Two Degree of Freedom PID Controller, Its Equivalent Forms and Special Cases

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Article Info	ABSTRACT
	The design of control systems is a multi-objective problem so, a two-degree-of-freedom (abbreviated as 2DOF) control system naturally has advantages over a one degree- of-freedom (abbreviated as 1DOF) control system. The main objective of 2DOF control is to control both set point tracking and disturbance rejections.Various 2DOF PID controllers and its equivalent transformations were proposed for industrial use by different researchers. Most of the above researches were published in Japanese language and have not been translated into English language yet. An objective here is to provide detail analysis regarding structure of 2DOF controller, its equivalent forms and its special cases. A system transfer function having transport delay and load disturbance is considered as a test bench to verify various 2DOF control strategies. MATLAB is used as software tool to verify the various 2DOF control strategies.The analysis will be helpful to the engineers and researchers to understand the topic in detail for further exploration.

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1. INTRODUCTION

The process of optimizing simultaneously a collection of objective functions is called multiobjective optimization. Design of control systems is a multi-objective problem because; it involves the optimization of more than one objective functions like set point response, load disturbances and robustness to model uncertainty. In control system, the degree of freedom is defined as the number of closed-loop transfer functions that can be adjusted independently [1], [13]. 1DOF PID offers a feasible outcome either for reference tracking operation or disturbance rejection operation [12]. The problem with the conventional control system which has 1DOF control structure is that when the disturbance response is optimized, then the set-point response is found to be poor, and vice versa. For this reason some of the classical researches on the optimal tuning of PID controllers have shown two tables: one for the "disturbance optimal" parameters, and the other for the "set point optimal" parameters [2], [3]. Two objectives i.e. set point response and disturbance response conflict, and hence trade-off exist which results in Pareto set.

If we try to control simultaneously two control system objectives i.e. set point and load disturbance then it results in structure of two-degree-of-freedom control system. The various control techniques using 2DOF controller have been devised by various researchers like, A feedback linearization-based two-degree-of-freedom constrained controller [14], Control of Uncertain Input-delay Systems by using Input/output Linearization with A Two-degree-of-freedom Scheme[15], Data-driven design of two-degree-of-freedom controllers using reinforcement learning techniques[16], two-degree-of-freedom internal model control(2DOF-IMC) [17], a model reference design procedure to the robust tuning of two-degree-of-freedom

proportional integral derivative controllers with filter [18], Two degree of freedom based robust iterative Learning control for uncertain LTI systems[19]. An objective here is to provide detail analysis regarding structure of 2DOF controller, its equivalent forms and its special cases so that, it will be helpful to the engineers and researchers to understand the topic in detail for further exploration.

The paper is organized as follows; first analysis of conventional 1DOF feedback control system with its constraints was carried out. Next section describes detailed analysis of 2DOF controller, derivations of steady state error for both reference and disturbance input for step function. This section also derives constrains on design of 2DOF controller, plant and detector. The section III, IV & V contains analysis regarding variants of 2DOF controller, its equivalent forms and special cases of 2DOF controller respectively. Section VI & VII contains simulation results and conclusion.

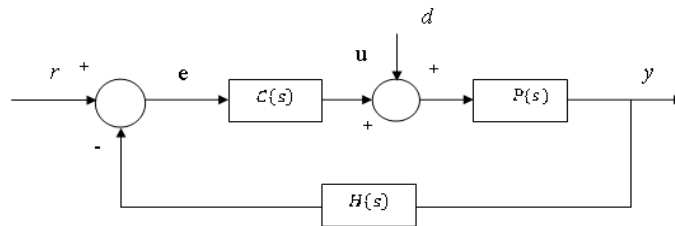## 2. CONVENTIONAL 1DOF FEEDBACK CONTROL SYSTEM

Figure 1. Conventional 1DOF Control System

Consider the conventional control system of Figure 1, having 1DOF structure. Where 'r' is set point, 'e' is error between set point & process variable, 'u' is controller output, 'd' is disturbance input, 'y' is process variable, $C(s)$ is controller , $P(s)$ is process or plant and $H(s)$ feedback gain.
In order to simplify the problem, we introduce the next two assumptions that are appropriate for many practical design problems with some exceptions.

Assumption 1: The detector has sufficient accuracy and speed for the given control purpose, i.e. $H(s) = 1$, and no detector noise present.

Assumption 2: The main disturbance enters at the manipulating point,

i.e. $P_d(s) = P(s)$. (1)

The responses of the controlled variable 'y' to the unit change of the set-point variable 'r' and to the unit step disturbance 'd' are called "set-point response" and "disturbance response," respectively. They have been traditionally used as measures of the performance in tuning the PID controllers. The closed-loop transfer function of this control system from the set-point variable 'r' to the controlled variable 'y' and that from the disturbance 'd' to 'y' are G_{yr1} and G_{yd1} respectively. Here, the subscript "1" means that the quantities are of the 1DOF control system.

Consider following two cases for structure of 1DOF controller to derive transfer function G_{yr1} and G_{yd1} respectively.

Case 1: Transfer function G_{yr1} , assuming $d = 0$.

Where, $G_{yr1} = \dfrac{y}{r}$ (2)

From Figure.1,

$$y = \left(r - y \cdot H(s)\right) \cdot C(s)P(s) \tag{3}$$

$$y \cdot [1 + C(s)P(s)H(s)] = r \cdot C(s)P(s) \tag{4}$$

By, rearranging above equations (3) & (4) to derive transfer function as in (5)

$$\frac{y}{r} = \frac{P(s)C(s)}{1+P(s)C(s)H(s)} \qquad (5)$$

Case 2: Transfer function G_{yd1} , assuming $r = 0$.

