function of this condition is to restrict the generation of new particle. When a new particle swarm is generating, the particles of parent swarm have not got affected. In children swarms, half of particles are randomly selected from parent swarms, the other half are randomly selected particles. If the number of a particle swarm can not reach the initial number of particles, then the swarm's other particles could be initialized randomly and added to a new particle swarm. When a particle swarm reaches a new optimal fitness value while the particle number of this swarm has not reached the maximum, a new particle swarm emerges [9].

Like the standard PSO algorithm, many parameters of Darwinian PSO also need to set reasonably to make the algorithm better performance, including:

- (1) Particle number of the initialization particle swarms.
- (2) The minimum and maximum particle number that a swarm allow to exist.
- (3) The number of initialization particle swarms.
- (4) The minimum and maximum number of the particle swarm that allowed to exist.
- (5) The threshold that the searching ends.

## 2.2. Improved Darwinian Particle Swarm Algorithm using Fractional Calculus

The time domain fractional-order calculus equation [10] defined by Grunwald-Letnikov is

$$D^\alpha [x(t)] = \lim_{h \rightarrow 0} \left[ \frac{1}{h^\alpha} \sum_{k=0}^{+\infty} \frac{(-1)^k (\alpha + 1) x(t - kh)}{\Gamma(k+1)\Gamma(\alpha - k + 1)} \right] \quad (2)$$

From the Equ (2), it has been shown that integer-order derivative contains finite series, while fractional-order derivative contains infinite series. The most significant difference between fractional-order calculus and integer-order calculus is that fractional-order calculus is related to all the points' past information, is a global operator.

Formula of fractional-order derivative used in discrete time is approximate as

$$D^\alpha [x(t)] = \frac{1}{T^\alpha} \sum_{k=0}^r \frac{(-1)^k \Gamma(\alpha + 1) x(t - kT)}{\Gamma(k+1)\Gamma(\alpha - k + 1)} \quad (3)$$

Where, T represents the sampling period, r represents the stop order.

Using the fractional-order calculus, the updating velocity of Darwinian PSO can be improved, and the velocity updating formula of Darwin PSO can be rearranged as [11]

$$v_{id}^{t+1} - \omega \cdot v_{id}^t = c_1 r_1 (p_{id}^t - x_{id}^t) + c_2 r_2 (p_{(i-1)d}^t - x_{id}^t) \quad (4)$$

Suppose that  $\omega = 1$ , the left side of the equation is a derivative in discrete form, order number  $\alpha = 1$  (suppose  $T = 1$ ), then the formula above turns to

$$D^\alpha [v^{t+1}] = c_1 r_1 (p_{id}^t - x_{id}^t) + c_2 r_2 (p_{(i-1)d}^t - x_{id}^t) \quad (5)$$

Using fractional-order calculus idea, velocity derivative's order can be extended to real number in the limits of  $0 \leq \alpha \leq 1$ , which will cause the changes more stable and the memory effect more longer. In order to discuss the effect of  $\alpha$ 's value on the performance of improved PSO algorithm, make  $\alpha$  changes from 0 to 1, step length  $\Delta\alpha = 0.1$ , to calculate the optimal solution of some functions. According to the experiments, the performance can achieve the best when  $\alpha = 0.6$ . As the fractional-order calculus is infinite dimensional, its "infinite memory" characteristics lead to the difficulties of digital realization. Existing simulation tools can't deal with non integer-order calculus directly, so when fractional-order calculus is used, it is necessary to approximate it with the finite dimensional function [12]. If we consume  $r = 4$ , in which only the first four terms are being considered, the equation above can turns into

$$v^{t+1} - \alpha v^t - \frac{1}{2} \alpha v^{t-1} - \frac{1}{6} \alpha (1 - \alpha) v^{t-2} - \frac{1}{24} \alpha (1 - \alpha) (2 - \alpha) v^{t-3} = c_1 r_1 (p_{id}^t - x_{id}^t) + c_2 r_2 (p_{(i-1)d}^t - x_{id}^t), \quad (6)$$

That is

$$v^{t+1} = \alpha v^t + \alpha v^t + \frac{1}{2} \alpha v^{t-1} + \frac{1}{6} \alpha (1 - \alpha) v^{t-2} + \frac{1}{24} \alpha (1 - \alpha) (2 - \alpha) v^{t-3} + c_1 r_1 (p_{id}^t - x_{id}^t) + c_2 r_2 (p_{(i-1)d}^t - x_{id}^t) \quad (7)$$

After many simulations we can conclude that the increase of  $r$  do little to improve the performance of the algorithm.

