❒ 252

A LQR Optimal Method to Control the Position of an Overhead Crane

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Article Info	ABSTRACT
	In this paper, a LQR (Linear Quadratic Regulation) optimal method is implemented to control position of an overhead carne. To do this, a tracking formulation of LQR is developed and applied to the system. Hence the dynamic model of the overhead crane is presented, the dynamic of the actuator motor of the trolley is considered. As the parameters of the optimal controller assigned, some simulations are done to show the efficiency of the proposed method.

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1. INTRODUCTION

Cranes are extensively applied in transportation and construction fields [1, 2]. Overhead cranes are one kind of Cranes used to transport arbitrary load form a position to another one. The overhead crane consists of a cart or trolley which moves along its rail. Moreover, a hoisting mechanism including a cable and a payload is attached to the cart. The overhead cranes have extensively used in many industries, because these systems exhibit novel features such as low cost, easy assembly, and less maintenance [3-5]. Therefore, the overhead cranes have attracted a great deal of interests, and the dynamic modeling and control of such systems are studied by some researchers. Hubbel et al. [6] used an open-loop method to control the motion of a gantry crane. In this method, the input control profile was determined in such way that unwanted oscillations and residual pendulations were avoided. However their approach was applicable, but the open-loop control scheme is not robust to disturbances and parameter uncertainties [7]. Moreover, a feedback PID anti-swing controller is developed in [8] to control of an overhead crane. Ahmad et al. [9] used a hybrid input-shaping method to control of the carne. Wahyudi and Jalani [10] employed fuzzy logic feedback controller to control an intelligent crane. Moreover, presented an optimal control method is used in [11] to control the dynamic motion of the system. Here in, minimum energy of system and integrated absolute error of payload angle are assumed as their optimization criterion. Zhao and Gao [12] studied the control of an overhead crane. They proposed a fuzzy method to control the input delay and actuator saturation of the system. Nazemizadeh et al. [13] studied tracking control of an underactuated gantry crane. Furthermore, Nazemizadeh [14] presented a PID tuning method for tracking control of a crane.

In this paper, a LQR (Linear Quadratic Regulation) optimal method is implemented to control position of an overhead carne. To do this, a tracking formulation of LQR is developed and applied to the system. Hence the dynamic model of the overhead crane is presented, the dynamic of the actuator motor of the trolley is considered. As the parameters of the optimal controller assigned, some simulations are done to show the efficiency of the proposed method.

In this article, the position control of the overhead crane is studied based on optimal control strategy. The dynamic equations of the system are derived, considering the motor voltage of a wheel of the trolley as the input, and displacement of the trolley as the output of the crane. To control the position of the cart, an LQR method is used. To verify the proposed method, some simulation results are done and presented.

2. LQR OPTIMAL CONTROL OF THE SYSTEM

In this section, the LQR control method is applied to the system. Presume the dynamic equation of the overhead crane in state-space from can be written as:

$$\begin{cases} \dot{X} = AX + Bu \\ \quad y = CX \end{cases}$$ (1)

Where X is the state vector of the system, u is the input effort, y is the output, and A, B, C are the coefficient matrices of the system.

Furthermore the final position of the cart can be defined as $r(t)$, and related to the final state vector X_e and final input of the system u_e by Eq. (2):

$$0 = AX_e + Bu_e$$
$$r(t) = CX_e$$ (2)

Thus, combing Eqs. (1) , (2) results in:

$$\dot{\overline{X}} = A\overline{X} + B\overline{u}$$
$$y = C\overline{X}$$ (3)

Where $\overline{X} = X - X_e, \overline{u} = u - u_e, \overline{y} = y - r$ are assumed as reformatted vectors.

Furthermore, to apply the LQR optimal controller, an objective function is considered as follows:

$$J = \int_0^\infty \left(\overline{X}^T Q \overline{X} + u^T R u \right) dt$$ (4)

Where Q and R are weighting matrices of optimal controller which is defined by the user.

Using LQR method, the optimal feedback law is $u = -K\overline{X} = -R^{-1}B^T P\overline{X}$, where can be achieved from Riccati's equation [15]:

$$A^T P + PA - PBR^{-1}B^T P + Q = 0$$ (5)

Where is defined as a positive matrix.