Where, $G_{yd1} = \frac{y}{d}$ $\qquad (6)$

From Figure 1,

$$y = (d - y \cdot H(s)C(s)) \cdot P(s) \qquad (7)$$

$$y \cdot [1 + P(s)C(s)H(s)] = d \cdot P(s) \qquad (8)$$

By, rearranging above equation (8) to derive transfer function as in (9)

$$G_{yd1} = \frac{P_d(s)}{1+P(s)C(s)H(s)} \qquad (9)$$

Now, multiplying G_{yr1} by *P(s)* & adding with G_{yd1} , assuming $P_d(s) = P(s)$.

$$G_{yr1}P(s) + G_{yd1} = \frac{P(s)P(s)C(s)}{1+P(s)C(s)H(s)} + \frac{P_d(s)}{1+P(s)C(s)H(s)} \qquad (10)$$

$$= P(s) \left[\frac{P(s)C(s)}{1+P(s)C(s)H(s)} + \frac{1}{1+P(s)C(s)H(s)} \right] \qquad (11)$$

$$= P(s) \qquad (12)$$

These two transfer functions include only one tunable element, i.e., *C(s)*, so they cannot be changed independently. To be concrete, the two functions are bound by

$$G_{yr1} \cdot P(s) + G_{yd1} = P(s) \qquad (13)$$

This equation shows explicitly that for a given *P(s)*, G_{yr1} (*s*) is uniquely determined if G_{yd1} (*s*) is chosen, and vice versa. This fact causes the following difficulty. If the disturbance response is optimized, the set-point response is often found to be poor, and vice versa. For this reason, some of the classical researches [2], [3] on the optimal tuning of PID controllers gave two tables: one for the "disturbance optimal" parameters, and the other for the "set point optimal" parameters.
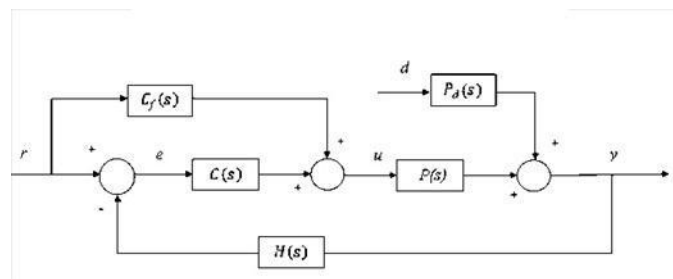
3.   TWO DEGREE OF FREEDOM CONTROLLER

Figure 2. Conventional 2DOF Control System.

A general form of the 2DOF control system is shown in Figure 2, where the controller consists of two compensators $C(s)$ and $C_f(s)$, the transfer function $P_d(s)$ from the disturbance '*d*' to the controlled variable '*y*' is assumed to be different from the transfer function $P(s)$ from the manipulated variable '*u*' to

'y'. $C(s)$ is called the serial (or main) compensator and $C_f(s)$, the feed forward compensator. The closed-loop transfer functions from 'r' to 'y' and 'd' to 'y' are, respectively, given by G_{yr2} and G_{yd2} derived below [4]. Here, the subscript "2" means that the quantities are of the 2DOF control system. Consider following two cases for structure of 2DOF controller to derive transfer function and steady state error for G_{yr2} and G_{yd2} respectively.

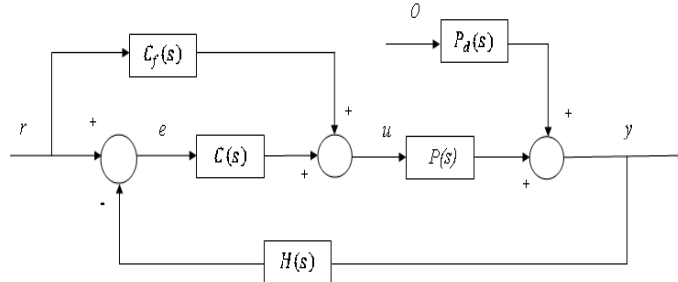*Case 1:* Transfer function G_{yr2}, assuming d_m & $d = 0$.

Figure 3. Conventional 2DOF Control System with reference input only

$$e = r - y \cdot H(s) \tag{14}$$

$$U = e \cdot C(s) + r \cdot C_f(s) \tag{15}$$

$$y = U \cdot P(s) \tag{16}$$

Now, Substituting values of equations (14) & (15) in (16) and then manipulating equations as under.

$$y = \{[(r - y(s)H(s)) \cdot C(s)] + r \cdot C_f(s)\} \cdot P(s) \tag{17}$$

$$y = \{[(r \cdot C(s) - y(s)H(s)C(s)] + r \cdot C_f(s)\} \cdot P(s) \tag{18}$$

$$y = \left\{ \left[\left(C(s) + C_f(s) \right) \cdot r \right] - y \cdot H(s)C(s) \right\} \cdot P(s) \tag{19}$$

$$y + y \cdot H(s)C(s)P(s) = r \cdot P(s)[C(s) + C_f(s)] \tag{20}$$

By, rearranging above equation (20) to derive 2DOF control set point response transfer function as in (21)

$$G_{yr2} \quad = \frac{y}{r} = \frac{P(s) \cdot [C(s) + C_f(s)]}{1 + P(s)C(s)H(s)} \tag{21}$$

Let's derive Steady state error for unit step input assuming zero disturbances.

$$e(s) = r(s) - y(s) \tag{22}$$

Substituting value of $y(s)$ in equation (22) in terms of $r(s)$

$$e(s) = r(s) - \frac{P(s) \cdot [C(s) + C_f(s)]}{1 + P(s)C(s)H(s)} \cdot r(s) \tag{23}$$

$$e(s) = r(s) \left[1 - \frac{P(s)[C(s) + C_f(s)]}{1 + P(s)C(s)H(s)} \right] \tag{24}$$