### 3. FUZZY ADAPTIVE EXTENDED KALMAN FILTER SLAM ALGORITHM

SLAM algorithm based on EKF mainly contains two steps: prediction process and observation update process. First, establish the system model. The spatial environment is expressed as

$$x(k) = [x_v(k), x_m(k)]^T \quad (8)$$

Where,  $x_v(k)$  represents robot's pose at time k,  $x_m(k)$  represents a feature location of the map.

Prediction process:  $u(k)$  is defined as robot control vector at any given time k.  $z(k)$  is the observation value.  $Q(k)$  and  $R(k)$  are covariance matrices of systematic noise and observation noise. The prediction equations are

$$x(k+1/k) = F(k)x(k/k) + u(k) \quad (9)$$

$$z(k+1/k) = H(k)x(k+1/k) \quad (10)$$

$$p(k+1/k) = F(k)p(k/k)F^T(k) + Q(k) \quad (11)$$

In these equations,  $F(k)$  and  $H(k)$  are the system state transition matrix and the observation matrix at time k.

Observation update process: observation value  $z(k+1)$  and prediction value at time k+1 are used to update the spatial environment.

$$v(k+1) = z(k+1) - z(k+1/k) \quad (12)$$

$$s(k+1) = H(k)p(k+1/k)H^T(k) + R(k+1) \quad (13)$$

$$x(k+1/k+1) = x(k+1/k) + K(k+1)v(k+1) \quad (14)$$

$$p(k+1/k+1) = p(k+1/k) - K(k+1) \times s(k+1)K^T(k+1) \quad (15)$$

$$K(k+1) = p(k+1/k)H^T(k)s^{-1}(k+1) \quad (16)$$

Where,  $v(k+1)$  is the innovation at time k+1;  $s(k+1)$  is the theoretical covariance matrix of innovation.

The difference of actual covariance matrix of innovation and theoretical covariance matrix of innovation is  $\Delta c_{lnnk}$ , use diagonal elements of  $\Delta c_{lnnk}$  to adjust the diagonal elements in covariance matrix of observation noise, that is the fuzzy adaptive extended kalman filter.

$$\Delta c_{lnnk} = v(k)v^T(k) - s(k) \quad (17)$$

Use fractional-order Darwinian PSO to train fuzzy system, there are three membership functions in fuzzy system, so the variable of membership functions has 9 dimensions, which is  $x = [a_1 \ a_2 \ a_3 \ b_1 \ b_2 \ b_3 \ w_1 \ w_2 \ w_3]^T$ . The purpose is to obtain the minimum difference of actual covariance and theoretical covariance. Therefore, the object function is

$$f = \frac{\sum_{n_{obs}}^{N_{obs}} \left( \frac{\sum_{j=1}^{j_{cnobs}} |\Delta c_{lnnk}|^2}{j_{cnobs}} \right)}{N_{obs}} \quad (18)$$

Where,  $N_{obs}$  is the total number of observation in iteration,  $j_{cnobs}$  is diagonal elements ' number of  $\Delta c_{lnnk}$ .

#### 4. ANALYSIS OF EXPERIMENT RESULTS

We can assume that the mobile robot 's simulation environment is a plane rectangular area, in this environment there are 28 feature points and 14 path points, which respectively represented by "x" and "o". Figure 1 is the map that gets by the use of fuzzy adaptive extended Kalman filter SLAM algorithm based on the geese, while figure 2 is the map that gets by using fuzzy adaptive extended Kalman filter SLAM algorithm based on the fractional-order Darwinian PSO. Red "+" in the picture are the feature points ' positions built by the robot.