3. DYNAMIC MODELING OF THE SYSTEM

In this section, the dynamic modeling of the overhead carne is presented. The dynamic equations of the system are derived using Lagrange's principle. Figure 1 shows an overhead crane. The crane is consists of a cart transverses in horizontal direction, while a pendulum connects on the cart and hoists the payload.
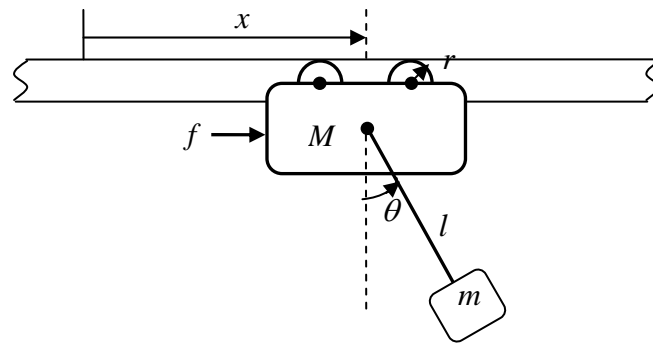
Figure 1. The overhead crane

The parameters of the system are: $x, \dot{x}$ are the cart position and speed, $\theta, \dot{\theta}$ are the pendulum angular displacement, l is the pendulum length, M is the mass of the cart, m is the payload mass, r is the radius of the wheels of the cart, e is the DC motor voltage of the cart, R is the motor armature resistance, k is the motor torque constant, B_p is the viscous damping coefficient of the pendulum axis, B_{eq} is the equivalent viscous damping coefficient, and g is the gravitational constant of earth.

To derive the equation of the motion, the kinetic energy of the cart T_1 and the kinetic energy of the pendulum T_2 are:

$$T_1 = \frac{1}{2} M \left(V_{1,x}^2 + V_{1,y}^2 \right) = \frac{1}{2} M \, \dot{x}^2$$
$$T_2 = \frac{1}{2} m \left(V_{2,x}^2 + V_{2,y}^2 \right) = \frac{1}{2} m \left[\dot{x}^2 + \dot{l}^2 + (l\dot{\theta})^2 + 2\dot{x}\dot{l} \sin\theta + 2\dot{x}l\dot{\theta}\cos \right.$$

(6)

Furthermore, the potential energy of the payload is:

$$U_2 = -mgl \cos\theta$$

(7)

To derive the dynamic equation of the system, the Lagrangian function is stated as:

$$L = T_1 + T_2 - U_2 = \frac{1}{2} m \left[\dot{x}^2 + \dot{l}^2 + (l\dot{\theta})^2 + 2\dot{x}\dot{l} \sin\theta + 2\dot{x}l\dot{\theta}\cos\theta \right]$$
$$+ \frac{1}{2} M\dot{x}^2 + mgl \cos\theta$$

(8)

And the damping force of the system is:

$$Q_{x,lost} = B_{eq}\dot{x}$$
$$Q_{\theta,lost} = B_p l\dot{\theta}$$

(9)

The Lagrange's principle is written as:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j} \right) - \frac{\partial L}{\partial q_j} = Q_j - Q_{j,lost}$$

(10)

Where q_j is the generalized coordinate of the system, Q_j is the generalized force exerted to the corresponding generalized coordinate, and $Q_{j,lost}$ is the damping force.

Then using Lagrange's principle, the nonlinear equations of the system can be achieved [16]:

$$(M+m)\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = f$$
$$\ddot{x}\cos\theta + l\ddot{\theta} + g\sin\theta = 0$$

(11)

On the other hand, the linear force of the cart f is originated from the torque of the DC motor. Therefore, the related equations are:

$$T = rf$$
$$T = \frac{-k^2}{R}\omega + \frac{k}{R}e$$
$$\dot{x} = r\omega$$

(12)

Where T is the torque of the actuator, and ω is the angular velocity of the motor. Thus, from Eqs. (11) and (12), we have:

$$(M+m)\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = \frac{1}{r}\left(\frac{-k^2}{Rr}\dot{x} + \frac{k}{R}e\right)$$
$$\ddot{x}\cos\theta + l\ddot{\theta} + g\sin\theta = 0$$

(13)

To present the nonlinear equations of the system in state-space form, the state vector is defined as $\vec{X} = \begin{bmatrix} x & \dot{x} & \theta & \dot{\theta} \end{bmatrix}$, and the nonlinear equations are:

$$
\begin{cases}
\dot{x} = \dfrac{d}{dt}(x) \\[2mm]
\ddot{x} = \dfrac{ml^2\dot{\theta}^2\sin\theta + (M+m)gl\tan\theta + l\left(\dfrac{+ke}{Rr} - \dfrac{k^2\dot{x}}{Rr^2}\right)}{M + m(1-\cos^2\theta)} - g\tan\theta \\[4mm]
\dot{\theta} = \dfrac{d}{dt}(\theta) \\[2mm]
\ddot{\theta} = \dfrac{-ml\dot{\theta}^2\sin\theta\cos\theta - (M+m)g\sin\theta - \cos\theta\left(\dfrac{+ke}{Rr} - \dfrac{k^2\dot{x}}{Rr^2}\right)}{Ml + ml(1-\cos^2\theta)}
\end{cases}
$$