For, step input $r(s) = 1/s$ substituting value of r(s) in equation (24)

$$e(s) = \left(\frac{1}{s} \right) \left[1 - \frac{P(s)[C(s) + C_f(s)]}{1 + P(s)C(s)H(s)} \right] \tag{25}$$

Deriving steady state error

$$ess = \lim_{s \to 0} s \cdot e(s) \tag{26}$$

Assume that, $\lim_{s \to 0} H(s) = 1$ \hfill (27)

$$ess = \lim_{s \to 0} s \cdot \left(\frac{1}{s}\right) \left[1 - \frac{P(s)[C(s) + C_f(s)]}{1 + P(s)C(s)H(s)}\right] \tag{28}$$

By, mathematically manipulating right hand side of equation (28) and canceling common terms, we get steady state error as in the form of (29)

$$ess = \lim_{s \to 0} s \cdot \left(\frac{1}{s}\right) \left[\frac{1 + P(s)C_f(s)}{1 + P(s)C(s)}\right] \tag{29}$$

Taking, $C_f(s)$ and $C(s)$ from numerator and denominator part common and cancelling common terms, we obtain equation (29) in the form of (30)

$$ess = \lim_{s \to 0} s \cdot \left(\frac{C_f(s)}{C(s)}\right) \left[\frac{1/C_f(s) + P(s)}{1/C(s) + P(s)}\right] \tag{30}$$

$$\lim_{s \to 0} C(s) = \infty, \lim_{s \to 0} P(s) \neq 0, \lim_{s \to 0} \frac{C_f(s)}{C(s)} = 0 \tag{31}$$

Above conditions imposes a constraints on the design of controller and process.

The cases that satisfy above conditions are that, $C(s)$ includes an integrator and C_f (s) does not include an integrator term and detector is accurate in the steady state. If the detector is not accurate i.e. $\lim_{s \to 0} H(s) \neq 1$, then the steady-state error is given by (35).

$$ess = \lim_{s \to 0} s \left(\frac{1}{s}\right) \left[1 - \frac{P(s)[C(s) + C_f(s)]}{1 + P(s)C(s)H(s)}\right] \tag{32}$$

$$ess = \lim_{s \to 0} s \left(\frac{1}{s}\right) \left[\frac{1 + P(s)C(s)H(s) - P(s)C(s) - P(s)C_f(s)}{1 + P(s)C(s)H(s)}\right] \tag{33}$$

$$ess = \lim_{s \to 0} \left[\frac{1/C(s) + P(s)H(s) - P(s) - P(s)C_f(s)/C(s)}{1/C(s) + P(s)H(s)}\right] \tag{34}$$

$$ess = \lim_{s \to 0} \left[\frac{H(0) - 1}{H(0)}\right] \tag{35}$$

The above equation gives value of steady state error for unit step input when $H(s) \neq 1$.

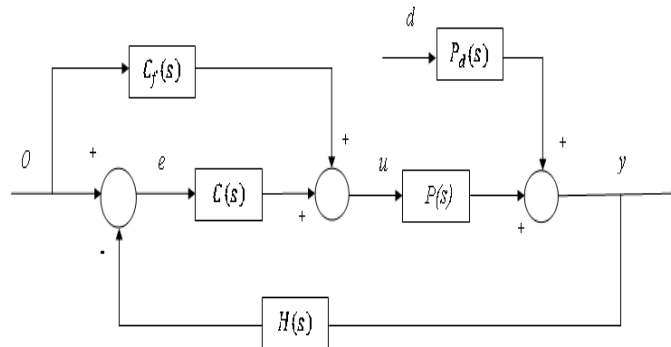**Case 2:** Transfer function G_{yd2}, assuming d_m & $r = 0$.

Figure 4. Conventional 2DOF Control System with disturbance input only

$$y = d \cdot [P_d(s)] - y \cdot [c(s)H(s)P(s)] \tag{36}$$

$$y[1 + C(s)P(s)H(s)] = d \cdot [P_d(s)] \tag{37}$$

By, rearranging above equation (37) to derive 2DOF control disturbance response transfer function as in (38)

$$G_{yd2} \quad = \frac{y}{d} = \frac{P_d(s)}{1+P(s)C(s)H(s)} \tag{38}$$

Let's derive Steady state error for unit step disturbance input assuming zero reference input.

$$e(s) = d(s) - y(s) \tag{39}$$

$$e(s) = d(s) - \left[\frac{P_d(s)}{1+P(s)C(s)H(s)}\right] d(s) \tag{40}$$

$$e(s) = d(s) \left[1 - \frac{P_d(s)}{1+P(s)C(s)H(s)}\right] \tag{41}$$

For, step disturbance input $d(s) = 1/s$ substituting value of $d(s)$ in above equation (41)

$$e(s) = \frac{1}{s}\left[1 - \frac{P_d(s)}{1+P(s)C(s)H(s)}\right] \tag{42}$$

$$\epsilon_{dstep} = \text{ess} = \lim_{s\to0} s[e(s)] \tag{43}$$

$$\lim_{s\to0} H(s) = 1 \tag{44}$$

$$\epsilon_{dstep} = \text{ess} = \lim_{s\to0} s \cdot \left(\frac{1}{s}\right)\left[1 - \frac{P_d(s)}{1+P(s)C(s)H(s)}\right] \tag{45}$$

$$\lim_{s\to0} \frac{P_d(s)}{P(s)} < \infty \tag{46}$$

The above equations put conditions on the plant, where the denominator of equation (46) requires that $P(s)$ is not of differentiating and the numerator equation (46) requires that the disturbance is not integrated more times than the manipulated variable in ordered to have $P_d(s) < \infty$. From the mathematical standpoint, conditions are nothing but sufficient conditions that make the steady-state errors zero robustly. But from the industrial viewpoint they can be regarded as necessary. Considering above conditions, $C(s)$ and $C_f(s)$ are derived as under.

$$C(s) = [K_p + \frac{K_p}{T_i s} + K_p T_D D(s)] \tag{47}$$

$$C_f(s) = -K_p [\alpha + \beta T_D D(s)] \tag{48}$$

Where $D(s)$ is the approximate derivative given by (49)

$$D(s) = \frac{s}{1+\tau \cdot s} \tag{49}$$

Three parameters of $C(s)$ i.e., the proportional gain K_p, the integral time T_i, and the derivative time T_D, are referred to as "basic parameters," and two parameters of $C_f(s)$ i.e., α and β are referred to as "2DOF parameters".