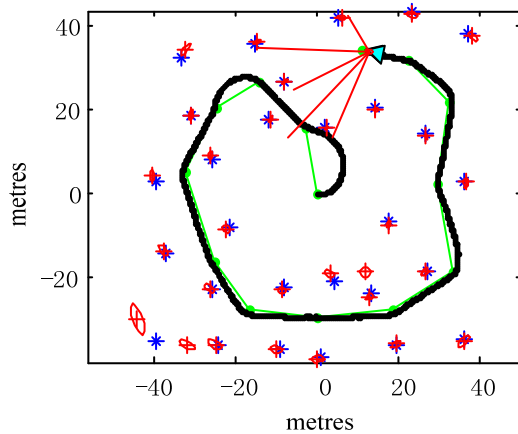


Figure 1. SLAM based on the geese PSO

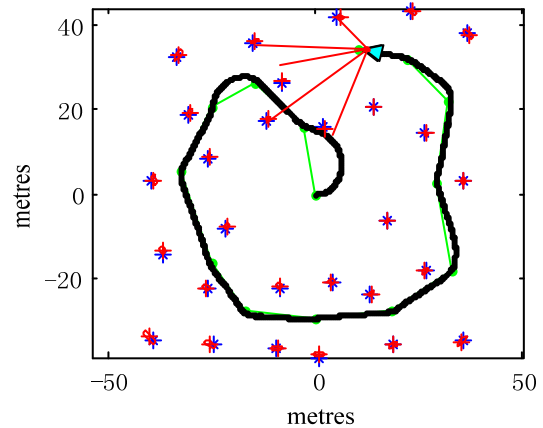


Figure 2. SLAM based on the fractional-order Darwinian PSO

As is shown from the two figures mentioned above, the fuzzy adaptive extended Kalman filter SLAM algorithm based on the fractional-order Darwinian PSO performs much better than fuzzy adaptive extended Kalman filter SLAM algorithm based on the geese in feature points estimation and the robot's pose. In figure 1, the location of estimated feature points has a large deviation to the actual feature points, and there are many points are mistaken as feature points. In figure 2, the location of estimated feature points and actual feature points are much improved in compared with figure 1, the number of error points decreases a lot.

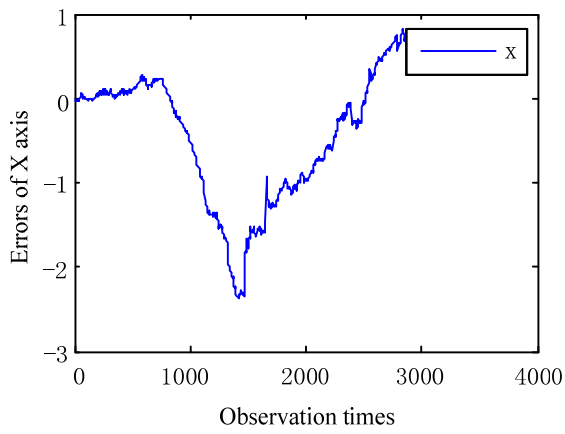


Figure 3. Location errors of SLAM the geese PSO in X direction based on

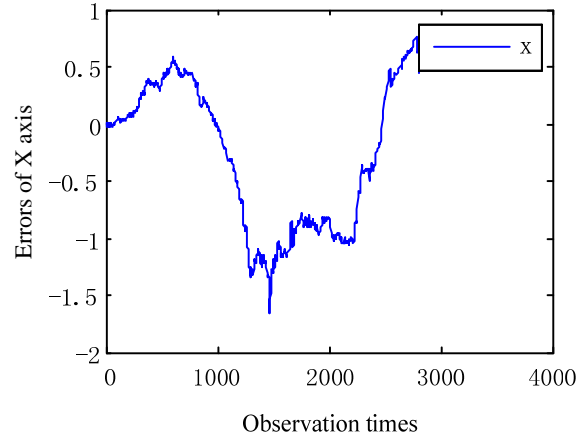


Figure 4. Location errors of SLAM based on the fractional-order Darwinian PSO in X direction