(14)

Moreover, to Use LQR method, the linearization of the nonlinear equations must be done. Using the linearization method, the state-space linearized Equation of the systems are obtained as:

$$\dot{X} = AX + Bu$$
$$y = CX + Du$$

(15)

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\dfrac{k^2}{Rr^2 M} & \dfrac{mg}{M} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \dfrac{k^2}{Rr^2 Ml} & -\dfrac{(M+m)g}{Ml} & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ \dfrac{k}{RrM} \\ 0 \\ -\dfrac{k}{RrMl} \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \qquad D = 0$$

4. SIMULATIONS AND RESULTS

In this section, the LQR control of the overhead crane is simulated. The parameters of the systems are: $l = 0.3302m$, $M = 1.073kg$, $m = 0.23kg$, $r = 0.006m$, $R = 2.6\Omega$ e, $k = 0.00767Vs/rad$, $e_{max} = 12V$, and $g = 9.81m/s^2$ [17].

As it is mentioned, the desired criterions of the control design are: the trolley can set the final position while the swaying of the pendulum is damped quickly, and the input voltage of the motor does not exceed its maximum value. For arbitrary Q and R weighting matrices, the LQR feedback controller gains are shown in Table 1.

Table 1. LQR controller gains

Q	R	Controller gain
$diag(1)$	1	$K = \begin{bmatrix} 1 & 1.5735 & -0.97281 & -0.83742 \end{bmatrix}$
$diag(0.1)$	1	$K = \begin{bmatrix} 0.31623 & 0.57139 & -0.17422 & -0.22028 \end{bmatrix}$
$diag(1)$	0.1	$K = \begin{bmatrix} 3.1623 & 4.3898 & -5.7902 & -2.7238 \end{bmatrix}$

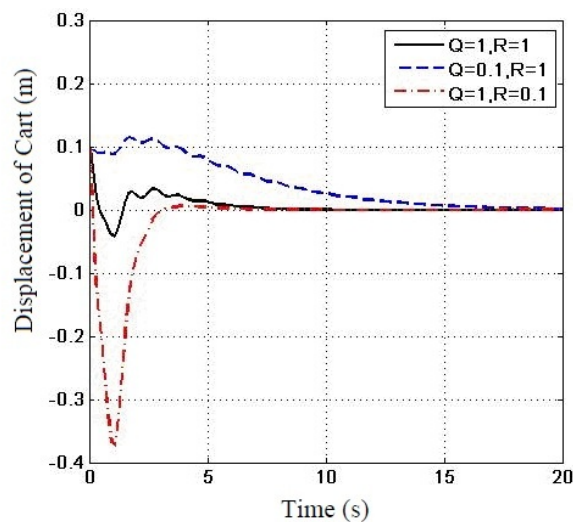And the simulation results are depicted.

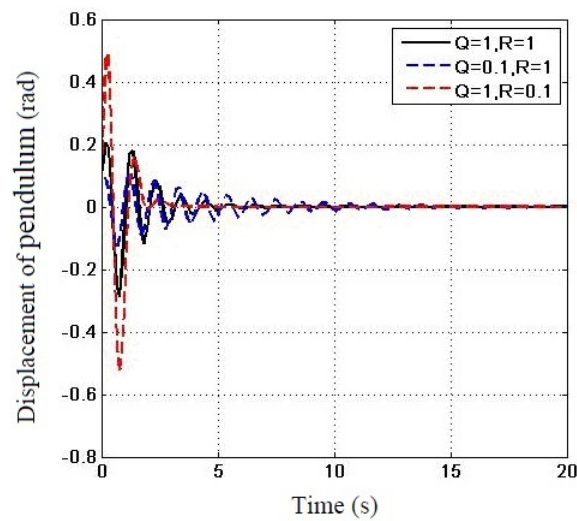Figure 2. The displacement of the cart
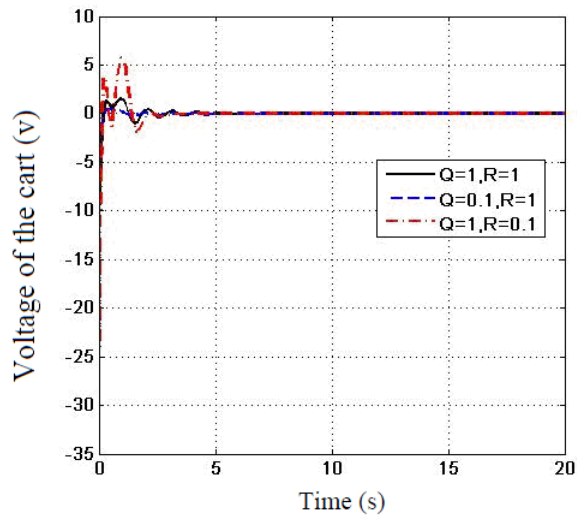
Figure 3. The displacement of the pendulum

Figure 4. The voltage of the cart

As it is seen in the forgoing figures, increasing the weighting of the state vector (Q), and decreasing the weighting of the input (R) leads to decreasing the maximum of the cart position and increasing of the maximum values of the input controller. Therefore, one can choose appropriate values of Q and R to obtain desired results.