The approximate derivative equation (49) is set as $\tau = \frac{T_D}{\sigma}$ Where, σ is called the derivative gain. It has been a traditional practice to use a fixed value. We follow this tradition, it has been done traditionally because of engineering convenience and partly because our numerical experiments indicated that the change of σ does not influence the optimal values of the other five parameters drastically, where some care must be

taken for certain types of plants. In order to simplify the problem, we introduce the next two assumptions that are appropriate for many practical design problems with some exceptions.

Assumption 1: The detector has sufficient accuracy and speed for the given control purpose, i.e., $H(s) = 1,\ dm = 0$.

Assumption 2: The main disturbance enters at the manipulating point, i.e., $Pd\ (s) = P(s)$ and plant or process has to be non differential type.

4. EQUIVALENT FORMS OF 2DOF CONTROLLER

The 2DOF controller is shown in above Figure 2. , derived under assumptions as above. Transforming this controller part, Figure. 2 can be changed equivalently in Figure. 5 to Figure. 9. The controllers in these figures are nothing but different expressions of the same 2DOF PID controller shown in above Figure 2. They are referred as follows:

4.1. Feed Forward Type (FF type)

The conventional Feed forward type of 2DOF controller is shown in following Figure.5. It is called as feed forward type (FF type), because it is obtained by adding a feed forward path from 'y' to 'u' in the conventional PID as shown below. Now, deriving the output of 2DOF controller 'u' from below Figure.5 so, output 'u' is addition of two controller outputs i.e. feed forward and normal PID controller.
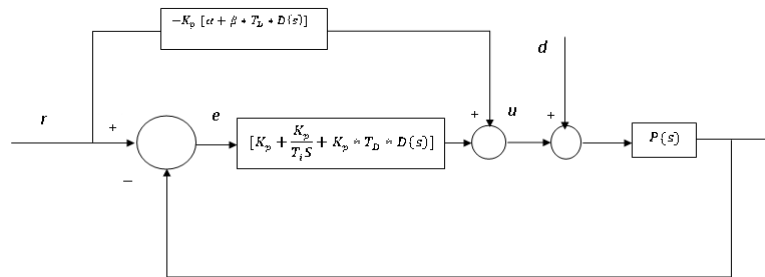

Figure 5. Feed forward type 2DOF Controller.

Where, normal PID controller is multiplied by error 'e' and feed forward controller is multiplied by reference input 'r' respectively shown in equation (50). After mathematical manipulation and cancelling common terms in equation (50) & (51), we get resultant controller output 'u' shown in equation (52).

$$U = (r - y)[K_p + \frac{K_p}{T_i \cdot S} + K_p T_D D(s)] - [K_p\,\alpha + K_p\,\beta T_D D(s)]r \tag{50}$$

$$U = r \cdot \left[K_p + \frac{K_p}{T_i \cdot S} + K_p T_D D(s)\right] - r \cdot \left[K_p\,\alpha\ +\ K_p\,\beta\ T_D D(s)\right] - y \cdot \left[K_p + \frac{K_p}{T_i \cdot S} + K_p T_D D(s)\right] \tag{51}$$

$$U = r\left[K_p(1 - \alpha) + \frac{K_p}{T_i \cdot S} + K_p T_D D(s)(1 - \beta)\right] - y \cdot \left[K_p + \frac{K_p}{T_i \cdot S} + K_p T_D D(s)\right] \tag{52}$$

4.2. Feedback Type (FB Type) of 2DOF Controller

It is called as feedback type (FB type), because it is obtained by adding a feedback path from 'y' directly to 'u' to the conventional PID, where $K_p \{\alpha\ +\ \beta\ T_D D(s)\}$ called as "feedback compensator."
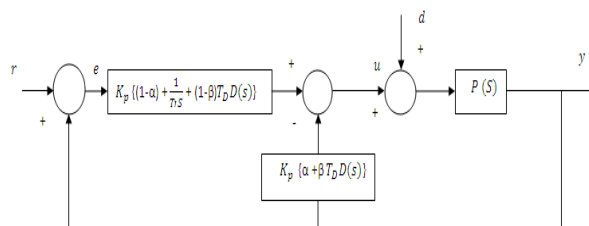

Figure 6. Feedback type 2DOF Controller

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Deriving the output of 2DOF controller 'u' from above Figure.6 so, output 'u' is addition of two controller outputs i.e. forward and feedback controller. Where, forward controller is multiplied by error 'e' and feedback controller is multiplied by process output 'y' respectively shown in equation (53). After mathematical manipulation and cancelling common terms in equation (53), (54) & (55), we get resultant controller output 'u' as shown in (56).