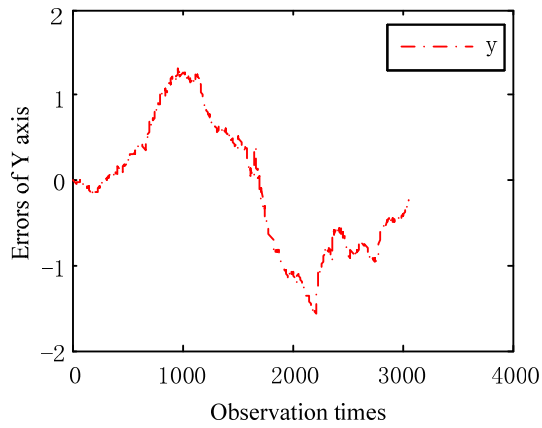


Figure 5. Location errors of SLAM based on the geese PSO in Y direction

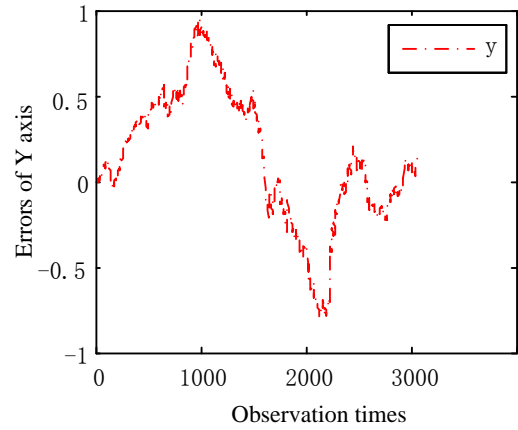


Figure 6. Location errors of SLAM based on the fractional-order Darwinian PSO in X direction

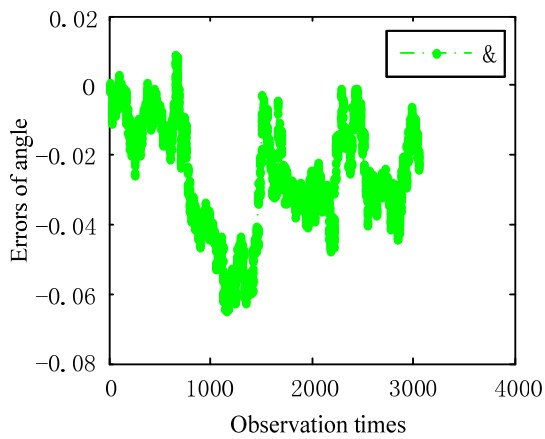


Figure 7. Location errors of SLAM based on the geese PSO in angle

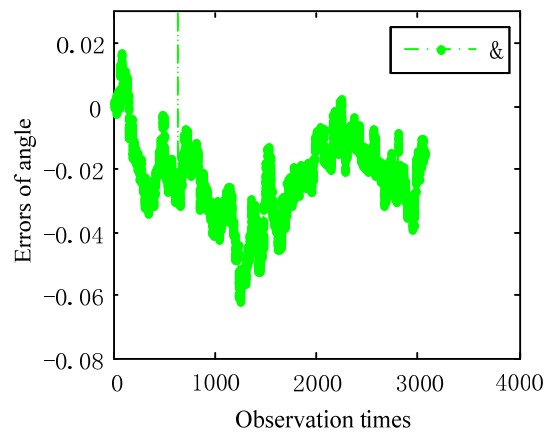


Figure 8. Location errors of SLAM based on the fractional-order Darwinian PSO in angle

As is shown in Figure 3 to Figure 8, comparing the pose (or angle) error estimated by fuzzy adaptive extended Kalman filter SLAM algorithm based on the fractional-order Darwinian PSO and fuzzy adaptive extended Kalman filter SLAM algorithm based on the geese algorithm, the maximum of robot location errors in X direction has decreased from 2.5 to 1.5, while maximum value of errors in Y direction has dropped from 1.5 to 1.0, and the maximum angle error has decreased from 0.07 to 0.06, the performance of fuzzy adaptive extended Kalman filter SLAM algorithm has been improved greatly.

## 5. CONCLUSION

To overcome the shortcoming of the inaccuracy for the prior characteristics of system noise and observation noise is inaccurate in extended Kalman filter SLAM, using fuzzy adaptive extended Kalman filter SLAM algorithm, the noises could be fuzzily regulated online by detecting the differences of covariance matrix of prediction and actual value difference and ideal covariance matrix, therefore the effect that time-varying noises posed on filter stability can be restrained. However, fuzzy logic not only has a lower accuracy, but also lacks systematic parameter design methods. For this defect, natural selection in biology and fractional calculus are used to improve particle optimization, and fractional-order Darwinian particle swarm algorithm is used to optimize the key parameters of fuzzy logic, thus regulation accuracy of fuzzy logic could be improved, as well as the performance of Kalman filter SLAM. According to the simulations, the performance of fuzzy adaptive extended Kalman filter SLAM algorithm based on the fractional-order Darwinian PSO has been improved obviously, and this improved strategy can be applied to the optimization of other swarm intelligence algorithms.

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