5. CONCLUSION

In this paper, the position control of the overhead crane has been investigated using LQR optimal control method. At first, the nonlinear dynamic equations of the system have been derived via Lagrange's principle, and then the dynamic of the DC motor has been applied to the system. The voltage of the actuator of the trolley has been assumed as the input, and displacement of the trolley has been presumed as the output of the system. To control the position of the cart, the LQR method has been developed and some simulations have been done. It is concluded that increasing the weighting of the state vector (Q) or decreasing the weighting of the input (R) leads to decreasing the maximum of the cart position and increasing of the maximum values of the input controller. Furthermore, simulation results properly demonstrated the power and efficiency of the proposed approach.

REFERENCES

[1] Jingsong, L.I., Su, H., & Liu, Q. (2013). A Three Dimensional Simulation Method of the Gantry Crane. *TELKOMNIKA Indonesian Journal of Electrical Engineering*, 11(4), 2187-2194.

[2] Zheng, X., Zeng, J., Zhang, H., & Xie, Z. (2014). Research of Embedded Tower Crane Monitoring System Based on FCS. *TELKOMNIKA Indonesian Journal of Electrical Engineering*, 12(2), 1046-1052.

[3] Zhao, Y., & Gao, H. (2012). Fuzzy-model-based control of an overhead crane with input delay and actuator saturation. *Fuzzy Systems, IEEE Transactions on*, 20(1), 181-186.

[4] Vukov, M., Van Loock, W., Houska, B., Ferreau, H.J., Swevers, J., & Diehl, M. (2012, June). Experimental validation of nonlinear MPC on an overhead crane using automatic code generation. In *American Control Conference (ACC)*, 2012 (pp. 6264-6269). IEEE.

[5] Moon, S.C., Lee, W.G., & Lee, S.G. (2013). Adaptive sliding mode control of overhead cranes with varying cable length. *Journal of Mechanical Science and Technology*, 27(3), 885-893.

[6] Hubbell, J.T., Koch, B. and McCormick, D. 1992. Modern crane control enhancements. ASCE, USA: Seattle

[7] Masoud, Z.N. 2009. Effect of hoisting cable elasticity on anti-sway controllers of quay-side container cranes. *Journal of Nonlinear Dynamics*, 58: 129-140.

[8] Solihin, M.I., & Legowo, A. (2010). Fuzzy-tuned PID anti-swing control of automatic gantry crane. *Journal of Vibration and Control*, 16(1), 127-145.

[9] Ahmad, M. A., & Mohamed, Z. (2009). Hybrid fuzzy logic control with input shaping for input tracking and sway suppression of a gantry crane system. *American Journal of Engineering and Applied Sciences*, 2(1), 241.

[10] Wahyudi, M. and Jalani, J. 2005. *Design and implementation of fuzzy logic controller for an intelligent gantry crane system*. Proceedings of International Conference on Mechatronics, pp. 345- 351.

[11] Wang, Z. and Surgenor, B. 2004. Performance evaluation of the optimal control of a gantry crane. *ASME Conference on Engineering Systems Design and Analysis*, pp. 869-874.

[12] Zhao, Y., & Gao, H. (2012). Fuzzy-model-based control of an overhead crane with input delay and actuator saturation. *Fuzzy Systems, IEEE Transactions on*, 20(1), 181-186.

[13] Bakhtiari-Nejad F., Nazemizadeh M., and Arjmand H. (2013). Tracking control of an underactuated gantry crane using an optimal feedback controller. *International Journal of Automotive & Mechanical Engineering*, 7, 830- 839.

[14] Nazemizadeh, M. (2013). A PID Tuning Method for Tracking Control of an Underactuated Gantry Crane. *Universal Journal of Engineering Mechanics*, 1, 45-49.

[15] Ogata K (2002) Modern Control Engineering. Aeeizb Publisher.

[16] Dey, R., Sinha, N., Chaube, P., Ghosh, S. and Ray, G. 2010. Active sway control of a single pendulum gantry crane system using output-delayed feedback control technique. International Conference on Control, Automation, Robotics and Vision, pp. 532-536.

[17] Iles, S., Koloni, F. and Matusko, J. 2011. Linear matrix inequalities based H1 control of gantry crane using tensor product transformation. International Conference on Process Control, pp. 92- 99.