$$U = (r - y)[K_p(1 - \alpha) + \frac{K_p}{T_i \cdot S} + K_p T_D D(s)(1 - \beta)] - y[K_p\{\alpha + \beta T_D D(s)\}] \tag{53}$$

$$U = r \cdot [K_p(1 - \alpha) + \frac{K_p}{T_i \cdot S} + K_p(1 - \beta) T_D D(s)] - y \cdot [K_p(1 - \alpha) + \frac{K_p}{T_i \cdot S} +$$

$$(1 - \beta)K_p T_D D(s) - y \cdot [K_p \alpha + K_p \beta T_D D(s)] \tag{54}$$

$$U = r \cdot [K_p(1 - \alpha) + \frac{K_p}{T_i \cdot S} + K_p(1 - \beta)T_D D(s)] - y \cdot [K_p - \alpha K_p + \frac{K_p}{T_i \cdot S} + K_p T_D D(s) - \beta K_p T_D D(s)$$

$$+ \alpha K_p + K_p \beta T_D D(s) \tag{55}$$

$$U = r[K_p(1 - \alpha) + \frac{K_p}{T_i \cdot S} + K_p T_D D(s)(1 - \beta)] - y[K_p + \frac{K_p}{T_i \cdot S} + K_p T_D D(s)] \tag{56}$$

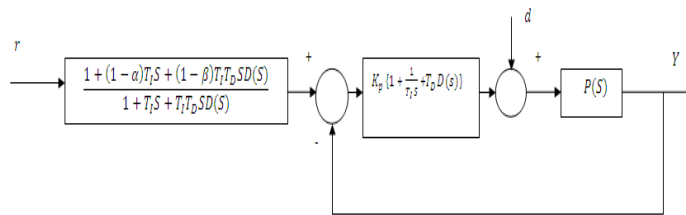### 4.3. Set Point Filter Type of 2DOF Controller

Figure 7. Set-point filter type 2DOF Controller

It is called a set-point filter type (Filter type), because it is obtained by inserting a filter in the set point path of the conventional PID controller. Deriving the output of 2DOF controller 'u' from above Figure.7 so, output of 2DOF controller is multiplied by error 'e' where, error 'e' is difference between set point filter output and process output shown in equation (57). After mathematical manipulation and cancelling common terms in equation (58), (59) & (60), we get resultant controller output 'u' as shown in (61).

$$U = r\left[\left[\frac{1 + (1 - \alpha)T_I S + (1 - \beta)T_I T_D SD(S)}{1 + T_I S + T_I T_D SD(S)}\right] - y\right]\left[K_p + \frac{K_p}{T_i S} + K_p T_D D(s)\right] \tag{57}$$

$$U = [r\left[\frac{1 + (1 - \alpha)T_I S + (1 - \beta)T_I T_D SD(S)}{1 + T_I S + T_I T_D SD(S)}\right] - y]K_p[\frac{1 + T_I S + T_I T_D SD(S)}{T_I S}] \tag{58}$$

$$U = r \cdot \frac{K_p}{T_i S}[1 + (1 - \alpha)T_I S + (1 - \beta)T_I T_D SD(S)] - y[K_p + \frac{K_p}{T_i S} + K_p T_D D(s)] \tag{59}$$

$$U = r \cdot \left[\frac{K_p}{T_i S} + (1 - \alpha)T_I S \frac{K_p}{T_i S} + (1 - \beta)\frac{K_p}{T_i S}T_I T_D SD(S)\right] - y \cdot [K_p + \frac{K_p}{T_i S} + K_p T_D D(s)] \tag{60}$$

$$U = r \cdot [K_p(1 - \alpha) + \frac{K_p}{T_i S} + K_p T_D D(s)(1 - \beta)] - y \cdot [K_p + \frac{K_p}{T_i S} + K_p T_D D(s)] \tag{61}$$

4.4. Filter With Preceded-Derivative Type Expression Of 2DOF Controller

It is filter and preceded-derivative type, because it is obtained by inserting a filter in the set-point

path of the preceded- derivative type controller. Deriving the output of 2DOF controller 'u' from Figure.8 so, output of 2DOF controller is obtained as shown in equation (62). After mathematical manipulation and cancelling common terms in equation (63), (64), (65), (66) & (67), we get resultant controller output 'u' as shown in (68).
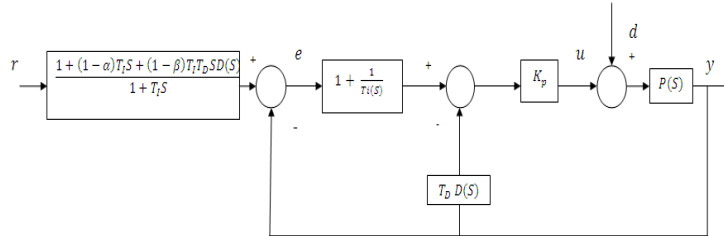

Figure 8. Filter with preceded-Derivative Type 2DOF controller

$$F(S) = \frac{1+(1-\alpha)T_I S+(1-\beta)T_I T_D SD(S)}{1+T_I S} \tag{62}$$

$$U = \{ [r.F(S) - y] [1 + \frac{1}{T_i \cdot S}] - y.T_D D(S)\} K_p \tag{63}$$

$$U = [r.F(S) [1 + \frac{1}{T_i \cdot S}] - y [\frac{1+T_i \cdot S}{T_i \cdot S}] - y.T_D D(S)] K_p \tag{64}$$

$$U = \{ r.F(S) [\frac{1+T_i \cdot S}{T_i \cdot S}] - y[1 + \frac{1}{T_i \cdot S} + T_D D(S)] \} K_p \tag{65}$$

$$U = \left\{ r. \left[\frac{1+(1-\alpha)T_I \cdot S+(1-\beta)T_I T_D SD(S)}{1+T_I \cdot S}\right] \left[\frac{1+T_i \cdot S}{T_i \cdot S}\right] - y \left[1 + \frac{1}{T_i S} + T_D D(S)\right]\right\} K_p \tag{66}$$

$$U = r. \left\{ K_p \left[\frac{(1-\alpha)T_I \cdot S}{T_I \cdot S} + \frac{1}{T_i \cdot S} + \frac{(1-\beta)T_I T_D SD(S)}{T_{I} \cdot S}\right] - y \left[K_p + \frac{K_p}{T_i \cdot S} + K_p T_D D(S)\right]\right\} \tag{67}$$

$$U = r \left[K_p (1 - \alpha) + \frac{K_p}{T_i \cdot S} + K_p T_D D(s) (1 - \beta)\right] - y \left[K_p + \frac{K_p}{T_i \cdot S} + K_p T_D D(s)\right] \tag{68}$$

4.5 Component Separated Type Expression of 2DOF Controller

It is component-separated type, because the three functional components (i.e., proportional, integral and derivative components) are separately built in and connected as shown in following Figure 9. Deriving the output of 2DOF controller 'u' from Figure 9 so, output 2DOF controller is obtained as shown in (69). After mathematical manipulation and cancelling common terms in equations (70), (71), (72), (73), (74), (75), (76) & (77), we get resultant controller output 'u' as shown in (78).
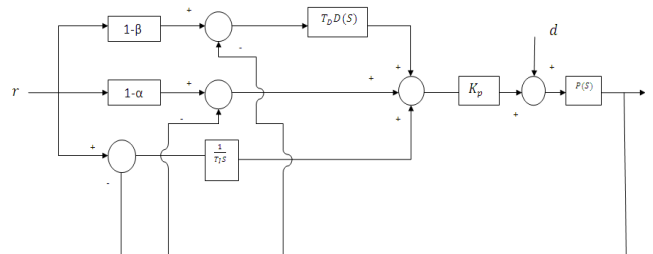

Figure 9. Component Separated Type Expression of 2DOF Controller

$$U = K_p [A + B + C] \tag{69}$$

$$A = [r (1 - \beta) - y] T_D D(S) \tag{70}$$

$$A = r\,(1-\beta)\,T_D\,D(S) - y\,T_D\,D(S) \tag{71}$$

$$B = [\,r\,(1-\alpha) - y\,] \tag{72}$$

$$C = (r-y)\left[\frac{1}{T_i \cdot S}\right] \tag{73}$$

$$A + B + C = r(1-\beta)T_D\,D(S) - yT_D\,D(S) + r(1-\alpha) - y + (r-y)\left[\frac{1}{T_i \cdot S}\right] \tag{74}$$

$$A + B + C = r\,(1-\beta)\,T_D\,D(S) - y\,T_D\,D(S) + r\,(1-\alpha) - y + \frac{r}{T_i \cdot S} - \frac{y}{T_i \cdot S} \tag{75}$$

$$A + B + C = r\left[(1-\beta)\,T_D\,D(S) + (1-\alpha) + \frac{1}{T_i \cdot S}\right] - y\left[\frac{1}{T_i \cdot S} + 1 + T_D D(S)\right] \tag{76}$$

$$U = r\left[K_p(1-\alpha) + \frac{K_p}{T_i S} + K_p(1-\beta)\,T_D\,D(S)\right] - y\left[K_p + \frac{K_p}{T_i S} + K_p\,T_D\,D(S)\right] \tag{77}$$

$$U = r\left[K_p\,(1-\alpha) + \frac{K_p}{T_i \cdot S} + K_p T_D D(s)\,(1-\beta)\right] - y\left[K_p + \frac{K_p}{T_i \cdot S} + K_p T_D D(s)\right] \tag{78}$$

The above equivalent transformations of 2DOF controller gives basic understanding regarding the effects of the 2DOF structure from various viewpoints like it is useful for developing an efficient algorithm in digital implementation [5][8][6][9] introducing nonlinear operations on the manipulated variable such as magnitude limitation, rate limitation, directional gain adjustment, [5][8][10] realizing bumpless switching, implementing an antireset-windup mechanism, managing the feed forward signals coming from other systems, utilizing predictable disturbances, etc. [5][6][7][8][9], and converting the conventional PID controller already built in to the 2DOF PID [5][6][9][11].

5.   SPECIAL CASES OF 2DOF CONTROLLER

Variants and its equivalent forms of 2DOF controllers are discussed above and it has been observed that output of controller remains same irrespective of type of 2DOF controller which is as below equation (79). If we select values of α and β either zero or one then four combinations are possible, substituting these combinations in (79) which results in special cases of 2DOF controller.

$$U = r\left[K_p\,(1-\alpha) + \frac{K_p}{T_i \cdot S} + K_p T_D D(s)\,(1-\beta)\right] - y\left[K_p + \frac{K_p}{T_i \cdot S} + K_p T_D D(s)\right] \tag{79}$$

Case 1: α =0 and β = 0, PID Controller.
If we substitute values of α =0 and β = 0 in (79) it reduced in the form of simple PID controller as shown below in (80) & (81).

$$U = r\left[K_p\,(1-0) + \frac{K_p}{T_i S} + K_p T_D D(s)\,(1-0)\right] - y\left[K_p + \frac{K_p}{T_i S} + K_p T_D D(s)\right] \tag{80}$$

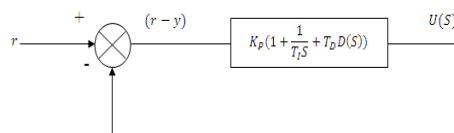$$U = (r-y)\left[K_p + \frac{K_p}{T_i \cdot S} + K_p T_D D(s)\right] \tag{81}$$

Figure 10. Special case of 2DOF Controller α =0 and β = 0, PID Controller.

Case 2: α =0 and β = 1, PI-D controller.
If we substitute values of α =0 and β = 0 in equation (79) it reduced as shown below in (82) & (83).

$$U = r\left[K_p\,(1-0) + \frac{K_p}{T_i\cdot S} + K_p T_D D(s)\,(1-1)\right] - y\left[K_p + \frac{K_p}{T_i\cdot S} + K_p T_D D(s)\right] \tag{82}$$

$$U = (r-y)\left[K_p + \frac{K_p}{T_i\cdot S}\right] - y\left[K_p T_D D(s)\right] \tag{83}$$

The equation (83) results in the form of PI-D controller as shown in following Figure.11.

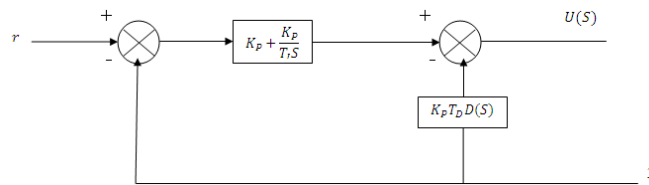Figure 11. Special case of 2DOF Controller α =0 and β = 1, PI-D controller.

Case 3: α =1 and β = 0, ID-P Controller.
If we substitute values of α =1 and β = 0 in equation (79) it reduced as shown below in (84) & (85).

$$U = r\left[K_p\,(1-1) + \frac{K_p}{T_i S} + K_p T_D D(s)\,(1-0)\right] - y\left[K_p + \frac{K_p}{T_i S} + K_p T_D D(s)\right] \tag{84}$$

$$U = (r-y)\left[\frac{K_p}{T_i\cdot S} + K_p T_D D(s)\right] - y\cdot\left[K_p\right] \tag{85}$$

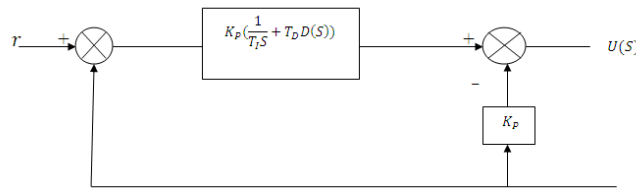The equation (85) results in the form of ID-P controller as shown in following Figure.12.

Figure 12. Special case of 2DOF Controller α =1 and β = 0, ID-P Controller

Case 4: α =1 and β = 1, I-PD Controller.
If we substitute values of α =1 and β = 0 in equation (79) it reduced as shown below in (86), (87), & (88).

$$U = r\left[K_p\,(1-1) + \frac{K_p}{T_i S} + K_p T_D D(s)(1-1)\right] - y\cdot\left[K_p + \frac{K_p}{T_i\cdot S} + K_p T_D D(s)\right] \tag{86}$$

$$U = r\left[\frac{K_p}{T_i\cdot S}\right] - y\left[K_p + \frac{K_p}{T_i\cdot S} + K_p T_D D(s)\right] \tag{87}$$

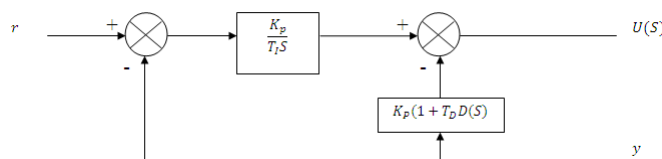$$U = (r-y)\left[\frac{K_p}{T_i\cdot S}\right] - y\left[K_p + K_p T_D D(s)\right] \tag{88}$$

Figure 13. Special case of 2DOF Controller α =1 and β = 1, I-PD Controller

The equation (86) results in the form of I-PD controller as shown in following Figure.13. The special cases of above 2DOF Controllers have been used in practice since long, they have already established their name like PID, PI-D, ID-P, and I-PD Controller. Interesting thing is even though they are special cases of 2DOF controller they are represented by their own identities.

6.   SIMULATION RESULTS

The transfer function of system is derived using following assumptions.
1.   DC gain of the term =1.
2.   Transport delay $e^{-\tau_d s}$ is, $e^{-0.2s}$.
3.   Time constant τ =1.

Hence Plant model can be derived as $\frac{e^{-0.2s}}{(S+1)}$.

A complete system model with load disturbance having 2DOF controller is implemented as shown in following Figure 14.
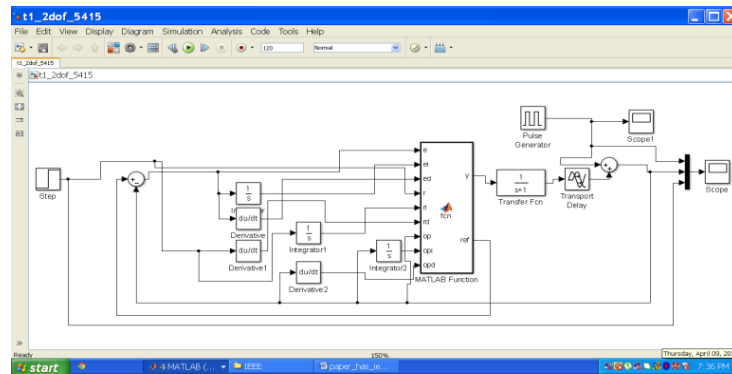

Figure 14. system with load disturbance and 2DOF controller

A step input having unity amplitude is applied as reference input; variants of 2DOF controller are implemented in the MATLAB function, and load disturbance is added in the form of pulse generator. All variants of 2DOF controller are used to control system model considering following 2DOF control parameters i.e. K_p = 3.6, T_i = 2.8; T_D = 0.012, α = 0.448, and β =0.5. System response of five variants of 2DOF controller is as shown below, where blue waveform is of set point or reference input, magenta waveform is of process value and yellow waveform is of load disturbance in all the graphs of Figure 15 to Figure 21.
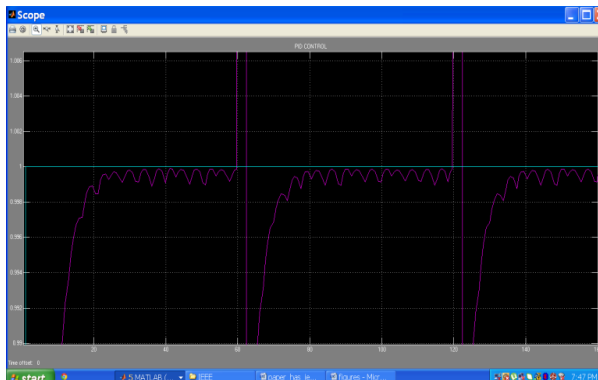

Figure 15. system responses with PID controller

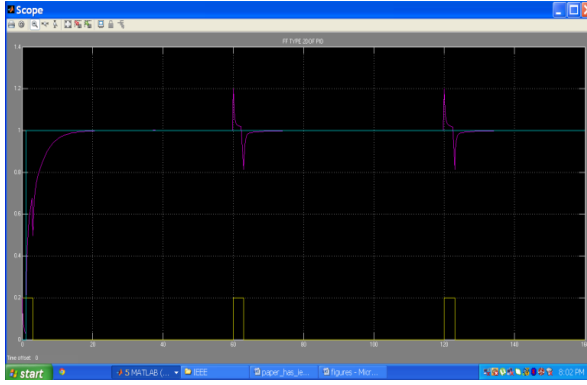Figure 16. system responses with feed forward type 2DOF controller with Zoom

Figure 17. system responses with feed forward type 2DOF controller
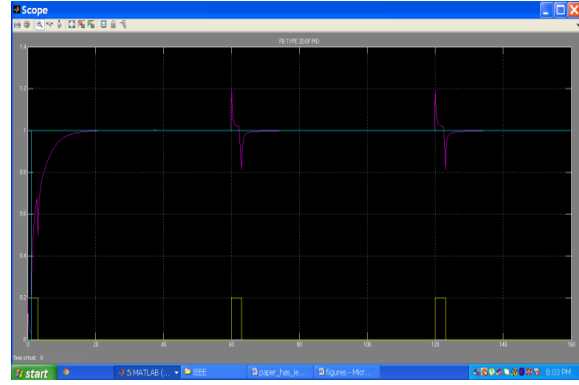

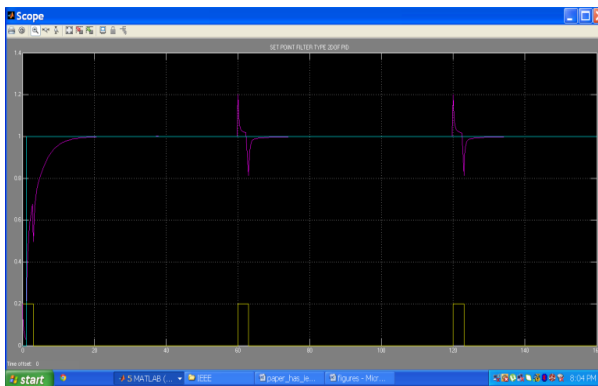Figure 18. system responses with feedback type 2DOF controller

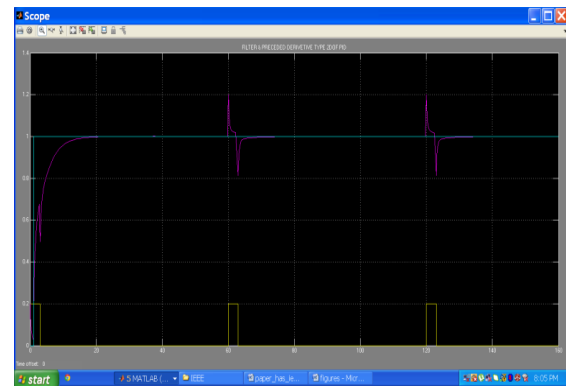Figure 19. system responses with Set point filter type 2DOF controller

Figure 20. system responses with Filter and Preceded derivative type 2DOF controller
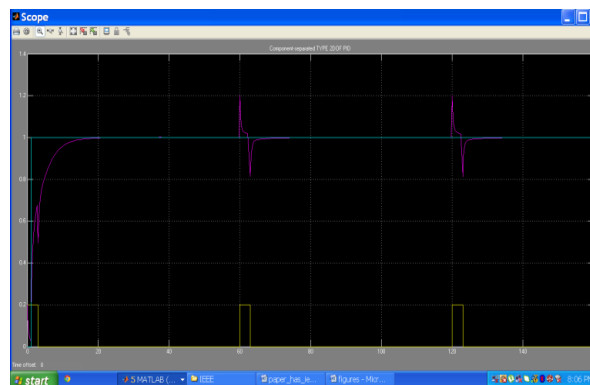

Figure 21. system responses with component separated type 2DOF controller

In all the above responses of variants of 2DOF controller it is observed that system step response is identical under same load disturbance condition.

7. CONCLUSION

The main aim of this paper is to provide detail mathematical analysis starting from conventional 1DOF PID control, 2DOF controller, its equivalent forms, and special cases of 2DOF Controller, to understand the topic in detail for further exploration. From the mathematical analysis it has been concluded that naturally 2DOF controller has advantages over the 1DOF controller. Analysis also derives constrains on

the design of 2DOF controller, plant and detector. Finally, we conclude that variants of 2DOF controller are nothing but different expressions of the same 2DOF PID controller. Fine tuning of five variables (K_p, T_i, T_D, α, and β) can be done using multiobjective optimization algorithm for better result.

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